1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Design Example

Analysis of Columns Subject to Biaxial Bending

Objective: To demonstrate the adequacy of a given column section subject to a given biaxial loading using the approximate contour method and the theoretical method.

Problem: A square cross section is as illustrated in Fig. 1 providing material strengths shown below:

Concrete

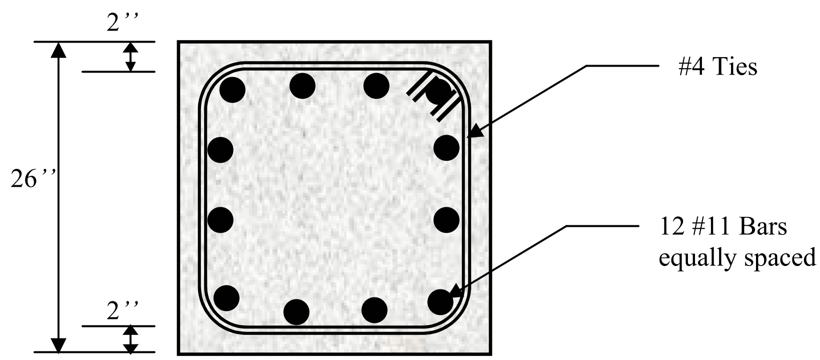
Uniaxial compressive strength: $f_c^{'} = 4000$ psi;

Maximum concrete strain: $\varepsilon_{cu} = 0.003$;

Steel

Yield stress: $f_y = 60$ ksi;

Yield strain: $\varepsilon_{sy} = 0.002$.



[Notice that the distance between concrete outer surface and the edge of longitudinal reinforcement is 2 in.]

Figure 1. Configuration of the reinforced concrete column section

Task: Consider both unaxial and biaxial loading situations described as follows and accomplish the tasks.

1. Uniaxial Column Interaction

For the cross section shown in Fig. 1, construct the axial load-moment interaction diagram. As a minimum calculation, establish the points corresponding to

(a) pure axial load ($P_o^{'}$),

(b) balanced loads ($P_b^{'}$, $M_b^{'}$), and

(c) pure moment ($M_o^{'}$).

You may assume straight lines between these points. Also, plot the axial load-ultimate curvature diagram.

2. Biaxial Column Interaction

For the same column cross section, assume that the neutral axis is oriented at 45 degree to the principal axes, and has a depth equal to 12 inches. At the extreme compression fiber, the compression strain is equal to 0.003. Accomplish the following tasks:

(a) Calculate the axial load and bending moment acting on the section,

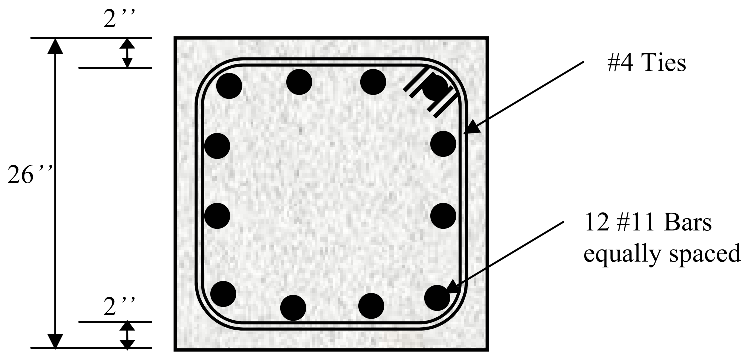
(b) Plot the load contour (P vs. M_x vs. M_y) using the Bresler load contour method with $\alpha = 1.5$, and the uniaxial interaction diagram from part 1.

(c) Show the loading points on the load contour and comment on the adequacy of the section for the loading considered.

[Design Procedures]

1. Uniaxial Column Interaction

Given the following information and configuration of the reinforced concrete column section:



Concrete

$f_c^{'} = 4000$ psi

$\varepsilon_{cu} = 0.003$

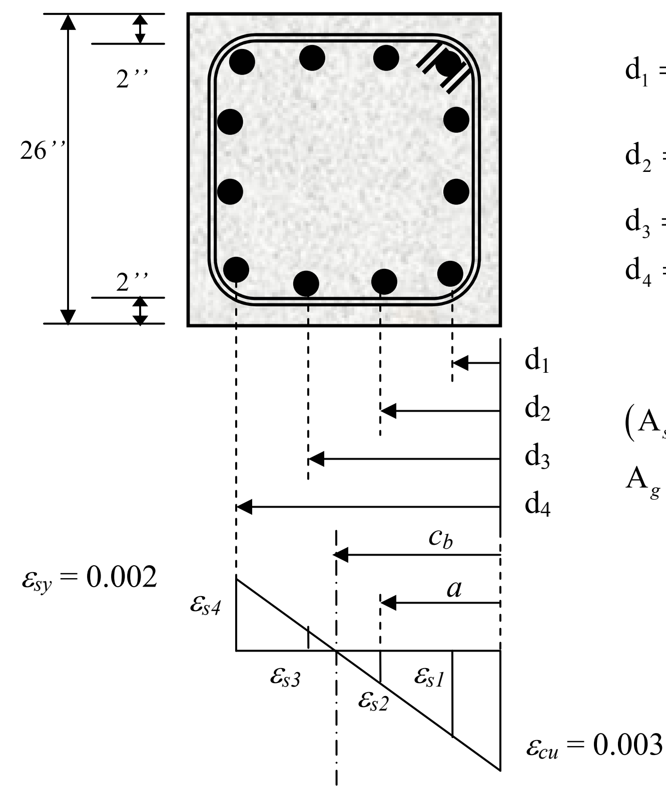
Steel

$f_y = 60$ ksi

$\varepsilon_{sy} = 0.002$

To examine the capacity of the reinforced concrete column section, we need to find out the stresses of concrete and reinforcements based on strain distribution.

Diameter of #11 is 1.41 in and area of #11 is 1.56 in^2. Consider the following geometry,



$$d_1 = 2 + \frac{1.41}{2} = 2.705 \text{ in}$$

$$d_2 = d_1 + \frac{26-4-1.41}{3} = d_1 + 6.863 = 9.568 \text{ in}$$

$$d_3 = d_2 + 6.863 = 16.432 \text{ in}$$

$$d_4 = d_3 + 6.863 = 23.295 \text{ in}$$

$$\left(A_s\right)_{total} = 12 \times 1.56 = 18.72 \text{ in}^2$$

$$A_g = 26 \times 26 = 676 \text{ in}^2$$

(a) Pure axial load, $P_o^{'}$:

$$P_o^{'} = 0.85 f_c^{'} \left(A_g - A_{st}\right) + f_y A_{st}$$

$$= 0.85 \cdot 4000 \cdot \left(676 - 18.72\right) + 60000 \cdot 18.72$$

= 3357.6 kips

Ultimate curvature $\phi_{P_0} = \dfrac{\varepsilon_u}{c} = \underline{\underline{0}}$ (for pure compression)

(b) Balanced loads ($P_b^{'}$, $M_b^{'}$):

From the geometry, it is found that

$$\frac{c_b}{0.003} = \frac{23.295}{0.005} \Rightarrow c_b = 13.98 \text{ in } \Rightarrow a = 0.85 \cdot c_b = 11.88 \text{ in}$$

Compute ε_{s1}, ε_{s2}, ε_{s3}, ε_{s4}:

$$\varepsilon_{s1} = \frac{0.003 \times (13.98 - 2.705)}{13.98} = 0.0024 > 0.002$$

$$\varepsilon_{s2} = \frac{0.003 \times (13.98 - 9.568)}{13.98} = 0.000947$$

$$\varepsilon_{s3} = \frac{0.003 \times (13.98 - 16.432)}{13.98} = 0.000526$$

$$\varepsilon_{s4} = \frac{0.003 \times (13.98 - 23.295)}{13.98} = -0.002$$

then

$$f_{s1} = 60 \text{ ksi (C)}$$

$$f_{s2} = \varepsilon_{s2} E_s = 0.000947 \times 29000 = 27.46 \text{ ksi (C)}$$

$$f_{s3} = \varepsilon_{s3} E_s = 0.000526 \times 29000 = -15.25 \text{ ksi (T)}$$

$$f_{s4} = \varepsilon_{s4} E_s = -0.002 \times 29000 = -60 \text{ ksi (T)}$$

$$\begin{aligned} P_b^{'} &= 0.85 f_c^{'} ab + A_s^{'} f_s^{'} - A_s f_s \\ &= 0.85(4)(11.88)(26) + (4 \times 1.56)(60 - 0.85 \times 4) \\ &\quad + (2 \times 1.56)(27.46 - 0.85 \times 4) \end{aligned}$$

= 1056.5 kips

$$\begin{aligned} M_b^{'} &= 0.85 f_c^{'} (ab - 6 \times 1.56)\left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n} A_{si} f_{si} \left(\frac{h_i}{2} - d_i\right) \\ &= 0.85(4)(11.88 \times 26 - 6 \times 1.56)\left(13 - \frac{11.88}{2}\right) + 60(4 \times 1.56)(13 - 2.705) \\ &\quad + 27.46(2 \times 1.56)(13 - 9.568) - 15.25(2 \times 1.56)(13 - 16.432) - 60(4 \times 1.56)(13 - 23.295) \end{aligned}$$

= 15355.9 kips-in

Ultimate curvature $\phi_b = \dfrac{0.003}{c_b} = \dfrac{0.003}{13.98} = \underline{\underline{0.0002146}}$ (for pure compression)

(c) Pure moment, $M_o^{'}$:

In case of pure moment, $\sum C = \sum T$. Find the balanced position, c_b, by trial and error. From the strain diagram shown below, there are more compressive stress than tensile. Thus, try $c < c_b$.

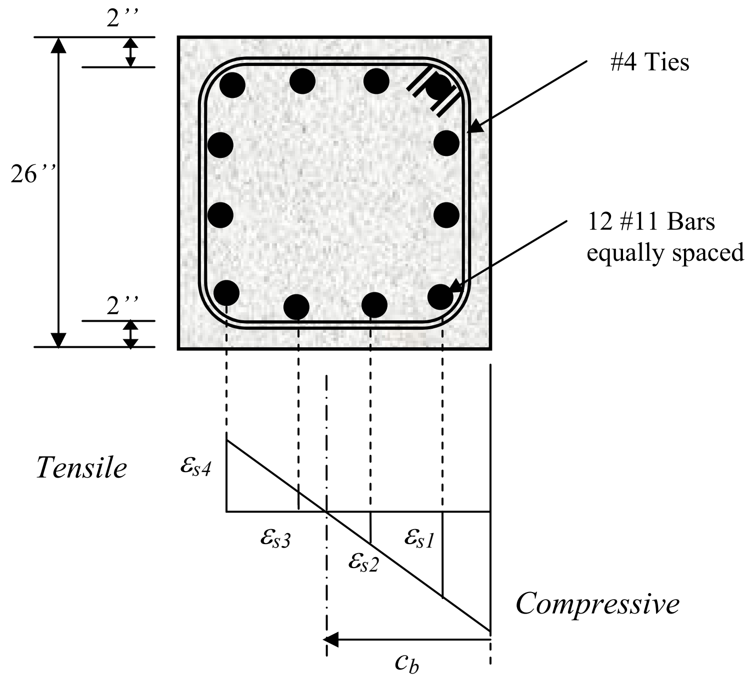


Fig. 1 Strain distribution of the reinforcements

<u>Trial 1</u>

Assume $c = 6$ in, then $a = 0.85\ c = 5.1$ in.

$$\varepsilon_{s1} = \frac{1}{6}(0.003)(6-2.705) = 1.648\times10^{-3} \qquad f_{s1} = \varepsilon_{s1}E_s = 47.79 \text{ ksi}$$

$$\varepsilon_{s2} = \frac{1}{6}(0.003)(6-9.568) = -1.784\times10^{-3}, \text{ then } f_{s2} = \varepsilon_{s2}E_s = -51.74 \text{ ksi}$$

$$\varepsilon_{s3} > \varepsilon_y \qquad f_{s3} = \varepsilon_{s3}E_s = 60 \text{ ksi}$$

$$\varepsilon_{s4} > \varepsilon_y \qquad f_{s4} = \varepsilon_{s4}E_s = 60 \text{ ksi}$$

$$P = 0.85(4)\left[(5.1)(26) - 4\times1.56\right] + (47.79)(4\times1.56)$$
$$-(51.74)(2\times1.56) - (60)(2\times1.56 + 4\times1.56)$$

= 4.74 kips

<u>Trial 2</u>

Assume $c = 5.95$ in, then $a = 0.85\ c = 5.06$ in.

$\varepsilon_{s1} = \frac{1}{5.95}(0.003)(5.95-2.705) = 1.636\times10^{-3}$ $f_{s1} = \varepsilon_{s1}E_s = 47.44$ ksi

$\varepsilon_{s2} = \frac{1}{5.95}(0.003)(5.95-9.568) = -1.824\times10^{-3} > \varepsilon_y$, then $f_{s2} = \varepsilon_{s2}E_s = -52.9$ ksi

$\varepsilon_{s3} > \varepsilon_y$ $f_{s3} = \varepsilon_{s3}E_s = 60$ ksi

$\varepsilon_{s4} > \varepsilon_y$ $f_{s4} = \varepsilon_{s4}E_s = 60$ ksi

$$P = 0.85(4)\left[(5.06)(26)-4\times1.56\right]+(47.44)(4\times1.56)$$
$$-(52.9)(2\times1.56)-(60)(2\times1.56+4\times1.56)$$

= -4.53 kips

Trial 3

Assume c = 5.975 in, then a = 0.85 c = 5.08 in.

$\varepsilon_{s1} = \frac{1}{5.975}(0.003)(5.975-2.705) = 1.642\times10^{-3}$ $f_{s1} = \varepsilon_{s1}E_s = 47.61$ ksi

$\varepsilon_{s2} = \frac{1}{5.975}(0.003)(5.975-9.568) = -1.804\times10^{-3} > \varepsilon_y$, then $f_{s2} = \varepsilon_{s2}E_s = -52.32$ ksi

$\varepsilon_{s3} > \varepsilon_y$ $f_{s3} = \varepsilon_{s3}E_s = 60$ ksi

$\varepsilon_{s4} > \varepsilon_y$ $f_{s4} = \varepsilon_{s4}E_s = 60$ ksi

$$P = 0.85(4)\left[(5.08)(26)-4\times1.56\right]+(47.61)(4\times1.56)$$
$$-(52.32)(2\times1.56)-(60)(2\times1.56+4\times1.56)$$

= 0.104 kips $\cong 0$

Take c = 5.975 in.

$$M_b^{'} = 0.85(4)(5.08\times26-4\times1.56)\left(13-\frac{5.08}{2}\right)+47.61(4\times1.56)(13-2.705)$$
$$-52.32(2\times1.56)(13-9.568)-60(2\times1.56)(13-16.432)$$
$$-60(4\times1.56)(13-23.295)$$

= 11470.6 kips-in

Ultimate curvature $\phi_{M_0} = \frac{0.003}{c} = \frac{0.003}{5.975} = \underline{\underline{0.0005021}}$ (for pure bending)

(d) Axial load – bending moment interaction diagram:

The axial load vs. bending moment interaction diagram is shown in Fig. 2.

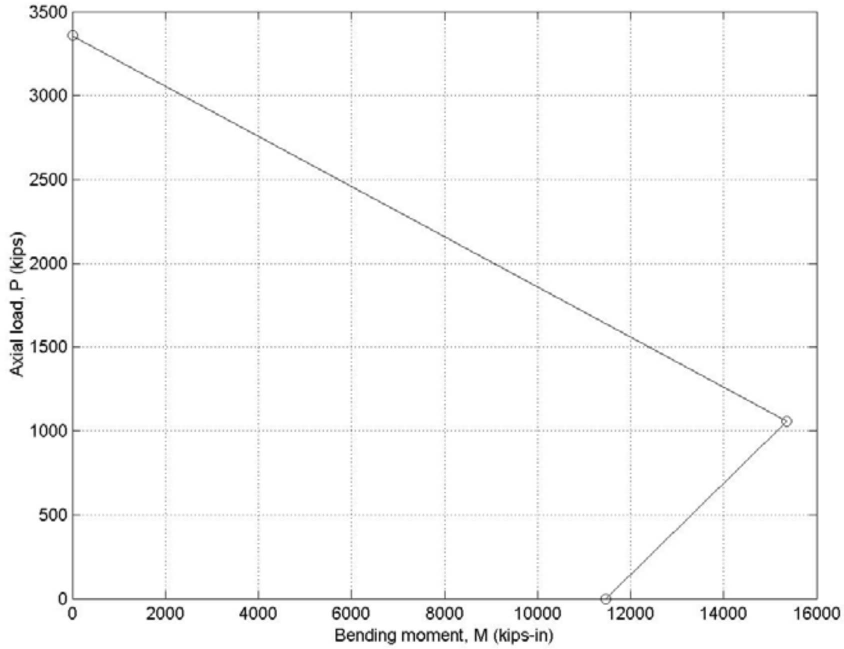


Fig. 2 Axial load – bending moment interaction diagram

(e) Axial load – ultimate curvature diagram

The axial load vs. ultimate curvature diagram is shown in Fig. 3.

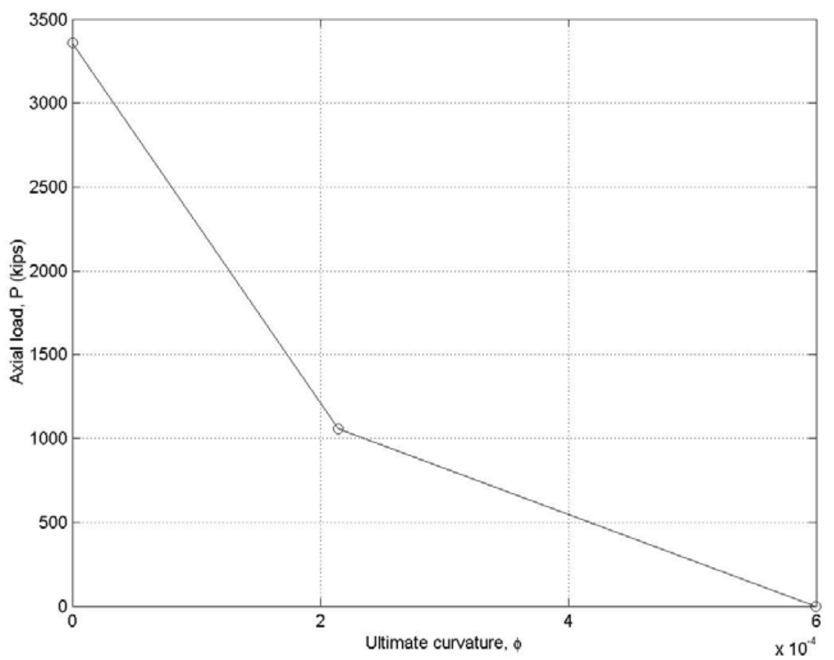
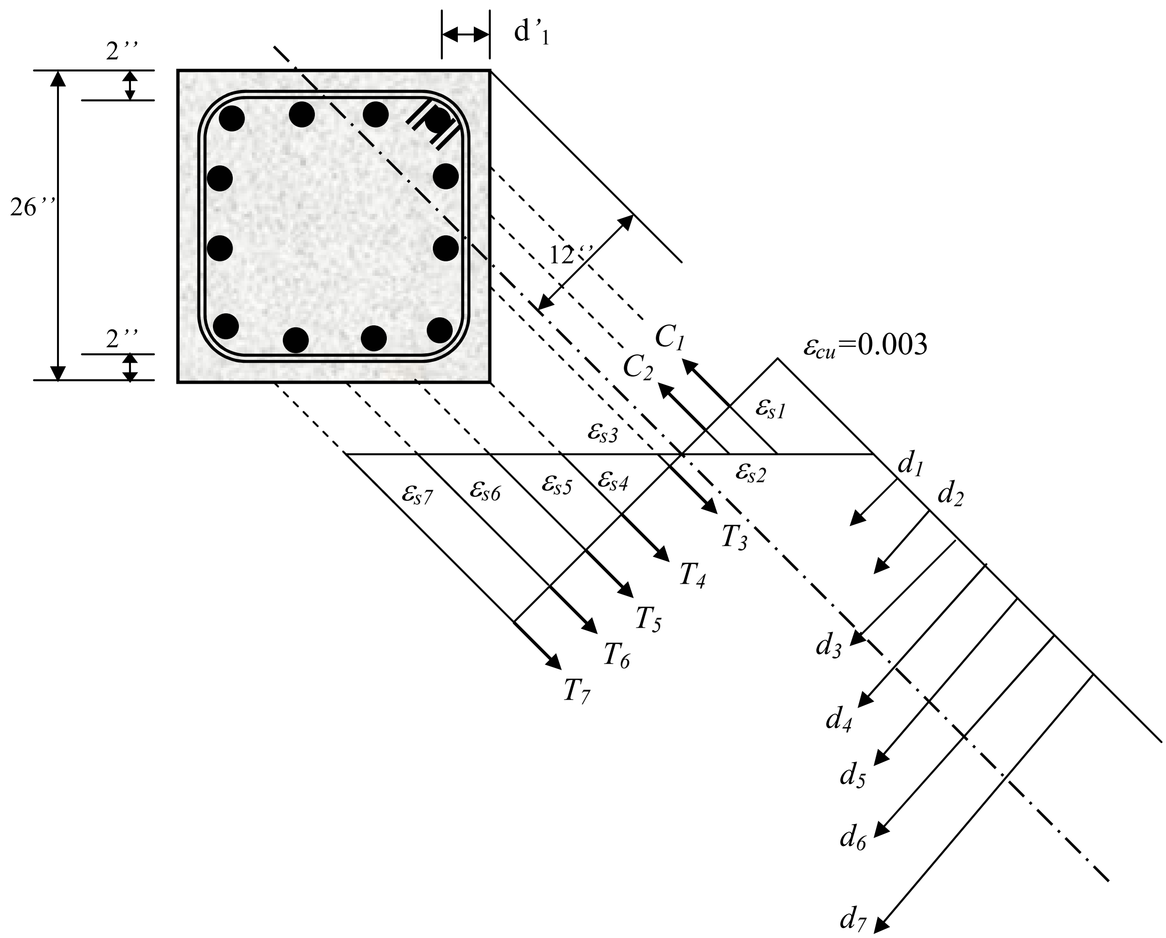


Fig. 3 Axial load – ultimate curvature diagram

2. Biaxial Column Interaction

Consider the strain distribution shown below, the distance between C_1 and C_2 is (26-4-1.41)/3 = 6.863 in., which is also the distance between any other two adjacent reinfrecement bars in the axis of strain distribution.



Given c = 12 in, the effective range of concrete in compression is $a = 0.85\ c = 10.2$ in. Calculate the distances from reinforcement location to upper right corner of the section below, respectively.

$$d_1 = d_1'\sqrt{2} = 2.705\sqrt{2} = 3.825 \text{ in}$$

$$d_2 = 3.825 + \frac{6.863}{\sqrt{2}} = 3.825 + 4.853 = 8.678 \text{ in}$$

$$d_3 = 8.678 + 4.853 = 13.531 \text{ in}$$

$$d_4 = 13.531 + 4.853 = 18.384 \text{ in}$$

$$d_5 = 18.384 + 4.853 = 23.237 \text{ in}$$

$$d_6 = 23.237 + 4.853 = 28.09 \text{ in}$$

$$d_7 = 28.09 + 4.853 = 32.943 \text{ in}$$

Calculae the strains of reinforcement bars:

$$\varepsilon_{s1} = \frac{1}{12}(0.003)(12-3.825) = 2.044\times10^{-3} > \varepsilon_y$$

$$\varepsilon_{s2} = \frac{1}{12}(0.003)(12-8.678) = 0.831\times10^{-3}$$

$$\varepsilon_{s3} = \frac{1}{12}(0.003)(12-13.531) = -0.383\times10^{-3}$$

$$\varepsilon_{s4} = \frac{1}{12}(0.003)(12-18.384) = -1.596\times10^{-3}$$

$$\varepsilon_{s5} = \frac{1}{12}(0.003)(12-23.237) = -2.809\times10^{-3} > \varepsilon_y$$

$$\varepsilon_{s6} > \varepsilon_y$$

$$\varepsilon_{s7} > \varepsilon_y$$

Compute the forces of the reinforcement:

$$f_{s1} = 60 \text{ ksi} \Rightarrow C_1 = (60)(1.56) = 93.6 \text{ kips}$$

$$f_{s2} = E_s\varepsilon_{s2} = 24.1 \text{ ksi} \Rightarrow C_2 = (24.09)(2\times1.56) = 75.16 \text{ kips}$$

$$f_{s3} = E_s\varepsilon_{s3} = -11.11 \text{ ksi} \Rightarrow T_3 = (-11.11)(2\times1.56) = -34.66 \text{ kips}$$

$$f_{s4} = E_s\varepsilon_{s4} = -46.28 \text{ ksi} \Rightarrow T_4 = (-46.28)(2\times1.56) = -144.39 \text{ kips}$$

$$f_{s5} = -60 \text{ ksi} \Rightarrow T_5 = (-60)(2\times1.56) = -187.2 \text{ kips}$$

$$f_{s6} = -60 \text{ ksi} \Rightarrow T_6 = (-60)(2\times1.56) = -187.2 \text{ kips}$$

$$f_{s7} = -60 \text{ ksi} \Rightarrow T_7 = (-60)(1.56) = -93.6 \text{ kips}$$

Therefore, the axial load, P, and bending moment, M, are

$$\begin{aligned} P &= C_c + C_1 + C_2 + T_3 + T_4 + T_5 + T_6 + T_7 \\ &= 0.85(4)\left[\left(10.2\sqrt{2}\right)^2 \big/ 2 - 3\times1.56\right] + 93.6 + 75.16 - 34.66 - 144.39 - 187.2 - 187.2 - 93.6 \\ &= \underline{\underline{-140.4 \text{ kips}}} \text{ (T)} \end{aligned}$$

$$\begin{aligned} M &= 0.85 f_c'\left[\left(10.2\sqrt{2}\right)^2 \big/ 2 - 3\times1.56\right]\left(\frac{26\sqrt{2}}{2} - \frac{10.2}{3}\right) + \sum_{i=1}^{n} \mathrm{A}_{si} f_{si}\left(\frac{26\sqrt{2}}{2} - d_i\right) \\ &= 0.85(4)\left[\left(10.2\sqrt{2}\right)^2 \big/ 2 - 3\times1.56\right](18.385-3.4) + (93.6)(18.385-3.825) \\ &\quad + (75.16)(18.385-8.678) - (34.66)(18.385-13.531) \\ &\quad - (144.39)(18.385-18.384) - (187.2)(18.385-23.237) \\ &\quad - (187.2)(18.385-28.09) - (93.6)(18.385-32.943) \\ &= \underline{11074 \text{ kips-in}} \end{aligned}$$

Use Bresler load contour method with $\alpha = 1.5$,

$$\left(\frac{M_{nx}}{M_{ox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{oy}}\right)^{\alpha} = 1 \Rightarrow \left(\frac{M_{ux}}{M_{ux_0}}\right)^{\alpha} + \left(\frac{M_{uy}}{M_{uy_0}}\right)^{\alpha} = 1$$

$M = M_{ux} = M_{uy}$=11074 kips-in (due to the symmetry). To get M_{ux_0} and M_{uy_0}, extend the axial load vs. bending moment interaction diagram to tension zone. Locate the M corresponding to P= -140.4 kips. P_0 in tension zone is calculated by neglecting the contribution of compressive strength of concrete.

$$P_{0,tension} = -A_s f_y = -(12 \times 1.56)(60) = -1123.2 \text{ kips}$$

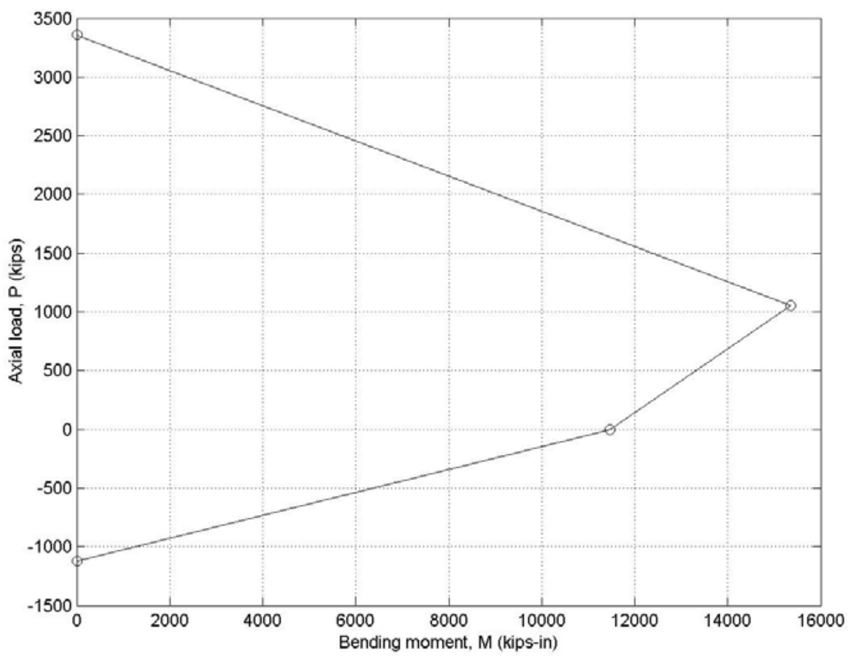


Fig. 4 Axial load – moment interaction diagram

For P = -140.4 kips,

$$M = \frac{1}{1123.2}(11470.6)(-140.4 + 1123.2) = 10036.8 \text{ kips-in} = M_{ux_0} = M_{uy_0}$$

Substitute them into the equation,

$$\left(\frac{M_{ux}}{10036.8}\right)^{1.5} + \left(\frac{M_{uy}}{10036.8}\right)^{1.5} = 1 \Rightarrow (M_{ux})^{1.5} + (M_{uy})^{1.5} = (10036.8)^{1.5}$$

For $M_{ux} = 0$, M_{uy} = 10036.8 kips-in.

For $M_{ux} = M_{uy}$, $M_{ux} = M_{uy}$ = <u>6322.8 kips-in</u>

For $M_{uy} = 0$, $M_{ux} = 10036.8$ kips-in.

Concluding the analysis above, it is found that the actual bending moment the column section subjected to is 11074 kips-in and the maximum biaxial bending capacity the column section can take, based on Bresler's method, is 6322.8 kips-in. The relationship between M and M_{ux} (or M_{uy}) is shown in Fig. 5.

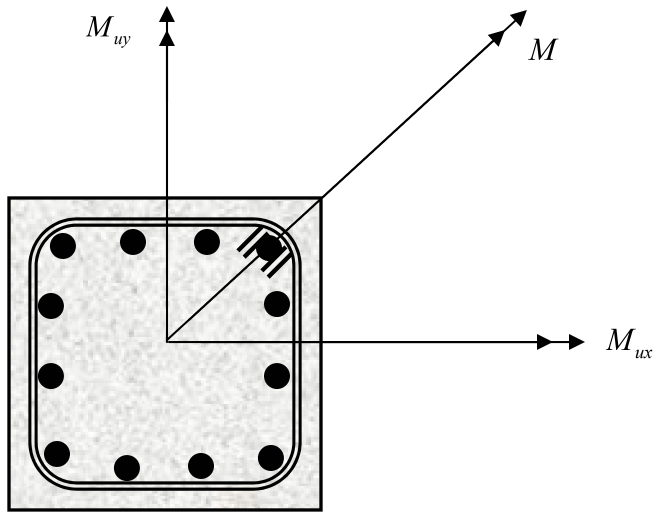


Fig. 5 M, M_{ux}, and M_{uy}

- **Comment:**

The three dimensional failure surface using Bresler's method is illustrated in Fig. 6. Notice that the failure surface is described by a nonlinear, concave function, although the approximated surface in Fig. 6 is piecewisely assembled. Since the actual load of the column is outside the failure curve, $M = 11074$ kips-in > 6322.8 kips-in , the column section is inadequate and will fail (Fig. 7).

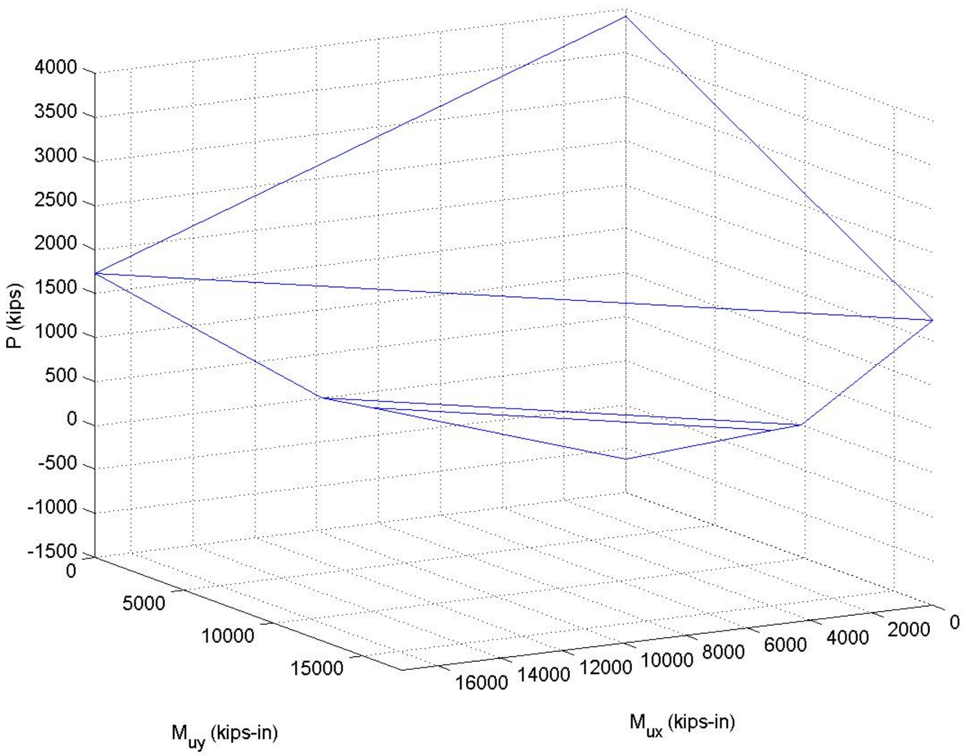


Fig. 6 The piecewise failure surface of the column section using Bresler's method

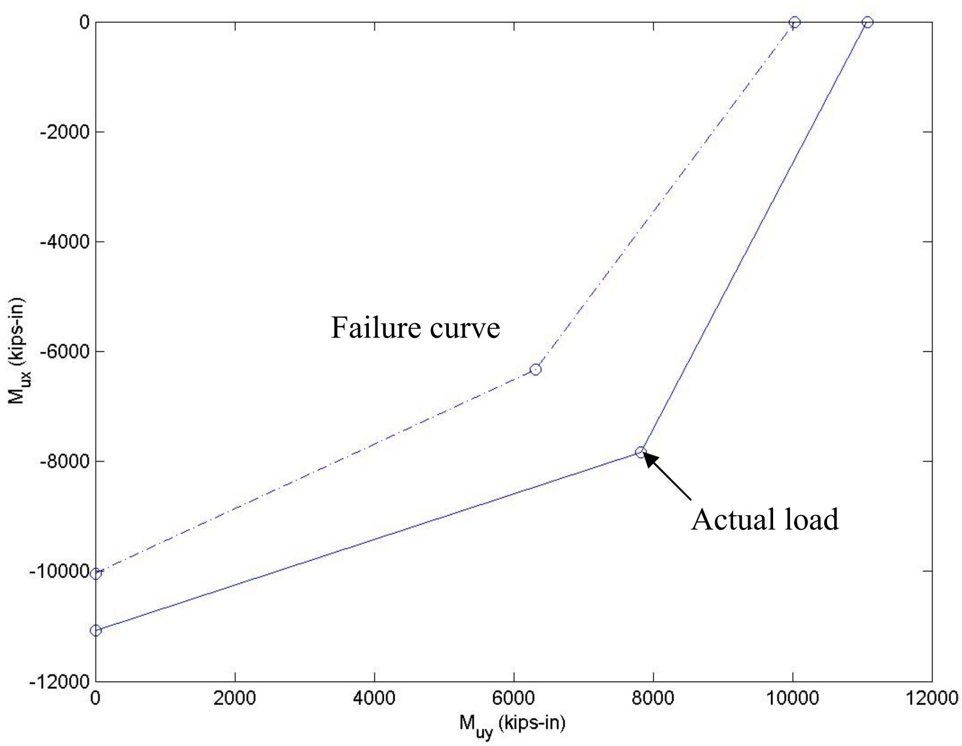


Fig. 7 Positions of failure curve and the actual load