# 1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

## Design Example

## Creep Strain of A Reinforced Concrete Column Frame During Construction

**Objective:** To investigate the effect of creep strain on a reinforced concrete column (column, concrete, and reinforcing steel) during construction.

**Problem:** An interior column located in the first floor of a 3-story reinforced concrete frame is considered, providing the load increments during construction. The load history for the first floor column from recorded dates of casting and form removal was determined and shown below.

| Time (days) | Event |
|---|---|
| 0 | Column cast |
| 30 | First floor shoring removed, $\Delta P$ = 200 kips |
| 60 | Second floor cast, $\Delta P$ = 200 kips |
| 120 | Penthouse and roof case, $\Delta P$ = 280 kips |

The column is 20x20 inches and contains #8 bars ($A_s$ = 6.32 in$^2$), the modulus of elasticity for the reinforcing steel is $E_s = 29 \times 10^6$ psi, and for the concrete $E_c = 2.9 \times 10^6$ psi. The attached data shows the specific creep function for the concrete used in this R/C frame. (See Attachment A)

**Task:** Consider only elastic and creep strains. Plot the following information at 30-day intervals for the first 180 days of the construction. Present all calculation results in tabular format.

1. Column strain (stress-strain curve)
2. Stress in the concrete (stress-time curve)
3. Stress in the reinforcing steel (stress-time curve)

To provide a basis for comparison, show on these plots simultaneously the elastic stress-strain ($\sigma$–$\varepsilon$) curve that neglects creep effects. The column is to be considered concentrically loaded for the purpose of this analysis.

# Attachment A

Specific creep, $C(t, \tau)$ X $10^{-6}$ (in/in/psi)

| t (days) / $\tau$ (days) | **15** | **30** | **45** | **60** | **75** | **90** | **105** | **120** | **135** | **150** | **165** | **180** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0 | 0.563 | 0.657 | 0.712 | 0.754 | 0.787 | 0.813 | 0.835 | 0.852 | 0.867 | 0.880 | 0.891 |
| **30** | | 0 | 0.364 | 0.424 | 0.460 | 0.487 | 0.508 | 0.525 | 0.539 | 0.550 | 0.560 | 0.568 |
| **45** | | | 0 | 0.276 | 0.322 | 0.349 | 0.370 | 0.386 | 0.399 | 0.410 | 0.418 | 0.426 |
| **60** | | | | 0 | 0.229 | 0.267 | 0.289 | 0.307 | 0.320 | 0.331 | 0.339 | 0.347 |
| **75** | | | | | 0 | 0.200 | 0.233 | 0.253 | 0.268 | 0.280 | 0.289 | 0.297 |
| **90** | | | | | | 0 | 0.181 | 0.211 | 0.229 | 0.242 | 0.253 | 0.261 |
| **105** | | | | | | | 0 | 0.168 | 0.196 | 0.212 | 0.225 | 0.253 |
| **120** | | | | | | | | 0 | 0.159 | 0.185 | 0.201 | 0.213 |
| **135** | | | | | | | | | 0 | 0.152 | 0.177 | 0.192 |
| **150** | | | | | | | | | | 0 | 0.147 | 0.171 |
| **165** | | | | | | | | | | | 0 | 0.143 |
| **180** | | | | | | | | | | | | 0 |

**[Design Procedures]**

1. Modified superposition method is applied as the numerical procedure. Define

$\Delta t_i$ = time step ($i$ = time index)

$\tau$ = age of concrete at loading

$\rho'$ = reinforcement ratio, $A_s / A_c$

$t_i$ = age of concrete at step $i$

$m$ = modular ratio, $E_s / E_c$

From the load history, initial conditions are given.

Initial strain in column: $\varepsilon_{column,-1} = \varepsilon_{col,-1} = 0$,

Initial stress in the concrete: $\sigma_{concrete,-1} = \sigma_{c,-1} = 0$,

Initial stress in the reinforcing steel: $\sigma_{steel,-1} = \sigma_{s,-1} = 0$

Changes in free creep strain are calculated by

$$\delta\varepsilon_n^{creep} = \delta\varepsilon_n^{cr} = \sum_{i=0}^{n-1} \Delta\sigma_{c,i} \times \left[ C\left(t_n, \tau_i\right) - C\left(t_{n-1}, \tau_i\right) \right]$$

Correction strain and stress:

$$\delta\varepsilon_{column,n}^{creep} = \delta\varepsilon_{col,n}^{cr} = \frac{\delta\varepsilon_n^{cr}}{1 + m \cdot \rho'}, \quad \delta\sigma_{concrete,n}^{creep} = \delta\sigma_{c,n}^{cr} = \frac{-\delta\varepsilon_n^{cr} \cdot E_c}{1 + \dfrac{1}{m \cdot \rho'}}, \quad \delta\sigma_{steel,n}^{creep} = \delta\sigma_{s,n}^{cr} = \frac{-\delta\sigma_{c,n}^{cr}}{\rho'}$$

Total change in strain and stress:

$$\Delta\varepsilon_{column,n} = \Delta\varepsilon_{col,n} = \delta\varepsilon_{col,n}^{cr} + \Delta\varepsilon_{c,n}^{load}$$

$$\Delta\sigma_{concrete,n} = \Delta\sigma_{c,n} = \delta\sigma_{c,n}^{cr} + \Delta\sigma_{c,n}^{load}$$

$$\Delta\sigma_{steel,n} = \Delta\sigma_{s,n} = \delta\sigma_{s,n}^{cr} + \Delta\sigma_{s,n}^{load}$$

Current state of strain and stress:

$$\varepsilon_{column,n} = \varepsilon_{col,n} = \varepsilon_{col,n-1} + \Delta\varepsilon_{col,n}$$

$$\sigma_{c,n} = \sigma_{c,n} = \sigma_{c,n-1} + \Delta\sigma_{c,n}$$

$$\sigma_{steel,n} = \sigma_{s,n} = \sigma_{s,n-1} + \Delta\sigma_{s,n}$$

2. From the provided information about the column, we have

$$\left.\begin{array}{l} A_s = 6.32in^2 \\ A_c = 20\times 20 - 6.32 = 393.68in^2 \end{array}\right\} \Rightarrow \rho' = \frac{A_s}{A_c} = 0.01605$$

$$\left.\begin{array}{l} E_s = 29\times 10^6\ psi \\ E_c = 2.9\times 10^6\ psi \end{array}\right\} \Rightarrow m = \frac{E_s}{E_c} = 10$$

From $\varepsilon_s = \varepsilon_c$, $\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s} \Rightarrow \sigma_s = m\cdot\sigma_c$

$$\Delta P = A_c\cdot\Delta\sigma_c + A_s\cdot\Delta\sigma_s$$
$$= A_c\cdot\Delta\sigma_c\cdot(1+m\cdot\rho')$$
$$\Rightarrow \Delta\sigma_c = \frac{\Delta P}{A_c\cdot(1+m\cdot\rho')}, \qquad \Delta\sigma_s = \frac{m\cdot\Delta P}{A_c\cdot(1+m\cdot\rho')}$$

| n | t (days) | ΔP (kips) | $\Delta\sigma_{c,n}^{load}$ (psi) | $\Delta\sigma_{s,n}^{load}$ (psi) | $\Delta\varepsilon_{col,n}^{load}\left(\times 10^{-6}\right)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 30 | 200 | 437.8 | 4378 | 151.0 |
| 2 | 60 | 200 | 437.8 | 4378 | 151.0 |
| 3 | 90 | 0 | 0 | 0 | 0 |
| 4 | 120 | 280 | 612.9 | 6129 | 211.3 |
| 5 | 150 | 0 | 0 | 0 | 0 |
| 6 | 180 | 0 | 0 | 0 | 0 |

(i) t = 30 days (n = 1)

No creep stress and strain. No previous stress and strain.

$$\Delta\varepsilon_{c,1} = 0 + \Delta\varepsilon_{col,1}^{load} = 151.0\left(\times 10^{-6}\right)$$
$$\Delta\sigma_{c,1} = 0 + \Delta\sigma_{c,1}^{load} = 437.8\ (\text{psi})$$
$$\Delta\sigma_{s,1} = 0 + \Delta\sigma_{s,1}^{load} = 4378\ (\text{psi})$$

$$\varepsilon_{col,1} = \varepsilon_{col,0} + \Delta\varepsilon_{col,1}^{load} = 151\left(\times 10^{-6}\right)$$
$$\sigma_{c,1} = \sigma_{c,0} + \Delta\sigma_{c,1}^{load} = 437.8\ (\text{psi})$$
$$\sigma_{s,1} = \sigma_{s,0} + \Delta\sigma_{s,1}^{load} = 4378\ (\text{psi})$$

(ii) t = 60 days (n = 2)

Change in free creep strain:

$$\delta\varepsilon_2^{cr} = 437.8\times(0.424-0) = 185.7\ \left(\times10^{-6}\right)$$

Correction strain and stress:

$$\delta\varepsilon_{col,2}^{cr} = \frac{\delta\varepsilon_2^{cr}}{1+m\rho'} = \frac{\delta\varepsilon_2^{cr}}{1.1605} = 160\ \left(\times10^{-6}\right)$$

$$\delta\sigma_{c,2}^{cr} = \frac{-\delta\varepsilon_2^{cr}\cdot E_c}{1+\frac{1}{m\rho'}} = \frac{-\delta\varepsilon_2^{cr}\cdot E_c}{7.2305} = -\delta\varepsilon_2^{cr}\cdot 0.4011\times10^6 = \text{-74.5 (psi)}$$

$$\delta\sigma_{s,2}^{cr} = \frac{-\delta\sigma_{c,2}^{cr}}{\rho'} = \frac{-\delta\sigma_{c,2}^{cr}}{0.01605} = 4641.7\ \text{(psi)}$$

| | | | | | |
|---|---|---|---|---|---|
| $\Delta\varepsilon_{col,2} =$ | 160 | + | 151 | = | $311\left(\times10^{-6}\right)$ |
| $\Delta\sigma_{c,2} =$ | -74.5 | + | 437.8 | = | 363.3 (psi) |
| $\Delta\sigma_{s,2} =$ | 4641.7 | + | 4378 | = | 9019.7 (psi) |

| | | | | | |
|---|---|---|---|---|---|
| $\varepsilon_{col,2} =$ | 151 | + | 311 | = | $462\ \left(\times10^{-6}\right)$ |
| $\sigma_{c,2} =$ | 437.8 | + | 363.3 | = | 801.1 (psi) |
| $\sigma_{s,2} =$ | 4378 | + | 9019.7 | = | 13397.7 (psi) |

(iii) t = 90 days (n = 3)

Change in free creep strain;

$$\delta\varepsilon_3^{cr} = 437.8\times(0.267-0) + 437.8\times(0.487-0.424) = 144.47\left(\times10^{-6}\right)$$

Correction strain and stress;

$$\delta\varepsilon_{col,3}^{cr} = 124.49\left(\times10^{-6}\right)$$

$$\delta\sigma_{c,3}^{cr} = -57.94\ \text{(psi)}$$

$$\delta\sigma_{s,3}^{cr} = 3609.97\ \text{(psi)}$$

| | |
|---|---|
| $\Delta\varepsilon_{col,3}$ (n=3) = | $124.49\left(\times10^{-6}\right)$ |
| $\Delta\sigma_{c,3}$ (n=3) = | –57.94 (psi) |
| $\Delta\sigma_{s,3}$ (n=3) = | 3609.97 (psi) |

| | | | | | |
|---|---|---|---|---|---|
| $\varepsilon_{col,3}$ (n=3) = | 462 | + | 124.49 | = | 586.49 $\left(\times 10^{-6}\right)$ |
| $\sigma_{c,3}$ (n=3) = | 801.1 | – | 57.94 | = | 743.16 (psi) |
| $\sigma_{s,3}$ (n=3) = | 13397.7 | + | 3609.97 | = | 17007.67 (psi) |

(ix) t = 120 days (n = 4)

Change in free creep strain;

$$\delta\varepsilon_4^{cr} = 437.8\times(0.386-0.349)+437.8\times(0.525-0.487) = 32.84\ \left(\times 10^{-6}\right)$$

Correction strain and stress;

$\delta\varepsilon_{col,4}^{cr} = 28.29\ \left(\times 10^{-6}\right)$

$\delta\sigma_{c,4}^{cr} =$ -13.17 (psi)

$\delta\sigma_{s,4}^{cr} = 820.56$ (psi)

| | | | | | |
|---|---|---|---|---|---|
| $\Delta\varepsilon_{col,4}$ (n=4) = | 28.29 | + | 211.3 | = | 239.59 $\left(\times 10^{-6}\right)$ |
| $\Delta\sigma_{c,4}$ (n=4) = | -13.17 | + | 612.9 | = | 599.73 (psi) |
| $\Delta\sigma_{s,4}$ (n=4) = | 820.56 | + | 6129 | = | 6949.56 (psi) |

| | | | | | |
|---|---|---|---|---|---|
| $\varepsilon_{col,4}$ (n=4) = | 586.49 | + | 239.59 | = | 826.08 $\left(\times 10^{-6}\right)$ |
| $\sigma_{c,4}$ (n=4) = | 743.16 | + | 599.73 | = | 1342.89 (psi) |
| $\sigma_{s,4}$ (n=4) = | 17007.67 | + | 6949.56 | = | 23957.23 (psi) |

(x) t = 150 days (n = 5)

Change in free creep strain;

$$\delta\varepsilon_5^{cr} = 612.9\times(0.185-0)+0+437.8\times(0.331-0.307)+437.8\times(0.550-0.525) = 134.83\left(\times 10^{-6}\right)$$

Correction strain and stress;

$\delta\varepsilon_{col,5}^{cr} = 116.18\ \left(\times 10^{-6}\right)$

$\delta\sigma_{c,5}^{cr} =$ –54.08 (psi)

$\delta\sigma_{s,5}^{cr} =$ 3369.47 (psi)

| | |
|---|---|
| $\Delta\varepsilon_{col,5}$ (n=5) = | 116.18 $\left(\times 10^{-6}\right)$ |
| $\Delta\sigma_{c,5}$ (n=5) = | –54.08 (psi) |

$\Delta\sigma_{s,5}$ (n=5) = 3369.47 (psi)

| | | | | | |
|---|---|---|---|---|---|
| $\varepsilon_{col,5}$ (n=5) = | 826.08 | + | 116.18 | = | 942.26 $\left(\times 10^{-6}\right)$ |
| $\sigma_{c,5}$ (n=5) = | 1342.89 | - | –54.08 | = | 1288.81 (psi) |
| $\sigma_{s,5}$ (n=5) = | 23957.23 | + | 3369.47 | = | 27326.70 (psi) |

(xi) t = 180 days (n = 6)

Change in free creep strain;

$$\delta\varepsilon_6^{cr} = 612.9\times(0.213-0.185)+0+437.8\times(0.347-0.331)+437.8\times(0.568-0.550)=32.05\left(\times 10^{-6}\right)$$

Correction strain and stress;

$\delta\varepsilon_{col,6}^{cr} = 27.62\left(\times 10^{-6}\right)$

$\delta\sigma_{c,6}^{cr} = -12.86$(psi)

$\delta\sigma_{s,6}^{cr} = 801.25$ (psi)

$\Delta\varepsilon_{col,6}$ (n=6) = 27.62 $\left(\times 10^{-6}\right)$

$\Delta\sigma_{c,6}$ (n=6) = –12.86(psi)

$\Delta\sigma_{s,6}$ (n=6) = 801.25 (psi)

| | | | | | |
|---|---|---|---|---|---|
| $\varepsilon_{col,6}$ (n=6) = | 942.26 | + | 27.62 | = | 969.88 $\left(\times 10^{-6}\right)$ |
| $\sigma_{c,6}$ (n=6) = | 1288.81 | - | 12.86 | = | 1275.95 (psi) |
| $\sigma_{s,6}$ (n=6) = | 27326.70 | + | 801.25 | = | 28127.95 (psi) |