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Progress Report

Shape Finding Methods and Design of Thin Reinforced Concrete Shell

Thin shell concrete structures were developed in the nineteen-sixties in response to the need for economy in large-span structures and to the design and aesthetic program of the modern movement in architecture. The unique characteristics of reinforced concrete shell – easily shaped, having both tensile as well as compressive strength, durable and fire resistant, has since made it popular with architects throughout the world. This paper looks into the history and development of thin reinforced concrete shell structure, examines various concrete shell forms that are commonly in use and analyses shape finding methods and the design of free edge reinforced concrete shells.

History and Development

The earliest form of a shell structure is the dome. The ancient spherical concrete or masonry dome supports load entirely through compression with zero tensile forces within its elements. Since a dome is essentially an arch rotated 360 degrees about a point, the dome's force carrying mechanism can be rationalized from the analysis of the way an arch supports its loads. The simplest way of visualizing how an arch works can be obtained from the hanging string experiment. The hanging string, which is an inversed arch, has negligible bending strength and all of the loads are carried by direct tensile stress. In an arch, and dome, load is thus carried in the opposite direction, by pure compression with no bending. An example of an ancient dome is the roof of the Pantheon in Rome, built in 124 AD. The unreinforced hemispherical dome roof has a height and radius of 21.6 meter. The upper part of the dome is 1.2 m thick and it gets progressively thicker towards the bottom. The dome is supported by a 43.3 meters tall Rotunda, a massive cylindrical wall to resist the horizontal thrust of the dome.

The invention of reinforced concrete in the early 1900s affected the design of concrete domes dramatically. Reinforced concrete, which has a higher strength per unit volume than pure concrete, allows for a much thinner shell structure to be formed. Reinforced concrete domes could have such thin shells, that the radius to thickness ratio of such shells is smaller than that of a hen's egg. An example of a reinforced concrete dome is the Kingdome in Washington State, which has a clear span diameter of 201.6 meters while having a shell thickness of only 120 mm. Reinforced concrete shell membrane also has the capacity to resist tensile as well as compressive stresses, unlike pure concrete structures which has only compressive strength. This property enables the necessary support to resist spread of the shell boundary, previously provided by massive walls or flying buttresses, to be incorporated within the tensile reinforced concrete boundary members. There are many ways this could be done. In the Kingdome, a post-tensioned concrete ring forms the exterior circumference of the dome, acting as a tensile ring that resists the horizontal thrust. In the Algeciras Market Hall, Eduardo Torroja used cylindrical vaults to resist the end thrust of the dome. In this ingenious design, the dome merges into cylindrical vaults that span between the point supports.

The interconnected elements of the dome and cylindrical vaults provide rigidity to the dome and concentrate the forces towards the supports.

During the earlier developments in reinforced concrete shells, there was a great tendency to use surfaces that are easily definable by mathematical expressions. This trend can be accounted by the fact that in shell theory, an analytical definition of the shell geometry is needed to obtain a closed form solution. Although linear elastic and geometrically non-linear and buckling finite element analysis theory of free-form shell structures were available then, modern computers to work out numerical solution were not readily available. It is therefore more convenient to make forms that are easily prescribed by mathematical equations than to design a free form shell.

Hyperboloids and Hyperbolic Paraboloids

Two widely used shell forms whose surface can be prescribed by mathematical equations are the hyperboloids and hyperbolic paraboloids. The equation for hyperboloids is $x^2/a^2 - y^2/b^2 - z^2/c = 1$ where a,b and c are constants. An example of a hyperboloids shell is the Zarzuela Hippodrome in Madrid, designed by Torroja.
The equation for hyperbolic paraboloids is $x^2/a^2 - y^2/b^2 = z$ for a negative Gaussian curvature and $x^2/a^2 + y^2/b^2 = z$ for a positive Gaussian curvature. A surface with a negative Gaussian curvature can be generated by translating a convex parabola over a concave parabolic curve. A surface with positive Gaussian curvature is formed by translating a convex parabola over a convex parabola. A true hyperbolic paraboloid form very closely resembles the shallow saddle shaped surfaces of hyperboloids. However, hyperbolic paraboloid, which is essentially ruled surfaces generated by the translation of one line against another, can be easily varied and its variations combined to form a variety of shell segments. These segments could in turn be put together to form dramatic structures such as the restaurant in Xochimilco, Mexico, which is formed out of 4 hyperbolic paraboloids.

Free Form Shell

With the advancement of analysis method and computer technology, shell of any shape could be easily designed and constructed. The shape of a shell canopy can be selected purely arbitrarily, made to conform to artistic requirements, or to minimize weight corresponding to prescribed loading and boundary conditions. In a typical form finding procedure for a canopy, the architects will specify rough concepts of desired surface form together with geometric constraints on parameters such as height curvature and span. Structural designers must then determine a smooth surface that conforms to these architectural constraints while additionally satisfying structural constraints. Form-finding exercise could be done experimentally or through a computer software program analysis.

Experimental methods provide an easy yet clear visualization of how the structure will work. They are most suitable for determining the optimum shell form when specific loading conditions are given. The most commonly used experimental method is the inverse principles of hanging models to determine the shape of structures in compression. This method involves hanging a membrane clamped at its boundary conditions as prescribed by the architectural constraints, and placed under a defined loading condition. This hanging model renders pure, tension, when if stiffened by any means, turns into pure compression after inversion.

Next

In the next paper, I will continue with the description of computer form finding methods. I would also analyze the procedures involved in the design of free edge thin-shell structures. The procedure can be outlined as follows: shape optimization, structural analysis, which is followed by the actual design of the pre-stressed concrete structures.