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# ANALYSIS OF STRUCTURAL MEMBER SYSTEMS

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## Preface

With the development over the past decade of computer-based analysis methods, the teaching of structural analysis subjects has been revolutionized. The traditional division between structural analysis and structural mechanics became no longer necessary, and instead of teaching a preponderance of solution details it is now possible to focus on the underlying theory.

What has been done here is to integrate analysis and mechanics in a systematic presentation which includes the mechanics of a member, the matrix formulation of the equations for a system of members, and solution techniques. The three fundamental steps in formulating a problem in solid mechanics—enforcing equilibrium, relating deformations and displacements, and relating forces and deformations—form the basis of the development, and the central theme is to establish the equations for each step and then discuss how the complete set of equations is solved. In this way, a reader obtains a more unified view of a problem, sees more clearly where the various simplifying assumptions are introduced, and is better prepared to extend the theory.

The chapters of Part I contain the relevant topics for an essential background in linear algebra, differential geometry, and matrix transformations. Collecting this material in the first part of the book is convenient for the continuity of the mathematics presentation as well as for the continuity in the following development.

Part II treats the analysis of an ideal truss. The governing equations for small strain but arbitrary displacement are established and then cast into matrix form. Next, we deduce the principles of virtual displacements and virtual forces by manipulating the governing equations, introduce a criterion for evaluating the stability of an equilibrium position, and interpret the governing equations as stationary requirements for certain variational principles. These concepts are essential for an appreciation of the solution schemes described in the following two chapters.

Part III is concerned with the behavior of an isolated member. For completeness, first are presented the governing equations for a deformable elastic solid allowing for arbitrary displacements, the continuous form of the principles of virtual displacements and virtual forces, and the stability criterion. Unrestrained torsion-flexure of a prismatic member is examined in detail and then an approximate engineering theory is developed. We move on to restrained torsion-flexure of a prismatic member, discussing various approaches for including warping restraint and illustrating its influence for thin-walled

open and closed sections. The concluding chapters treat the behavior of planar and arbitrary curved members.

How one assembles and solves the governing equations for a member system is discussed in Part IV. First, the direct stiffness method is outlined; then a general formulation of the governing equations is described. Geometrically nonlinear behavior is considered in the last chapter, which discusses member force-displacement relations, including torsional-flexural coupling, solution schemes, and linearized stability analysis.

The objective has been a text suitable for the teaching of modern structural member system analysis, and what is offered is an outgrowth of lecture notes developed in recent years at the Massachusetts Institute of Technology. To the many students who have provided the occasion of that development, I am deeply appreciative. Particular thanks go to Mrs. Jane Malinofsky for her patience in typing the manuscript, and to Professor Charles Miller for his encouragement.

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