

Linear Elastic Fracture Mechanics

Assigned Reading:

Atkinson, B.K., *Fracture Mechanics of Rock*, pp. 534, Academic Press, London UK, 1987. Chapter 1, 2, 4.

Resource reading:

General Linear Elastic Fracture Mechanics:

Lawn, B.R., and T.R. Wilshaw, *Fracture of brittle solids*, Cambridge University Press, Cambridge, UK, 1975.

Paterson, M.S., *Experimental Rock Deformation - The Brittle Field*, 254 pp., Springer, New York, 1978.

Effects of surfactants:

Dunning, J., B.J. Douglas, M. Miller, and S.K. McDonald, The role of the chemical environment in frictional deformation stress corrosion cracking and comminution, *Pure and Applied Geophysics*, 143, 151-178, 1994.

Dunning, J., R. Kornbrekke, and M. Miller, Surfactant Induced Weakening of Geologic Materials, in *Surfactants in Solution*, edited by Mittal, pp. 297-321, Plenum Press, 1989.

Dunning, J., S.K. McDonald, B.J. Douglas, and C. Dintaman, Measurement of the stress corrosion limit, K_{Ic} , in quartz and Na-lime glass, *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.*, 30, 687-690, 1993.

Static Fatigue, Stable Crack Growth, Stress Corrosion:

Lockner, D.A., Rock failure, in *Handbook of Physical Constants*, edited by T.A. Ahrens, pp. Sections 3-10, 1995.

Lockner, D.A., Brittle deformation law for Westerly Granite, *J. Geophys. Res.*, 103, 5107-5103, 1998.

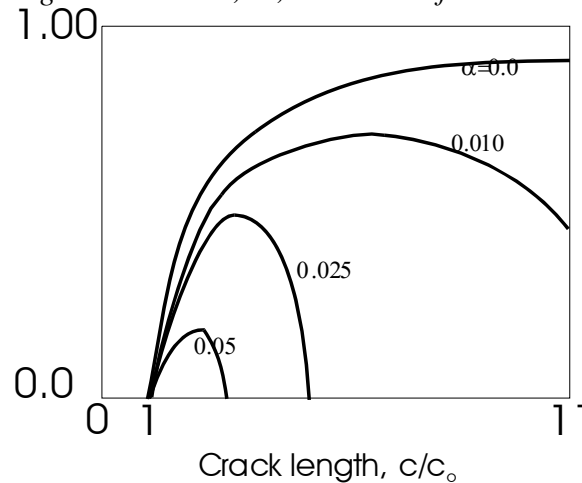
Fractography

Sources of Griffith Flaws

1. Thermal anisotropy and/or elastic anisotropy of small hard precipitate particles.
2. Surface damage by hard particles
3. Cracks generated from grain boundaries
(Chemical segregation at grain boundaries)
Precipitation at grain boundaries
4. Stroh Crack: Stresses generated by dislocation pileups

Details of dynamic fracturing depend critically on boundary conditions (esp. loading, i.e. constant load, constant displacement, ...)

Figure. 4.3 Lawn, B., *Fracture of Brittle Solids*



Crack front velocity as a function of crack length for a system subjected to constant load ($\alpha=0$), or constant displacement ($\alpha>0$), i.e. stress relaxation.

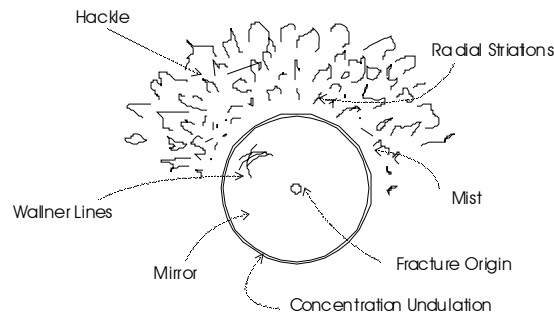
Note that cracking is unstable for constant load conditions whenever the critical stress intensity factor is exceeded.

Dynamic fracturing and fractography

If unstable crack growth occurs then:

1. Crack front initiates at critical stress intensity
2. Acceleration depends on the fracture stress, loading rate, geometry, and size of the sample.
3. Accelerates to a velocity which is 0.6 to 0.97 of the Rayleigh wave velocity, V_R .
4. Interference of stress waves reflected from surfaces of the sample with the stress field around the crack cause slight deviations from the crack front called Wallner lines.
5. Stress state around a moving crack is different from that of a stationary crack.
6. Consequently, small steps in the fracture surface begin to be formed. (Mist zone)
7. Eventually the dissipation of energy at the crack tip becomes large enough to cause the crack tip to bifurcate and two surfaces are formed. Crack branching begins. (Hackle zone).
8. With some assumptions about the stress field, arguments can be made, that the energy to form two surfaces simultaneously is available at a critical velocity. The length of the crack at that point is

$$\sigma_f c_b^{1/2} = \text{Constant}$$



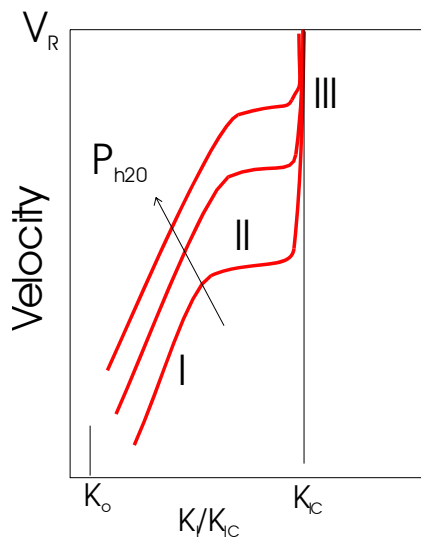
Schematic of fracture surface.

Stress Corrosion, Static Fatigue, Stable Crack Growth

Experiments sometimes show stable growth vs. dynamic fracture

Once the critical stress intensity factor is reached (exceeded) the crack extends unstably with velocities that approach and are limited by the sound (Rayleigh wave) velocity.

Experiments indicate that for stress intensities around 0.2-0.8 of the critical stress intensity that cracks will extend provided you are willing to wait.



As K_I approaches K_{Ic}
The crack tip velocity approaches the Rayleigh wave velocity. That is crack extension velocity is fixed.

As the partial pressure of water is changed, the fracture surface energy is reduced and so the fracture energy is reduced

Below K_o the crack will not extend at all. This threshold, perhaps owing to crack healing has not been measured experimentally.

When $K_o > K_I > K_{Ic}$ crack extends at a velocity that is less than K_{Ic} . Conversely, the strength becomes weakly rate-dependent.

Decrease in strength $\approx 33\%$ over a large range in velocity (or strain rate)

Mechanisms of Stress Corrosion

Empirically the relation between velocity and stress intensity can be given as:

$$v \propto K^n$$

The three regions observed empirically can be distinguished by the mechanism rate-limiting the growth of the crack tip.

Region I: Chemical reactions at the crack tip.

$$n \approx 20-50$$

Region II: Transport of reactive species.

$$n \approx 2-10$$

Region III: Mixture of reaction and mechanical (rate independent) crack extension

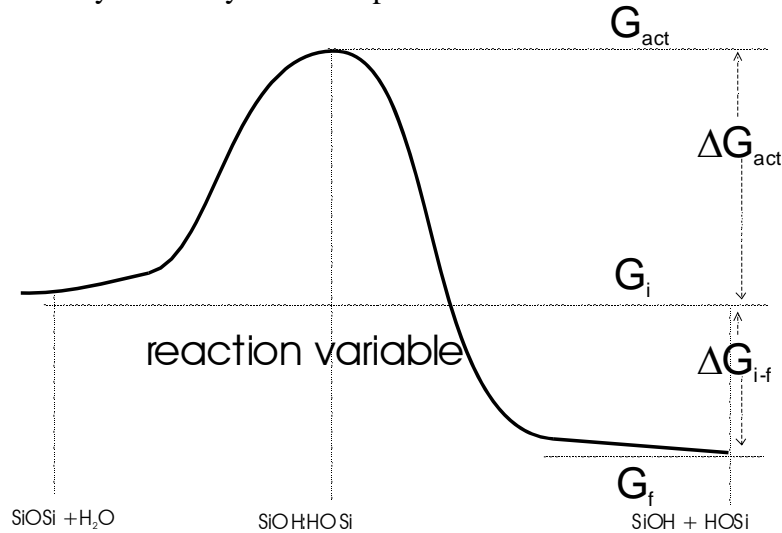
$$n > 50$$

Mechanisms

Stress Corrosion:
Diffusive Transport
Microplasticity
Dissolution
Ion Exchange
Lattice trapping

Stable crack growth: Kinetics

For any thermally activated process:



For classical particles, the number of particles with a given energy, G , is given by $\frac{N}{N_{\text{ave}}} \propto \exp\left(-\frac{\Delta G}{kT}\right)$ where $\Delta G = G - G_{(\text{ave})}$. Then the number of particles with excess energy ΔG_{act} can be calculated using the Boltzmann distribution.

For constant T and P conditions, the Gibbs free energy is $\Delta G = \Delta H + P\Delta V - T\Delta S$

The pressure around an elliptical, tensilely loaded crack is

$$P = -\frac{2}{3} \frac{K}{(\pi r_c)^{1/2}} \quad (\text{Wiederhorn et al., 1974})$$

So the added pressure reduces the amount of thermal energy necessary for the reaction to occur.

If the velocity of the crack is proportional to the rate of reaction, and if the rate of reaction is proportional to the number of atoms that have sufficient energy to achieve the transitional state, then

$$v = v_0 \exp\left\{ \left[-\Delta H + \frac{2}{3} \frac{K\Delta V}{(\pi r_c)^{1/2}} \right] / RT \right\}$$

An empirical relation often given is similar:

$$v = v_0 \exp\left\{ [-E^* + bK] / RT \right\}$$

Both equations are similar in that the numerator in the exponential contains the critical stress intensity factor (fracture toughness), K .

A second alternative expression can be arrived at by using a different expression for the stress dependence of ΔG (as opposed to pressure) (Charles, 1958)

$G \left[v=v_0 K^n \exp\{-\Delta H / RT\} \right]$ where the exponent n is called the stress corrosion factor (or index). This expression is commonly used to treat experimental data.

When the stress corrosion index is found experimentally to make an abrupt change, that change is typically taken to mean that the mechanism rate-controlling the crack extension has changed.

Conclusions:

- 1) *For rate-independent crack extension, the applied stress necessary to cause the crack to extend depends on the inverse of the square root of a linear dimension of the fracture.*
- 2) *The relevant material parameter is called the critical stress intensity factor. It depends on the mode of fracture and, of course, on the material.*
- 3) *The stress intensity factor can be related to the fracture surface energy, and therefore, chemical factors that lower or raise the fracture surface energy can be expected to affect K_c .*
- 4) *The crack extension velocity is a very strong function of the stress intensity factor (either exponential or power law with a very large exponent).*
- 5) *Those parameters that affect the stress intensity factor (e.g. chemical environment) will also affect the crack extension rate.*
- 6) *Slow, stable, crack extension can occur at applied stresses of 30-80 % of the stresses necessary to reach the critical stress intensity*

- Dunning, J., B.J. Douglas, M. Miller, and S.K. McDonald, The role of the chemical environment in frictional deformation stress corrosion cracking and comminution, Pure and Applied Geophysics, 143, 151-178, 1994.*
- Dunning, J., R. Kornbrenke, and M. Miller, Surfactant Induced Weakening of Geologic Materials, in Surfactants in Solution, edited by Mittal, pp. 297-321, Plenum Press, 1989.*
- Dunning, J., S.K. McDonald, B.J. Douglas, and C. Dintaman, Measurement of the stress corrosion limit, K_{Ic} , in quartz and Na-lime glass, Int. J. Rock Mech. Min. Sci. Geomech. Abstr., 30, 687-690, 1993.*