Diffusion Creep

Poirier, Chapter 2 and 7, 1985.

Gordon, 1985.

Fick's First Law: Driving Force

$$J = -D \bullet \nabla \mu \begin{cases} \nabla c \\ \nabla T \\ \nabla V \\ \sigma \end{cases}$$

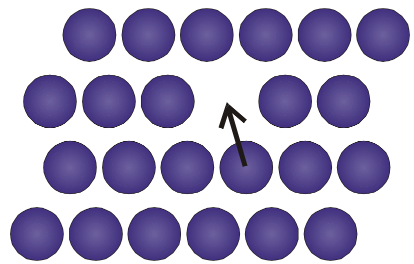
- Chemical Diffusion
- Thermal diffusion
- Electrical conduction
- Diffusion creep

Types of Diffusion

Mechanism	Path	Process
Isotope	Lattice	Interdiffusion
Self-diffusion	Pipe	Creep
Vacancy	Grain Boundary	Ambipolar
Interstitial	Surface	
Ring	Pore fluid	

Diffusion mechanisms

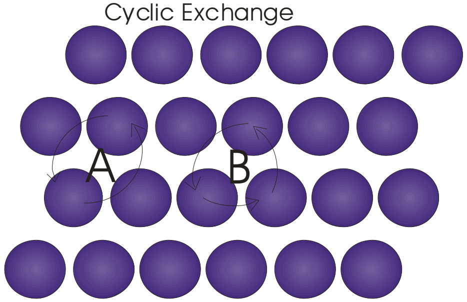
Vacancy Diffusion



Ring Diffusion

Direct Exchange

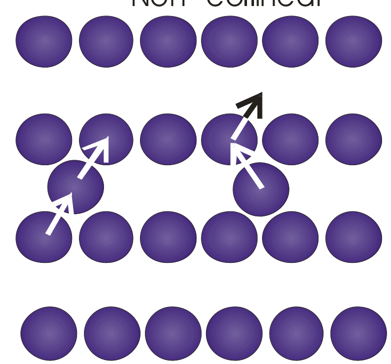
Cyclic Exchange



Interstitial

Collinear

Non -collinear

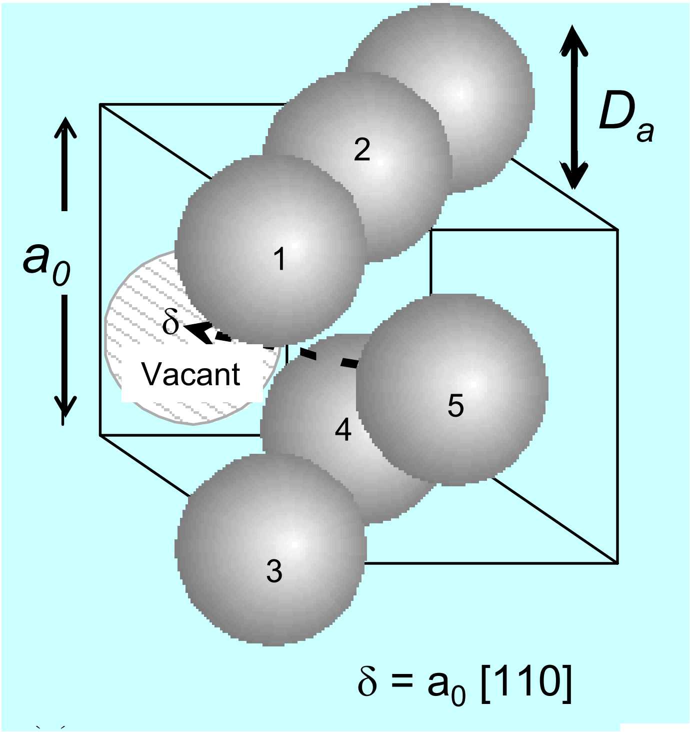


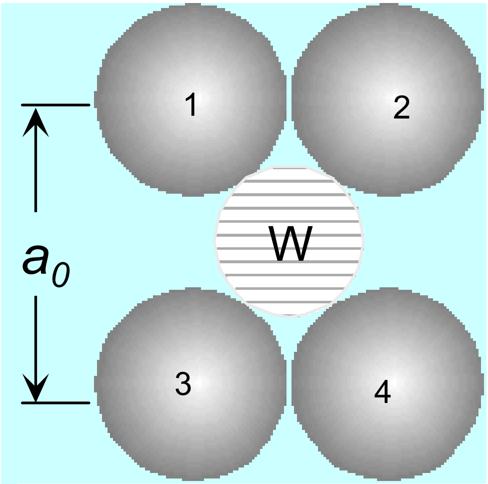
Vacancy–Assisted Diffusion

From Site by Glicksman and Lupulescu, RPI, 2003

The FCC lattice geometry requires

$$W = \left(\sqrt{3} - 1\right) D_a = 0.73 D_a$$





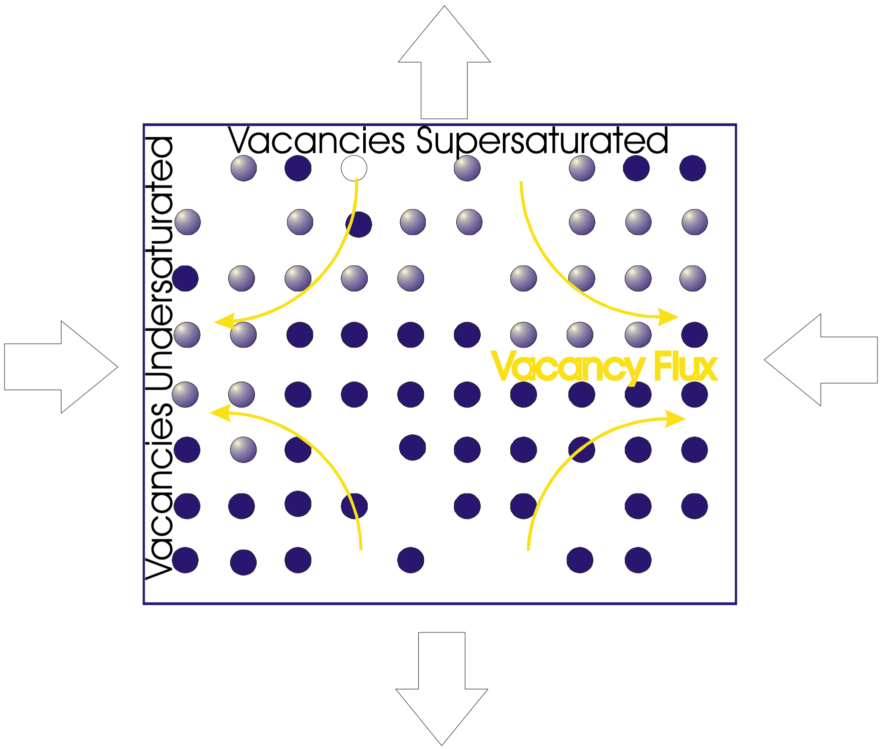
plan view

Kinetics Equation for Vacancy Diffusion

- Coefficient of Diffusivity for Self-diffusion not the same as Coefficient for Vacancy Diffusion

$$
\begin{aligned}
D_{sd} &= N_v \bullet D_{v\,migration} \\
&= N_{vo} \exp -(\Delta G_{vf} / kT) \bullet D_{v\,o} \exp -(\Delta G_{vm} / kT) \\
&= D_{sd\;o} \exp -(\Delta G_{vf} + \Delta G_{vm}) / kT
\end{aligned}
$$

Diffusion Creep in Monatomic Solid



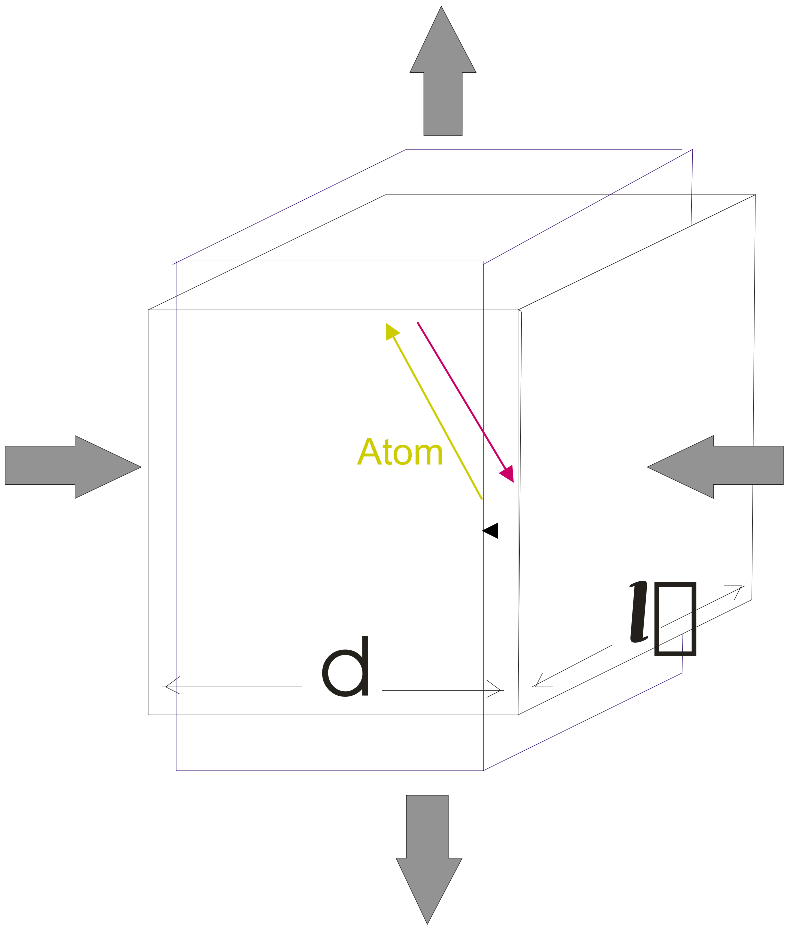
- Basic Ideas
  - Supersaturation of vacancies owing to stress
  - Diffusion results
  - Work done on mat'l by tractions
  - Energy dissipated in heat, entropy, and surface area

Diffusion Creep

- Nabarro-Herring Creep
  Lattice
- Coble Creep
  Grain Boundary

- Monatomic
- Quasi static
- Vacancy
- Increasing length; Poisoining

Critical Idea: Tension makes vacancy formation easier.



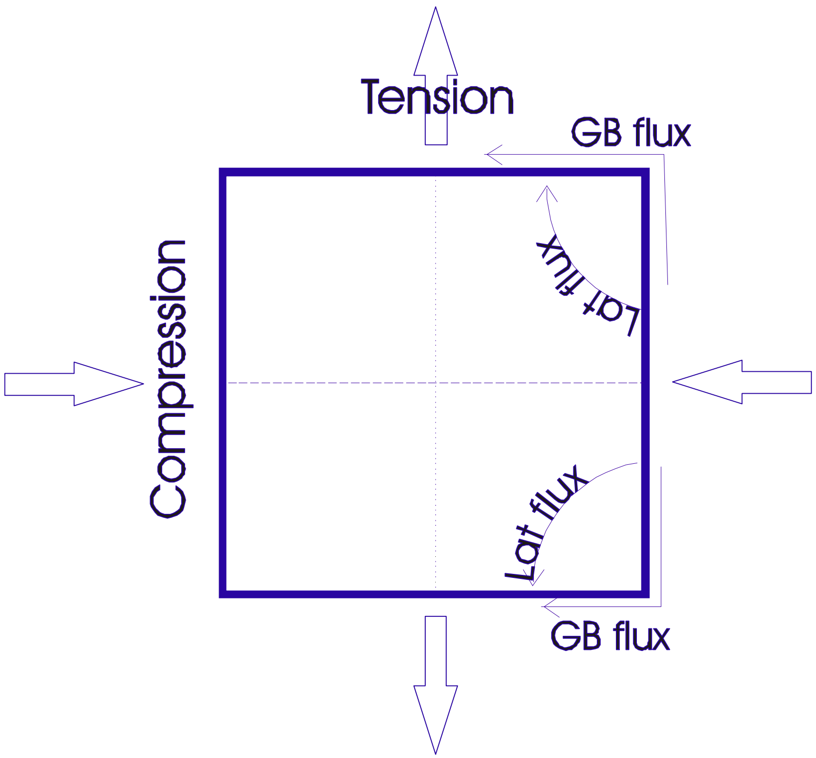
- Tension=supersaturation

$$\Delta G_{f_v}(\sigma) = \Delta G_{f_v}(0) - \sigma\Omega$$

$$C_{vo} = C \bullet \exp - \frac{\Delta G_{f_v}(0)}{kT}$$

$$C_v(\sigma) = C \exp - \left[\frac{\Delta G_{f_v}(0) - \sigma\Omega}{kT} \right]$$

Gradient in Composition



- Path Length:
  - Boundary: 2xd/4
  - Lattice: $(\pi/2)x(d/4)$
- Concentration Difference

$$C_o \exp-\left(\frac{\Delta G_{fv} - \sigma\Omega}{kT}\right)$$

$$-C_o \exp-\left(\frac{\Delta G_{fv} + \sigma\Omega}{kT}\right)$$

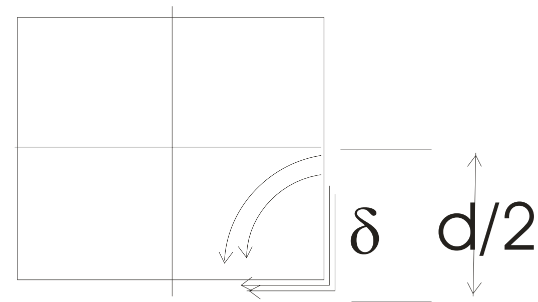
$$C_o \exp-\left(\frac{\Delta G_{fv}}{kT}\right)\left(\exp\left(\frac{\sigma\Omega}{kT}\right) - \exp\left(\frac{-\sigma\Omega}{kT}\right)\right)$$

$$\Delta C = 2C_o \exp-\left(\frac{\Delta G_{fv}}{kT}\right)\bullet\frac{\sigma\Omega}{kT}$$

- Quasi-static Approx.

Fick's 1st Law

$$J_{path} = -D_{path} \frac{\Delta C}{\Delta L_{path}} \qquad i.)$$



Total flux =ΣFlux on each Path:

$$\Phi_{vac\ flux} = J_L \cdot \frac{d}{2} \cdot l + J_B \cdot \delta \cdot l$$

$$J_{total} \triangleq total\ ave.\ flux = \frac{\Phi_{vac\ flux}}{d/2 \cdot l} = J_L + \frac{2\delta}{d} J_B \qquad ii.)$$

Plugging ΔC and ΔL into i.) and inserting fluxes in ii.):

$$J_{total} = -D_L\, 2C_o \exp-\left(\frac{\Delta G_{fv}}{kT}\right) \cdot \frac{\sigma\Omega}{kT} \frac{1}{\pi d/8} + \frac{2\delta}{d}\left(-D_b\right) \frac{2C_o \exp-\left(\frac{\Delta G_{fv}}{kT}\right) \bullet \frac{\sigma\Omega}{kT}}{d/2}$$

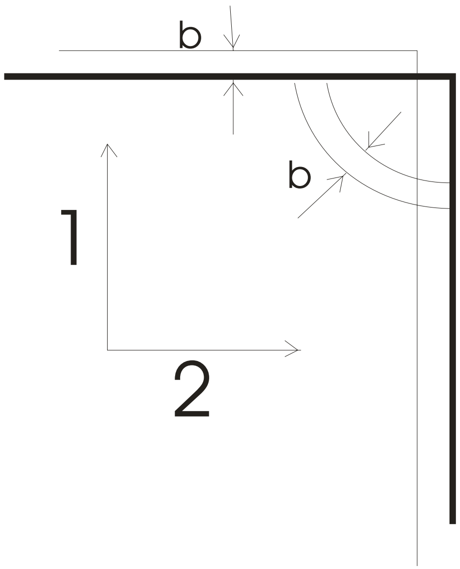
Total Flux on Both Paths

-

$$J_{Total} = \frac{8}{\pi d} D_L C_{vo} \exp - \frac{\Delta G_{vf}}{kt} \left\{ \frac{\sigma\Omega}{kT} \right\} + \frac{2\delta}{d} \frac{2 D_B C_{vo}}{d} \left(\exp - \frac{\Delta G_{vf}}{kt} \right) 2 \left\{ \frac{\sigma\Omega}{kT} \right\}$$

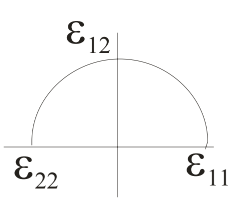
$$= \frac{16}{\pi d} \left[D_L C_{vo} \exp - \frac{\Delta G_{vf}}{kt} \right] \left\{ \frac{\sigma\Omega}{kT} \right\} \left\{ 1 + \frac{\pi\delta}{2d} \frac{D_B}{D_L} \right\}$$

Converting flux to strain



- Each vacancy that travels through the channel adds layer of depth b

$$\frac{\Delta l}{l \cdot t} = \frac{\# vacs}{s}\frac{b}{d} = 2 \cdot J_{total} \cdot b^2 \cdot \frac{b}{d} = \frac{2J_{total}\Omega}{d}$$



- $$\boxed{\dot{\varepsilon} = \frac{32}{\pi d^2}\frac{\sigma\Omega}{kT}D_L\left(1 + \frac{\pi\delta}{2d}\frac{D_B}{D_V}\right)}$$

Summary: N-H and Coble Creep

- Diffusion Creep Constitutive Law:

$$\dot{\varepsilon} = \frac{32}{\pi d^2}\frac{\sigma\Omega}{kT} D_L \left(1 + \frac{\pi\delta}{2d}\frac{D_B}{D_V}\right)$$

- $D_L = D_{VM} C_V$
- Strain rate linear in stress
- $\dot{\varepsilon} \propto 1/d^{2,3}$
- Other geometries change initial constant