5.5 Uncertainty in Lagrange Multiplier Method

When using the Lagrange multiplier approach, the issue of estimating the uncertainty of the solution remains, even if the system is linear. One approach is to calculate it from the covariance evolution equation of the filter/smoother. When one wishes to avoid that computational load, some limited information about it can be obtained from the Hessian of the cost function at the solution point.[170]

Understanding of the Hessian is central to quadratic norm optimization problems in general. Let $\boldsymbol{\xi}$ represent all of the variables being optimized, including $\mathbf{x}(t)$, $\mathbf{u}(t)$ for all t. Let $\boldsymbol{\xi}^*$ be the optimal value that is sought. Then, as with the static problems of Chapter 2, if we are close enough to ξ^* in the search process, the objective function is locally,

$$J = \text{constant} + (\boldsymbol{\xi} - \boldsymbol{\xi}^*)^T \mathcal{H}(\boldsymbol{\xi} - \boldsymbol{\xi}^*) + \Delta J$$

where $\mathcal{H}$ is the Hessian and ΔJ is a higher-order correction. The discussion of the behavior of the solution in the vicinity of the estimated optimal value proceeds then, exactly as before, with row and column scaling being relevant, and issues of ill-conditioning, solution variances, etc., all depending upon the eigenvalues and eigenvectors of $\mathcal{H}$.[171]

The only problem, albeit a difficult one, is that the dimensions of $\mathcal{H}$ are square of the dimensions of $\mathbf{x}(t)$ plus $\mathbf{u}(t)$ over the entire time history of the model and data. Finding ways to understand the solution structure and uncertainty with realistic fluids and large-scale datasets remains as one of the most important immediate challenges. (See the applications in Chapter 7.)