3.6 Kriging and Other Variants of Gauss-Markov Estimation

A variant of the Gauss-Markov mapping estimators, often known as "kriging" (named for David Krige, a mining geologist), addresses the problem of a spatially varying mean field. and is a generalization of the ordinary Gauss-Markov estimator.[89]

Consider the discussion on P. 134 of the fitting of a set of functions $f_i(\mathbf{r})$ to an observed field $y(\mathbf{r}_j)$. That is, we put,

{eq:54001} $$y(\mathbf{r}_j) = \mathbf{F}\boldsymbol{\alpha} + q(\mathbf{r}_j), \tag{3.55}$$

where $\mathbf{F}(\mathbf{r}) = \{f_i(\mathbf{r})\}$ is a set of basis functions, and one seeks the expansion coefficients, $\boldsymbol{\alpha}$, and q such that the data, y, are *interpolated* (meaning reproduced exactly) at the observation points, although there is nothing to prevent further breaking up q into signal and noise components. If there is only one basis function—for example a constant—one is doing kriging, which is the determination of the mean prior to objective mapping of q, as discussed in Chapter 3. If several basis functions are being used, one has "universal kriging." The main issue concerns the production of an adequate statement of the expected error, given that the q are computed from a preliminary regression to determine the $\boldsymbol{\alpha}$.[90]. The method is often used in situations where large-scale trends are expected in the data, and where one wishes to estimate and remove them before analyzing and mapping the q.

Because the covariances employed in objective mapping are simple to use and interpret only when the field is spatially stationary, much of the discussion of kriging uses instead what is called the "$v$ariogram," defined as $V = \langle \big(y(\mathbf{r}_i) - y(\mathbf{r}_j)\big)\big(y(\mathbf{r}_i) - y(\mathbf{r}_j)\big)\rangle$, which is related to the covariance, and which is often encountered in turbulence theory as the "structure function." Kriging is popular in geology and hydrology, and deserves wider use.