1.4 Importance of the Forward Model

Inference about the physical world from data requires assertions about the structure of the data and its internal relationships. One sometimes hears claims from people who are expert in measurements that "I don't use models." Such a claim is almost always vacuous. What the speaker usually means is that he doesn't use equations, but is manipulating his data in some simple way (e.g., forming an average) that seems to be so unsophisticated that no model is present. Consider, however, a simple problem faced by someone trying to determine the average temperature in a room. A thermometer is successively placed at different three-dimensional locations, $\mathbf{r}_i$, at times t_i. Let the measurements be y_i and the value of interest is,

$$\tilde{m} = \frac{1}{M}\sum_{i=1}^{M} y_i. \tag{1.35}$$ {mean1}

In deciding to compute, and use $\tilde{m}$, the observer has probably made a long list of very sophisticated, but implicit, model assumptions. Among them we might suggest: (1) Thermometers require assumptions about the quantity recorded (e.g., an oscillator frequency or a voltage) and the connection to the desired temperature as well as potentially elaborate calibration means. (2) That the temperature in the room is sufficiently slowly changing that all of the t_i can be regarded as effectively identical. A different observer might suggest that the temperature in the room is governed by shock waves bouncing between the walls at intervals of seconds or less.

Should that be true, $\tilde{m}$ constructed from the available samples might prove completely meaningless. It might be objected that such an hypothesis is far-fetched. But the assumption that the room temperature is governed, e.g., by a slowly evolving diffusion process, is a rigid, and perhaps incorrect model. (3) That the errors in the thermometer are such that the best estimate of the room mean temperature is obtained by the simple sum in Eq. (1.35). There are many measurement devices for which this assumption is a very poor one (perhaps the instrument is drifting, or has a calibration that varies with temperature), and we will discuss how to determine averages in Chapter 2. But the assumption that property $\tilde{m}$ is useful, is a strong model assumption concerning both the instrument being used and the physical process it is measuring.

This list can be extended (the interpretation of the mean is itself model-dependent), but more generally, the inverse problems listed earlier in this chapter only make sense to the degree that the underlying forward model is likely to be an adequate physical description of the observations. For example, if one is attempting to determine ρ in Eq. (1.15) by taking the Laplacian $\nabla^2\phi$, (analytically or numerically), this solution to the inverse problem is only sensible if this equation really represents the correct governing physics. If the correct equation to use were, instead,

$$\frac{\partial^2\phi}{\partial r_x^2} + \frac{1}{2}\frac{\partial\phi}{\partial r_y} = \rho, \tag{1.36}$$

where r_y is another coordinate, the calculated value of ρ would be incorrect. One might, however, have good reason to use Eq. (1.15) as the most likely hypothesis, but nonetheless remain open to the possibility that it is not an adequate descriptor of the required field, ρ. A good methodology, of the type we will develop in subsequent chapters, permits one to ask the question: is my model consistent with the data? If the answer to the question is "yes," a careful investigator would *never* claim that the resulting answer is the correct one and that the model has been "validated" or "verified." One claims only that the answer and the model are consistent with the observations, and remains open to the possibility that some new piece of information will be obtained that completely *invalidates* the model (e.g., some direct measurements of ρ showing that the inferred value is simply wrong). One can never validate or verify a model, one can only show consistency with existing observations.[8]

Notes

[1]See Lanczos (1961, Section 3.19)

[2]Whittaker and Robinson (1944)

[3]Lanczos (1961) has a much fuller discussion of this correspondence.

[4]Herman (1980).

[5]Herman (1980); Munk et al. (1995).

[6]Oceanographers will recognize this apparently highly artificicial problem as being a slightly simplified version of the so-called geostrophic inverse problem, and which is of great practical importance. It is a central subject in Chapter 5.

[7]Aki and Richards (1980). A famous two-dimensional version of the problem is described by Kac (1966); see also Gordon and Webb (1996).

[8]Oreskes et al. (1994).