Lecture 31 Nov 24, 03 (1)

Compressible TSL

A) Reynolds Analogy

B) Approximate Temperature Profile.

Last time we examined energy eqn:

$$\rho\left[u\frac{\partial h_o}{\partial x} + v\frac{\partial h_o}{\partial y}\right] = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial h}{\partial y}\right] + \frac{\partial}{\partial y}\left[\mu\left(1-\frac{1}{Pr}\right)u\frac{\partial u}{\partial y}\right]$$

ht cond — viscous disip (ht production)

$$Pr = \frac{\mu C_p}{k}$$

Examined special solutions for $Pr=1$: $h_o(y)=$ const, $\frac{dp}{dx}=0 \Rightarrow h_o(y)=Au+B$.
(balanced heat cond. & viscous disip

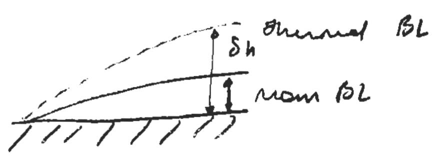
A) For near unity Pr (example air $Pr \approx 0.72$)

- momentum BL thickness $= \delta = \delta_m$
- enthalpy BL thickness $= \delta_h$

Interp of $Pr = \dfrac{\text{mom. diffusivity}}{\text{enthalpy diffusivity}}$

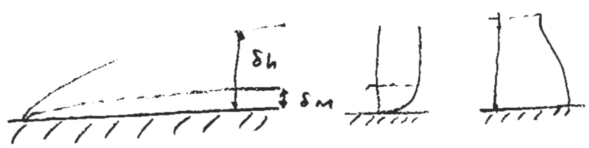
$Pr > 1$ $\quad \delta_m > \delta_h$
$Pr = 1$ $\quad \delta_m = \delta_h$
$Pr < 1$ $\quad \delta_m < \delta_h$

Gases:



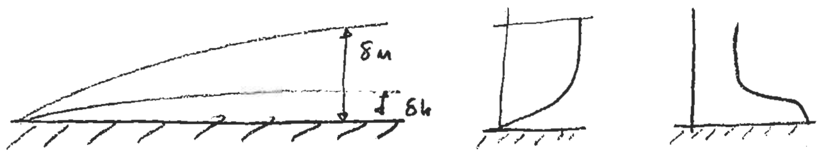
$Pr < 1$

liquid metals:



$Pr < 1$ (mercury 0.024)

θ is



$Pr > 1$

SAE 30 $Pr \approx 3500$.

for $Pr = 1$, $\frac{dp}{dx} = 0$

$$h_o = h_{o_w} + (h_{o_e} - h_{ow}) \frac{u}{U_e}$$

temp. profile given by

$$T = T_w + \left(T_e + \frac{U_e^2}{2C_p} - T_w\right)\frac{u}{U_e} - \frac{u^2}{2C_p}$$

If the wall is adiabatic $h_o = h_{ow} = \text{const}$

$$\therefore \quad T_e + \frac{U_e^2}{2C_p} \equiv T_{aw} \equiv \text{adiabatic wall temp}$$

$$\therefore \quad T = T_w + (T_{aw} - T_w)\, u/U_e - u^2/2C_p.$$

The wall ht flux is

$$q_w = k_w \left.\frac{\partial T}{\partial y}\right|_{wall} \quad \Big/ \quad \frac{\partial T}{\partial y} = \ldots$$

$$\therefore \quad q_w = \frac{(T_{aw} - T_w)\, k_w \tau_w}{U_e \mu_w}$$

$$\text{or} \quad C_h = \frac{q_w}{\rho_e U_e C_p (T_{aw} - T_w)} = \frac{C_f}{2 Pr}$$

- relates wall heat flux to the skin friction
- Reynolds analogy (effect called)
- Strictly valid for $Pr = 1$ $\Rightarrow$ $C_h = C_f/2$ (Stanton Number) (wall ht transf coeff

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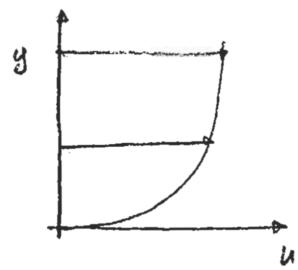
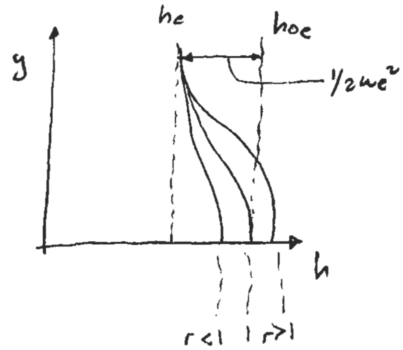
B) If $P_r \neq$ but close to unity as for gases; we can develop approximate wall temperature profile. which depends on P_r number.





Define temp recovery factor $r = f(P_r)$ where $f(1) = 1$

$$r \equiv \frac{T_{aw} - T_e}{T_{oe} - T_e}$$ — deviation from exact $P_r = 1$ where $h_o(y) = \text{const}$

Very good approximation for r

$$r \approx P_r^{1/2} \quad \text{— laminar}$$
$$\approx P_r^{1/3} \quad \text{— turbulent.}$$

for air $\quad r \approx 0.85$ laminar
$\quad\quad\quad \approx 0.9$ turbulent.

Adiabatic wall:

$$h_{oa} = h_{ow}\Big|_{ad} = h_e + 1/2\, r\, u_e^2 \qquad (q_w = 0)$$

Non adiabatic: $\quad h_{ow}$ prescribed

Note that $h_o(y)$ matches exact profile only at $y=0$ & $y=\delta$ for $P_r \neq 1$

Given some $u(y)$ we can obtain density profile which affects integral thicknesses δ^*, θ, etc.

Going back to Reynolds analogy.

$$C_h = \frac{C_f}{2 Pr}$$

In general,

$$C_h = C_f/2 \cdot f(Pr)$$

empirical data shows

$$C_h \approx C_f/2 \cdot \frac{1}{Pr^{2/3}} \quad \text{(laminar and turbulent)}$$

c) Integral BL Eqns:

$$\rho/\rho_e \neq 1$$

Compressible VKI eqn:

$$\frac{d\theta}{dx} + (H + 2 - M_e^2)\frac{\theta}{u_e}\frac{du_e}{dx} = C_f/2$$

Compressible K.E eqn:

$$\frac{1}{H^*}\frac{dH^*}{dx} = \frac{1}{\theta}\left(\frac{2C_D}{H^*} - C_f/2\right) - \left(\frac{2H^{**}}{H^*} + 1 - H\right)\frac{1}{u_e}\frac{du_e}{dx}$$

$$\delta^* \equiv \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \qquad \theta \equiv \int_0^\infty \left(1 - \frac{u}{u_e}\right)\frac{\rho u}{\rho_e u_e}\, dy.$$

$$\theta^* \equiv \int_0^\infty \left(1 - \left(\frac{u}{u_e}\right)^2\right)\frac{\rho u}{\rho_e u_e}\, dy \qquad \delta^{**} \equiv \int_0^\infty \left(1 - \rho/\rho_e\right)\frac{u}{u_e}\, dy \qquad O(M_e^2)$$

check

for adiab. flow

Note that $\qquad \frac{u_e}{\rho_e}\frac{d\rho_e}{du_e} \equiv - M_e^2$ (adiabatic freestream)

Shape parameter

$$H = \delta^* / \theta \qquad \text{(compressible)}$$

$$H^* = \theta^* / \theta$$

$$H^{**} = \delta^{**} / \theta \qquad \text{(density } \text{shape par} \text{)}$$

check

Compressible H and incompressible H_k are related (for adiab flow in air)

$$H_k = f(H, M_e^2)$$

$$H^{**} = f(H, M_e^2) \qquad \text{(Whitfield)}$$

See Drela AIAA paper.

Usual correlations apply with correction for density profile