from: GTL Report 187

for similar flows.

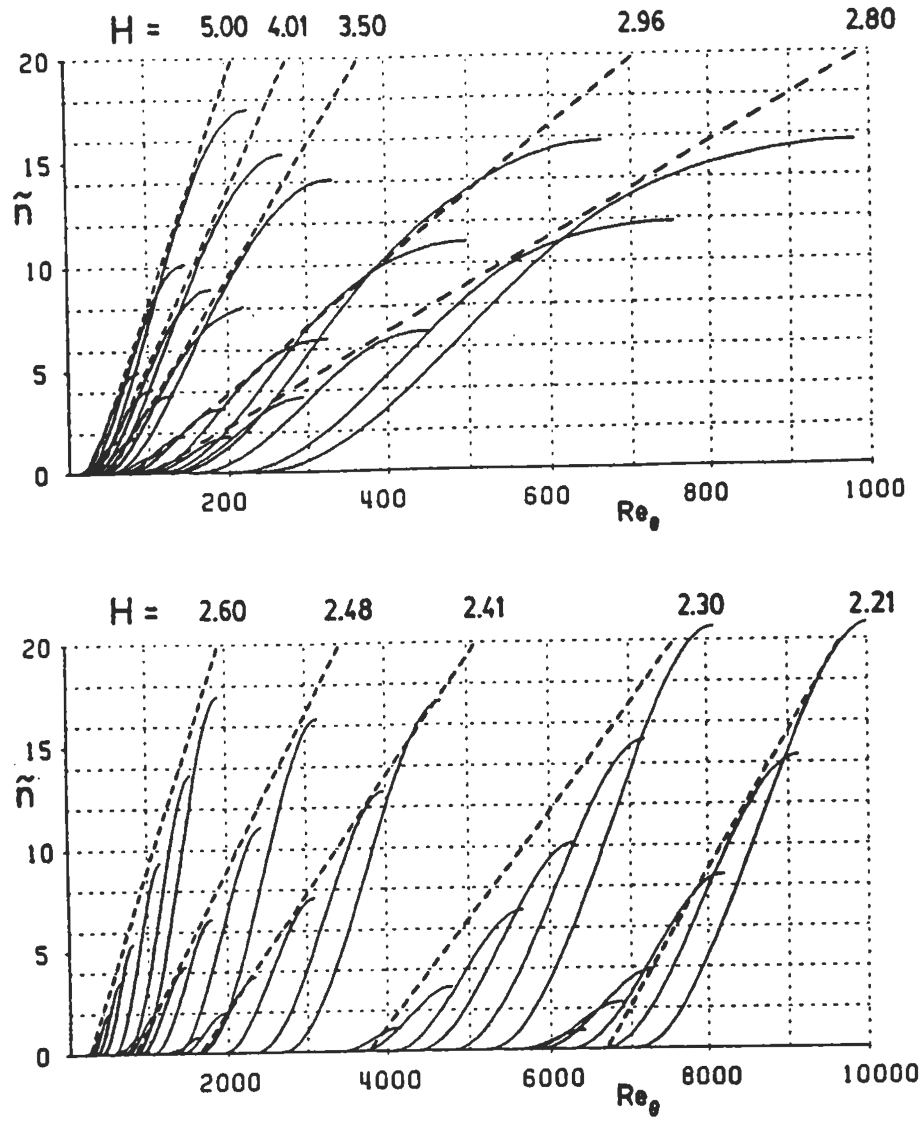


Figure 6.7 Orr-Sommerfeld spatial amplification curves

$$\ln(A/A_0) \equiv \tilde{n} = \frac{d\tilde{n}}{dRe_\theta}(H)\ \{Re_\theta - Re_{\theta_0}(H)\} \tag{6.41}$$

where the slope $d\tilde{n}/dRe_\theta$ and the critical Reynolds number Re_{θ_0} are given by

$$\frac{d\tilde{n}}{dRe_\theta} = 0.01\left(\left(2.4H - 3.7 + 2.5\tanh[1.5(H-3.1)]\right)^2 + 0.25\right)^{\frac{1}{2}} \tag{6.42}$$

$$\log_{10} Re_{\theta_0} = \left(\frac{1.415}{H-1} - .489\right)\tanh\left(\frac{20.}{H-1} - 12.9\right) + \frac{3.295}{H-1} + .440 \tag{6.43}$$

Figure 6.7 shows the envelopes defined by equations (6.41-43) together with the actual amplification curves calculated from the Orr-Sommerfeld equation.

$$\frac{d\tilde{n}}{dRe_\theta} = 0.01\left(\left(2.4H - 3.7 + 2.5\tanh[1.5(H-3.1)]\right)^2 + 0.25\right)^{\frac{1}{2}} \qquad (6.42)$$

$$\log_{10}Re_{\theta_0} = \left(\frac{1.415}{H-1} - .489\right)\tanh\left(\frac{20.}{H-1} - 12.9\right) + \frac{3.295}{H-1} + .440 \qquad (6.43)$$

Figure 6.7 shows the envelopes defined by equations (6.41-43) together with the actual amplification curves calculated from the Orr-Sommerfeld equation.

$$\frac{d\tilde{n}}{d\xi} = \frac{d\tilde{n}}{dRe_\theta}\frac{dRe_\theta}{d\xi} = \frac{d\tilde{n}}{dRe_\theta}\frac{1}{2}\left(\frac{\xi}{u_e}\frac{du_e}{d\xi} + 1\right)\frac{\rho_e u_e \theta^2}{\mu_e \xi}\frac{1}{\theta} \qquad (6.44)$$

Using the empirical relations

$$\frac{\rho_e u_e \theta^2}{\mu_e \xi} \equiv \ell(H) = (6.54H - 14.07)/H^2 \qquad (6.45)$$

and

$$\frac{\xi}{u_e}\frac{du_e}{d\xi} \equiv m(H) = \left(0.058\frac{(H-4)^2}{H-1} - 0.068\right)\frac{1}{\ell(H)} , \qquad (6.46)$$

the spatial amplification rate is expressed as a function of H and θ.

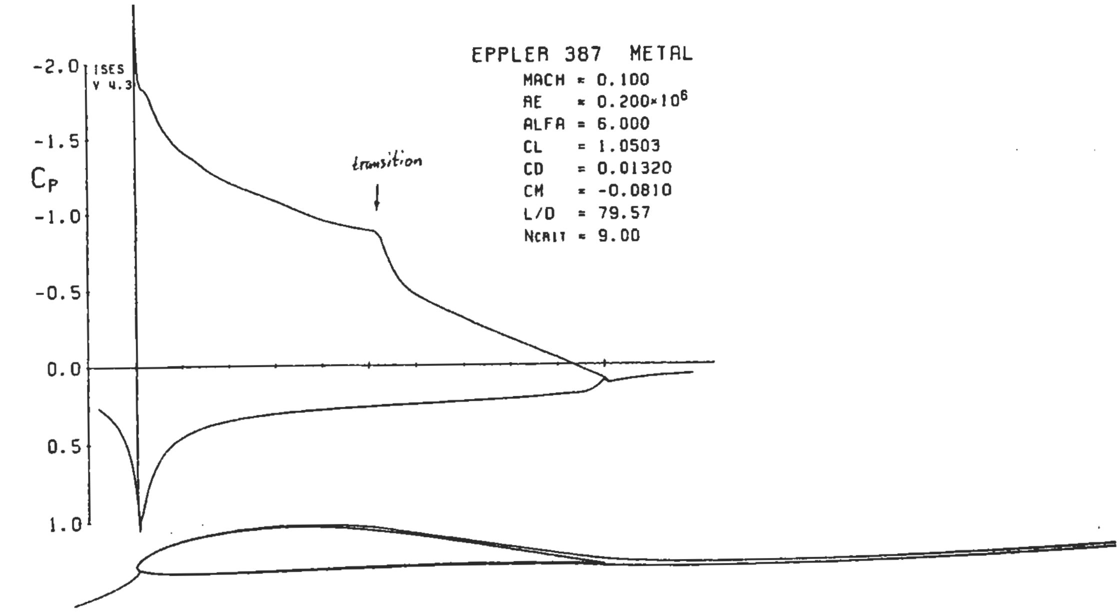
$$\frac{d\tilde{n}}{d\xi}(H,\theta) = \frac{d\tilde{n}}{dRe_\theta}(H)\,\frac{m(H)+1}{2}\,\ell(H)\,\frac{1}{\theta} \qquad (6.47)$$

This amplification rate can then be integrated downstream from the instability point ξ_{cr} (where $Re_\theta = Re_{\theta_0}$).

$$\tilde{n}(\xi) = \int_{\xi_{cr}}^{\xi}\frac{d\tilde{n}}{d\xi}\,d\xi \qquad (6.48)$$

Again, the onset of transition occurs at the point where $\tilde{n} = 9$.

In the actual implementation of the present transition criterion, equation (6.48) is not used directly. Instead, the differential amplification equation (6.47) is discretized and solved as part of the global Newton system. The transition onset location can thus be properly linearized. This is a much more robust procedure than if the integral equation (6.48) was used to explicitly set the transition onset location every iteration.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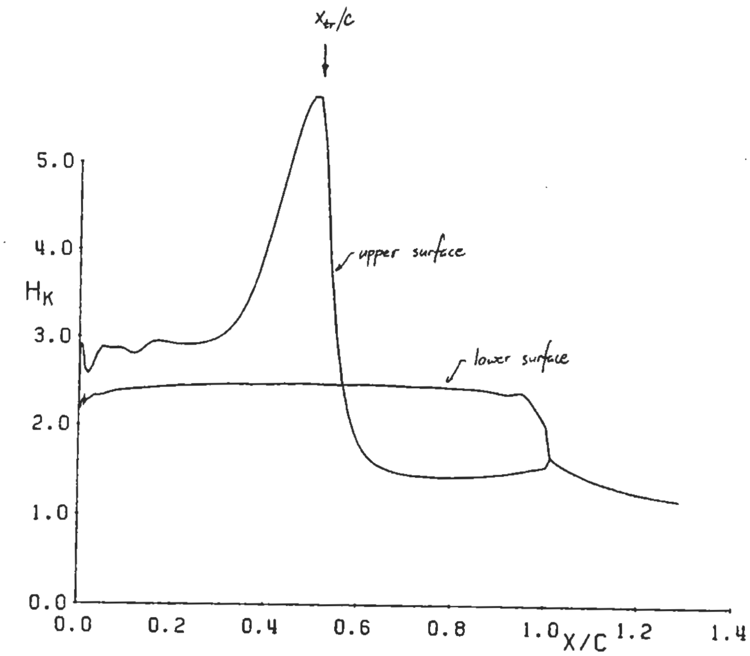
RE = 0.200×10^6
ALFA = 6.000
CL = 1.0503
CD = 0.01320
CM = -0.0810
L/D = 79.57
NCRIT = 9.00
ISES V 4.3
transition
CP
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0


Xtr/c
upper surface
lower surface
HK
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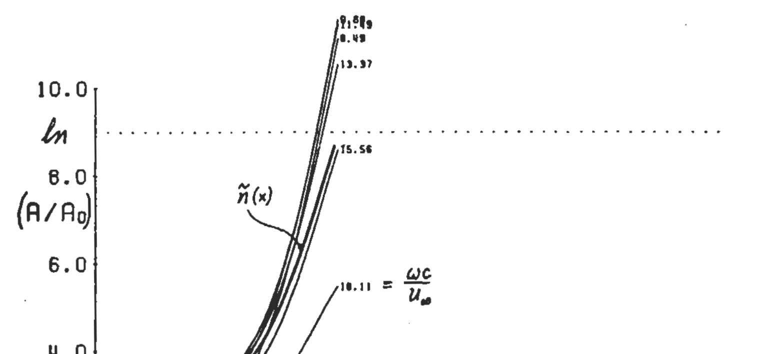
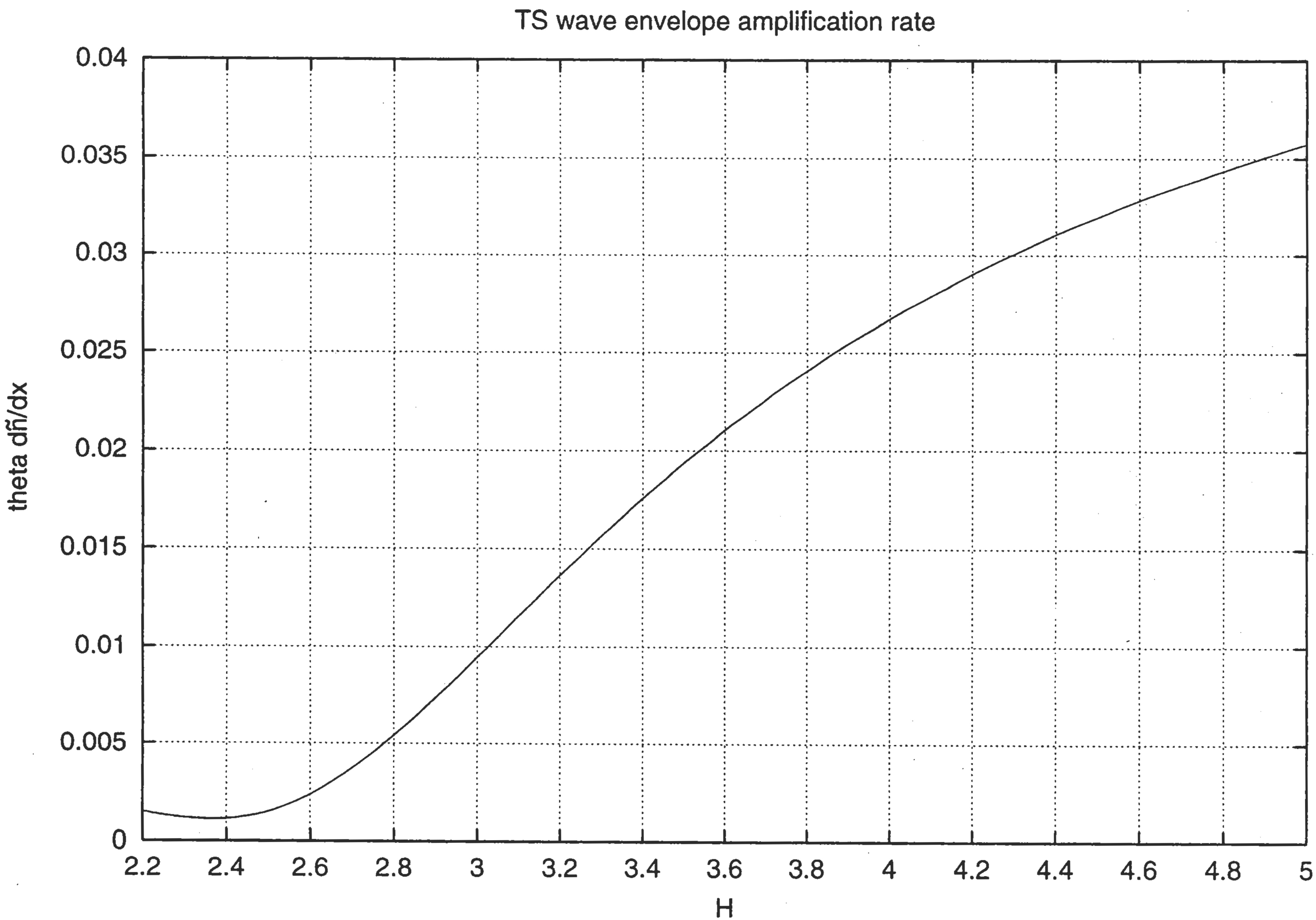
4.0
3.0
2.0
1.0
0.0
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
X/C


10.0
8.0
6.0
4.0
2.0
0.0
ln (A/A0)
ñ(x)
n(x;ω)
18.11 = ωc/u∞
13.37
15.56
21.07
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
X/C


TS wave envelope amplification rate
theta dñ/dx
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
4.8
5
H