FLOW AT LOW ROSSBY NUMBER

- Strongly rotating flows, i.e., flows at low Rossby number [$w/(\Omega L)<<1$], exhibit behavior that is strikingly counter-intuitive

- This remarkable behavior can be illuminated by thinking in terms of the vorticity equation, written in the relative system as,

$$\frac{D\vec{\omega}_{rel}}{Dt} = (\vec{\omega}_{rel} \cdot \nabla)\vec{w} - (2\vec{\Omega} \cdot \nabla)\vec{w}$$ Eq. (7.5.3) for inviscid flow

- The axis of rotation is along the z-direction. The vorticity equation thus reduces to

$$\frac{D\vec{\omega}_{rel}}{Dt} = (\vec{\omega}_{rel} \cdot \nabla)\vec{w} - 2\Omega\frac{\partial \vec{w}}{\partial z}$$ (7.5.4)

- The sizes of the terms in the equation can be characterized as

w^2/L^2 w^2/L^2 $w\Omega/L$

- Multiplying through by L/(wΩ), the relative magnitudes of the three terms are

  Ro Ro 1

- For Ro << 1 Eq. (7.5.4) becomes

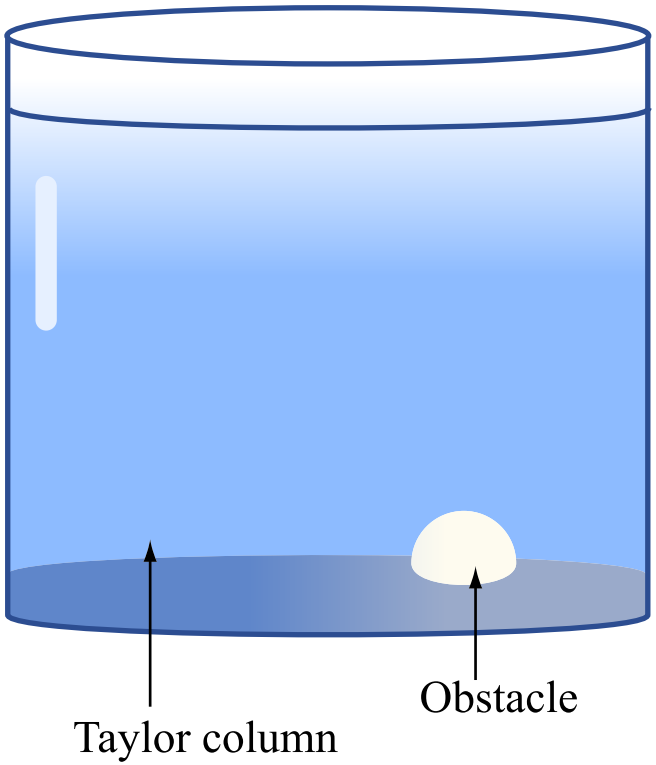
$$2\Omega\frac{\partial\vec{w}}{\partial z}\approx 0 \qquad (7.5.6)$$

- The content of Eq. (7.5.6) is that the relative velocity field is two-dimensional -- the relative velocity does not vary along the direction of the rotation axis

- **Thus: Slow steady motions of a strongly rotating inviscid flow are two-dimensional** -- ***This is an extraordinary conclusion indeed!***
  - The absolute vortex tubes tend to remain parallel to the axial direction, resisting bending or stretching
  - This result is known as "The Taylor-Proudman theorem"
  - It was first shown experimentally by Sir G. I. Taylor

LOW ROSSBY NUMBER FLOW [Greenspan]

STEADY FLOW PAST A SMALL OBSTACLE



DYE LINES AT A LEVEL WELL ABOVE OBSTACLE

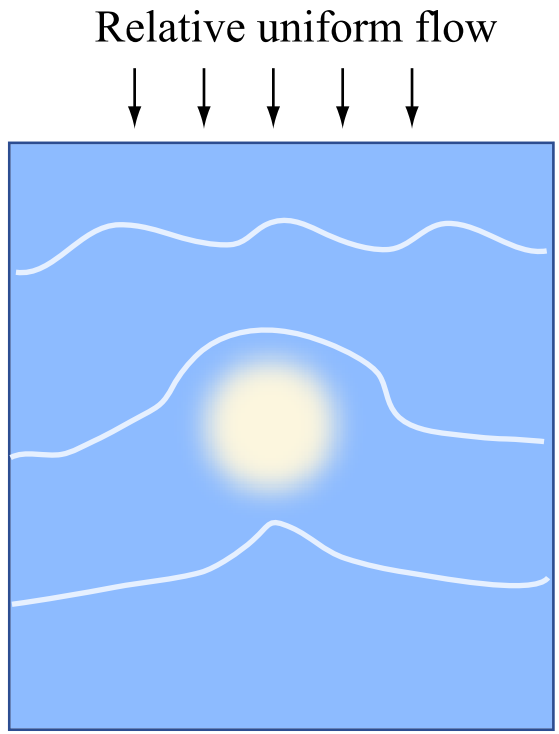
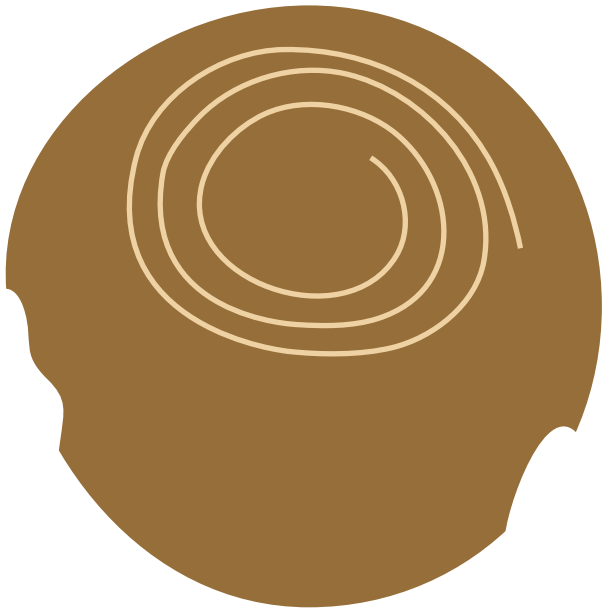


Figure by MIT OCW.

DROPS OF DYE ARE DRAWN OUT INTO TWO-DIMENSIONAL SHEETS IN A LOW ROSSBY NUMBER FLOW [Taylor]



Behavior of dye drops in a cylindrical rotating container

Behavior of dye drops in a rectangular rotating container

Figure by MIT OCW.

TAYLOR COLUMN ABOVE SPHERE MOVING PARALLEL TO THE ROTATION AXIS [*Illustrated Experiments in Fluid Mechanics* - Shapiro]

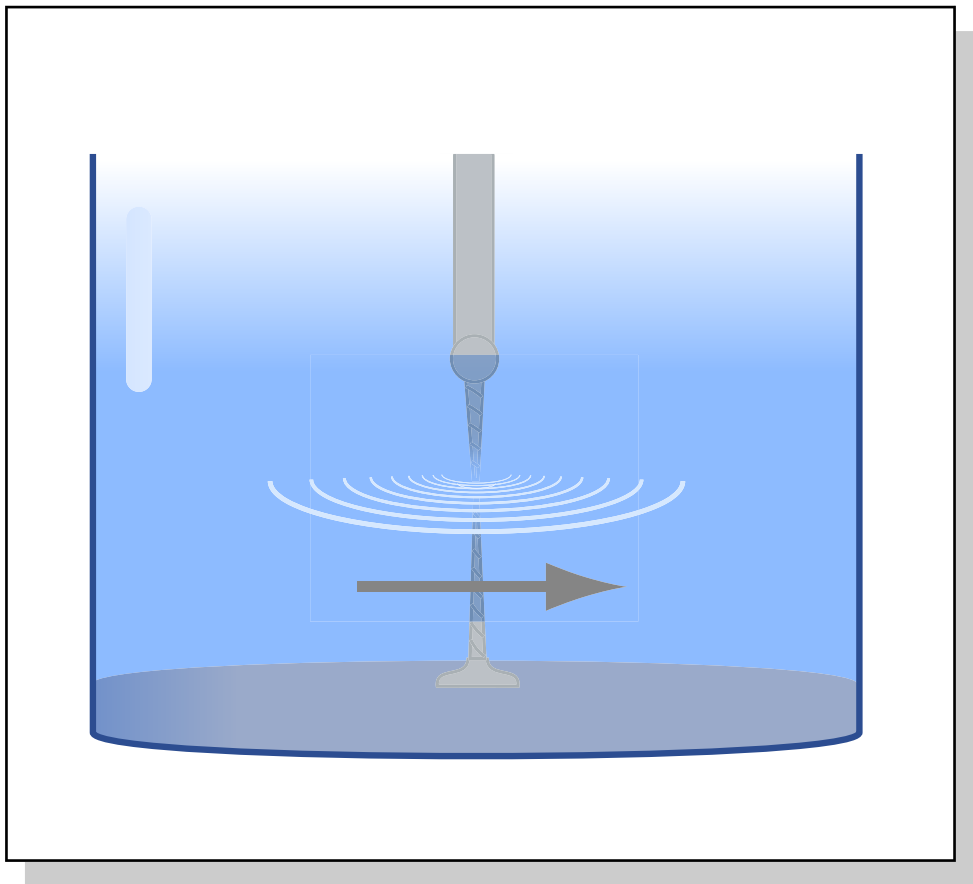


Figure by MIT OCW.

Taylor colums above and below the sphere. Vortex compression above, vortex stretching below (note helical motion showing strong counterclockwise spin)