16.540 Spring 2006

APPLICATIONS OF VORTICITY AND CIRCULATION IN DESCRIBING “REAL FLOWS”

MORE REMARKS ON VORTICITY

"There is no end to vorticity"

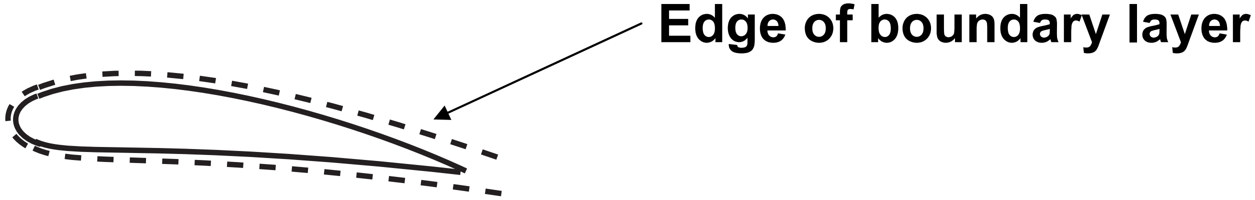
H. W. Liepmann

TRAILING VORTICITY

- What is it?
- Where does it come from
- What does it have to do with Mr. Crocco?
- Why do we care?

FLOW PAST A THREE-DIMENSIONAL BODY

- **For a real fluid the vorticity/circulation is associated with "boundary layers"**



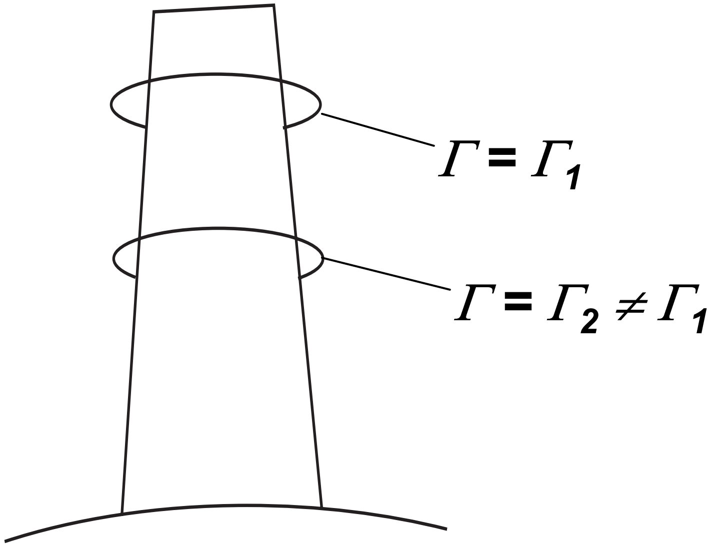
2-D First!

$\Gamma_{\textbf{contour right on body surface}}$ **= 0 (**$u_{\text{surface} = 0}$**)**

$\Gamma_{\textbf{contour just outside boundary layer}}$ **= circulation round the wing**

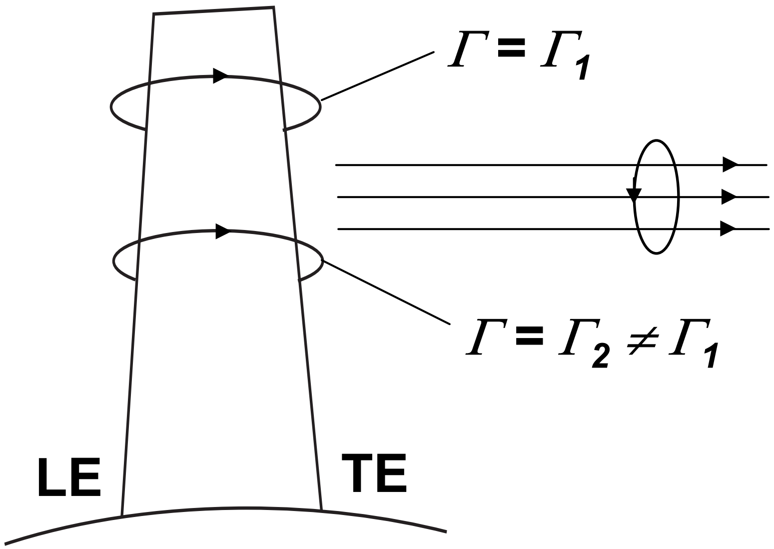
$\therefore$ **"Bound vorticity" is actually vorticity in fluid flowing over wing**

AIRFOIL WITH CIRCULATION VARYING ALONG SPAN



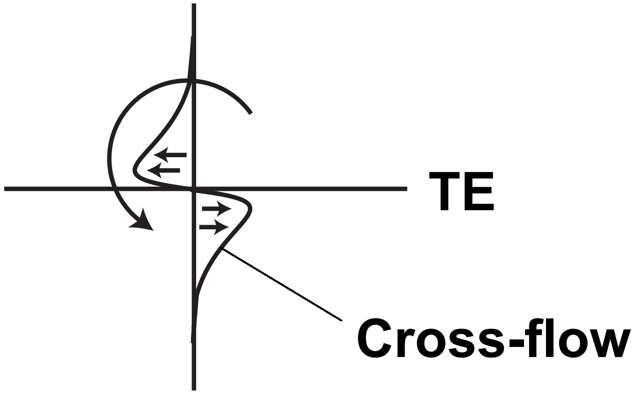
- **"All" vorticity is in boundary layers because vortex lines can't end in fluid**

- Γ **measures sum of all vortex lines threading contour**

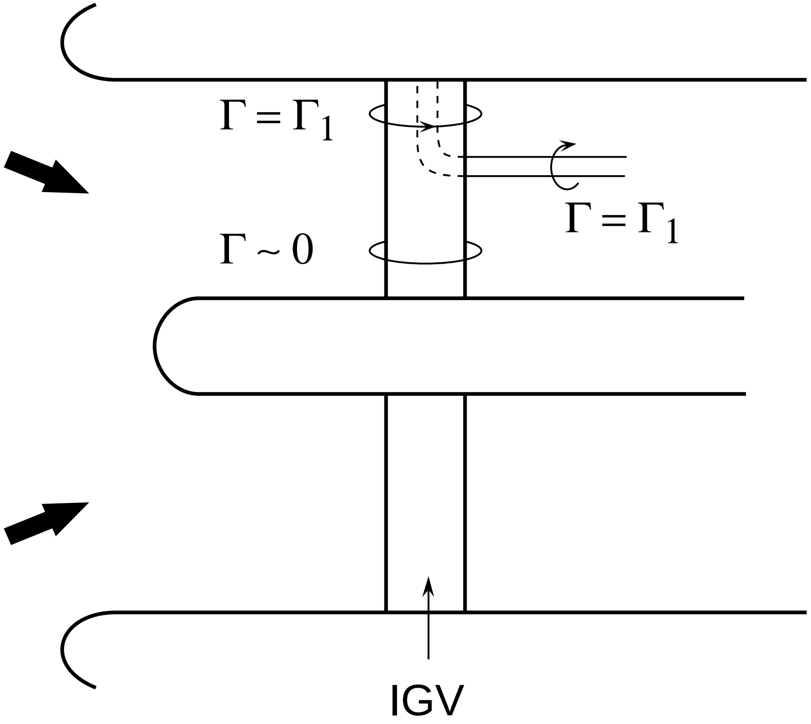


Vortex lines trailing off wing - must occur when circulation changes along the span

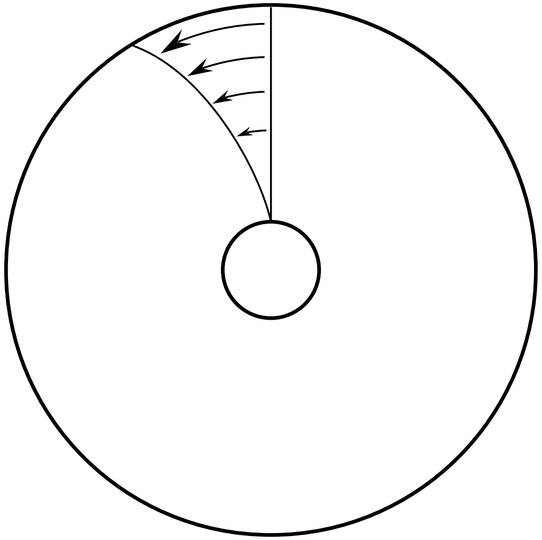
Downstream of trailing edge
Main stream going left to right



ROTATIONAL FLOW DOWNSTREAM OF IGV



Turbomachine Annulus and Inlet Guide Vane (IGV); Uniform Entropy and Stagnation Enthalpy



Rotational Swirl Flow Distribution Downstream of IGV

RECAP OF VORTICITY DYNAMICS AND KINEMATICS

Helmholtz's vortex laws - Inviscid, barotropic, fluid

1) Vortex lines never end in the fluid. The circulation all along vortex is the same for every contour enclosing the tube

Note: Kinematics, not dependent on fluid.

2) If fluid is barotropic, $\rho = \rho(p)$, conserv. $\bar{F}_{body}$, inviscid then vortex lines are material lines

A fluid line that once coincides with a vortex line does so forever

3) On a vortex line of fixed identity

$$\frac{\omega}{\rho \ell} = \text{constant}$$

A SIMPLE ESTIMATE FOR SECONDARY FLOW
The Secondary Flow Approximation

- **Consider a boundary layer vortex filament in a bend**
- **Flow outside boundary layer is irrotational**
- **Vortex filaments are convected by a "background flow" (vortex lines are material lines)**
- **Examine deformation of the vortex line as it convects through an bend of angle dθ**

DEFORMATION OF A FLUID CROSS

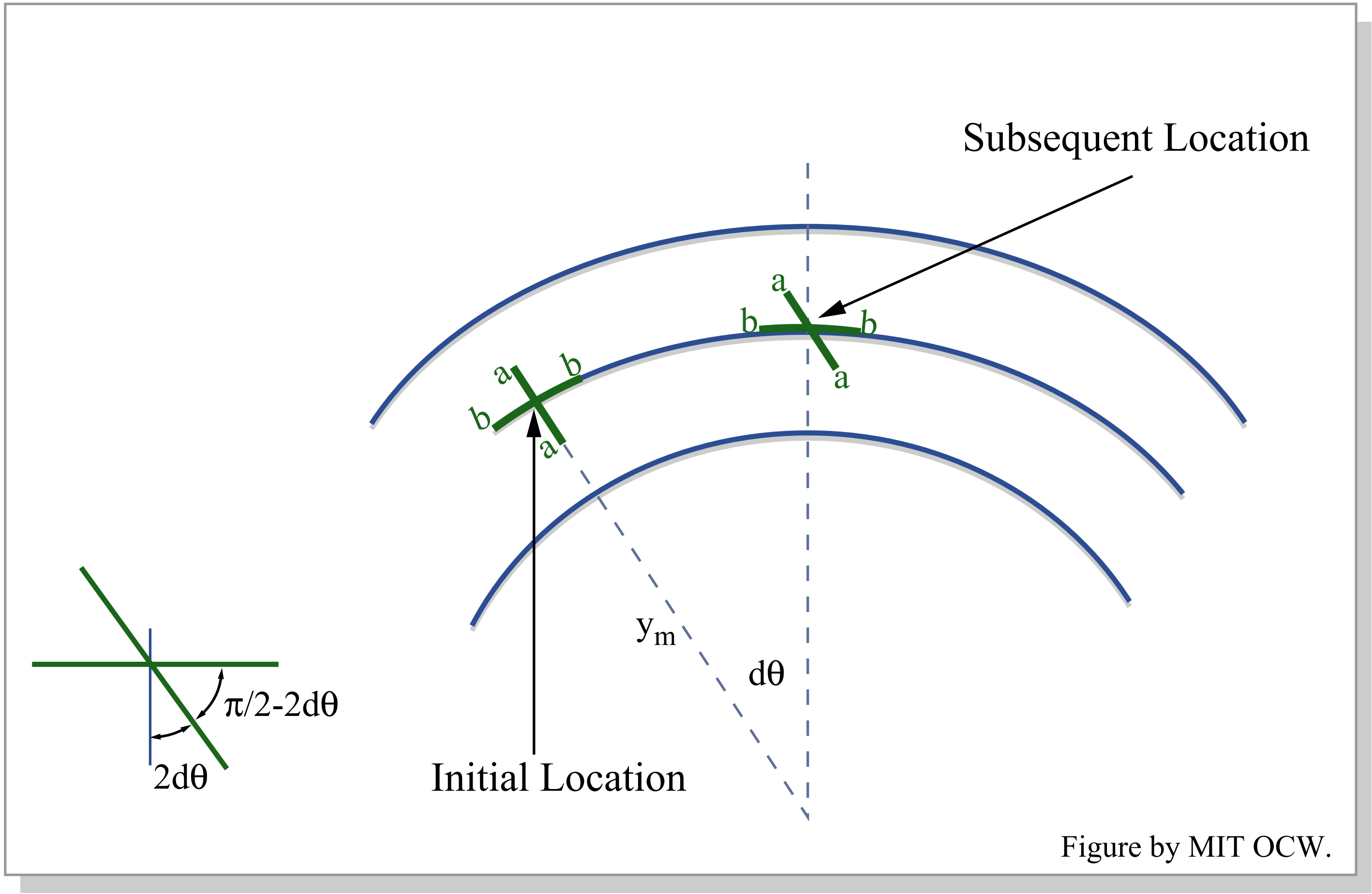


Figure by MIT OCW.

Normal leg swings by an angle $2d\theta$ into streamwise direction

Normal vorticity ω_n leads to a component $\omega_s = -\omega_n \sin(2d\theta) \approx -2\omega_n d\theta$

Squire-Winter formula for streamwise vorticity: $\boxed{\omega_s = -2\omega_n d\theta}$

SECONDARY FLOW IN A CURVED DUCT

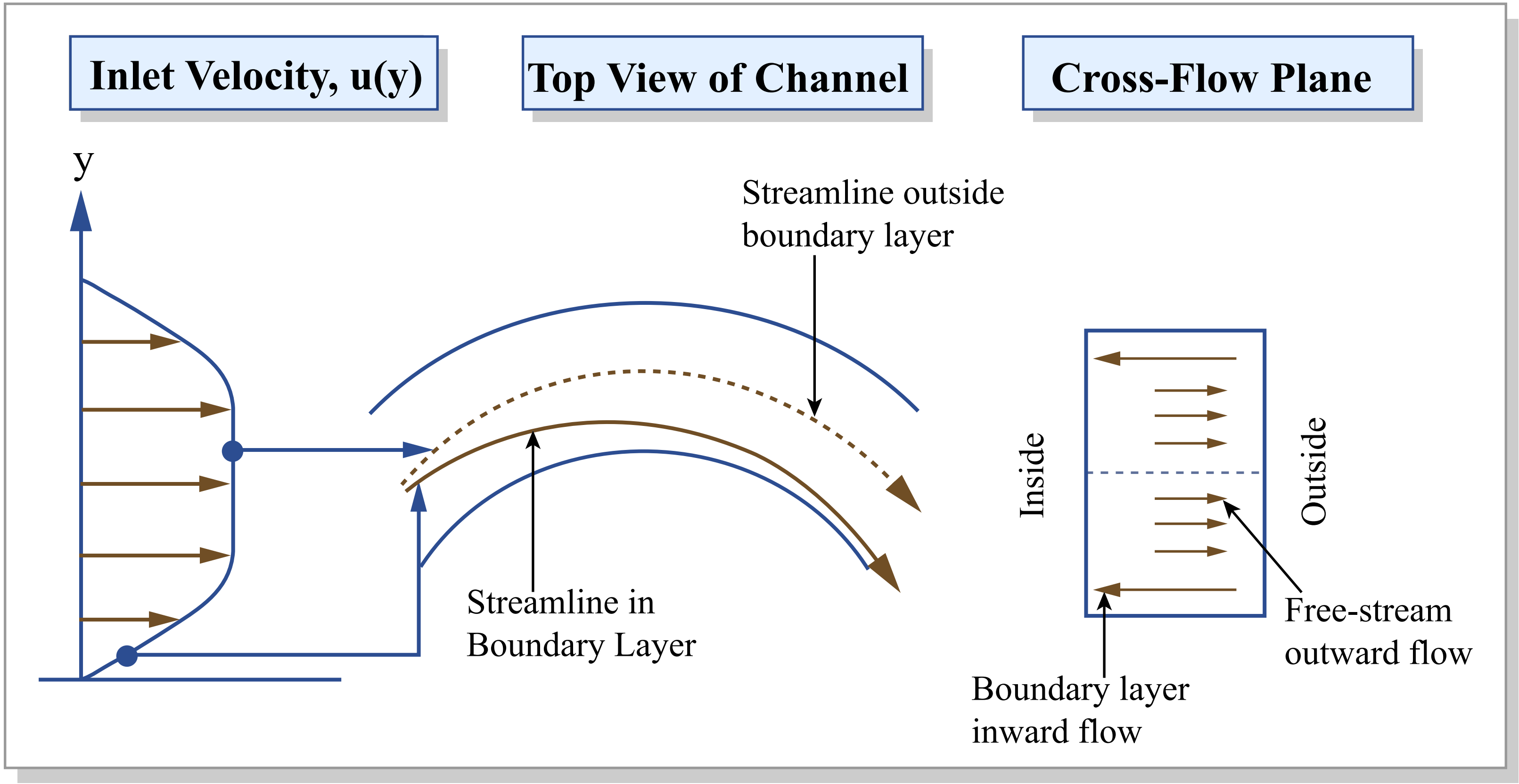
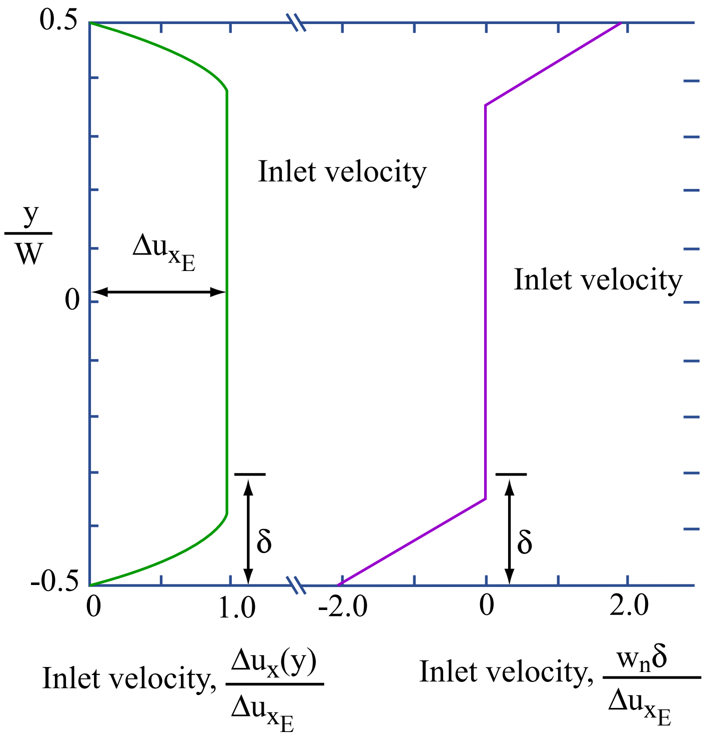


Figure by MIT OCW.

SECONDARY FLOW IN A CURVED DUCT -II

INLET CONDITIONS



CROSS-FLOW STREAMLINES

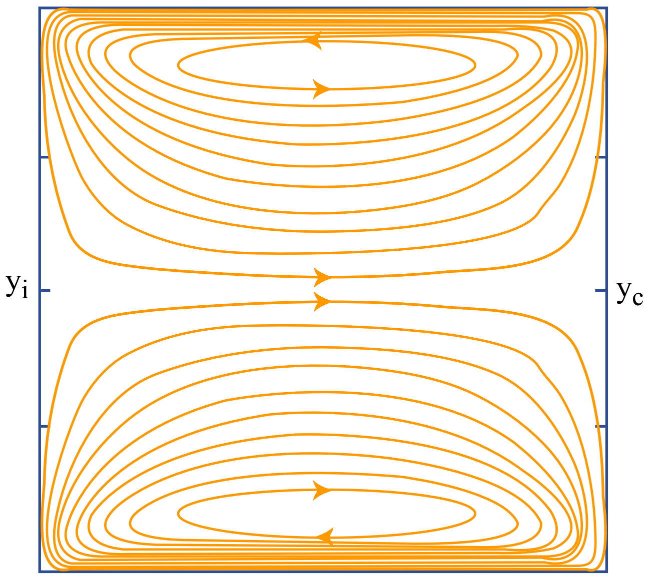


Figure by MIT OCW.

GENERATION OF NON-UNIFORMITY ON THE FLOOR OF A WIND TUNNEL CONTRACTION [Bradshaw, 1971]

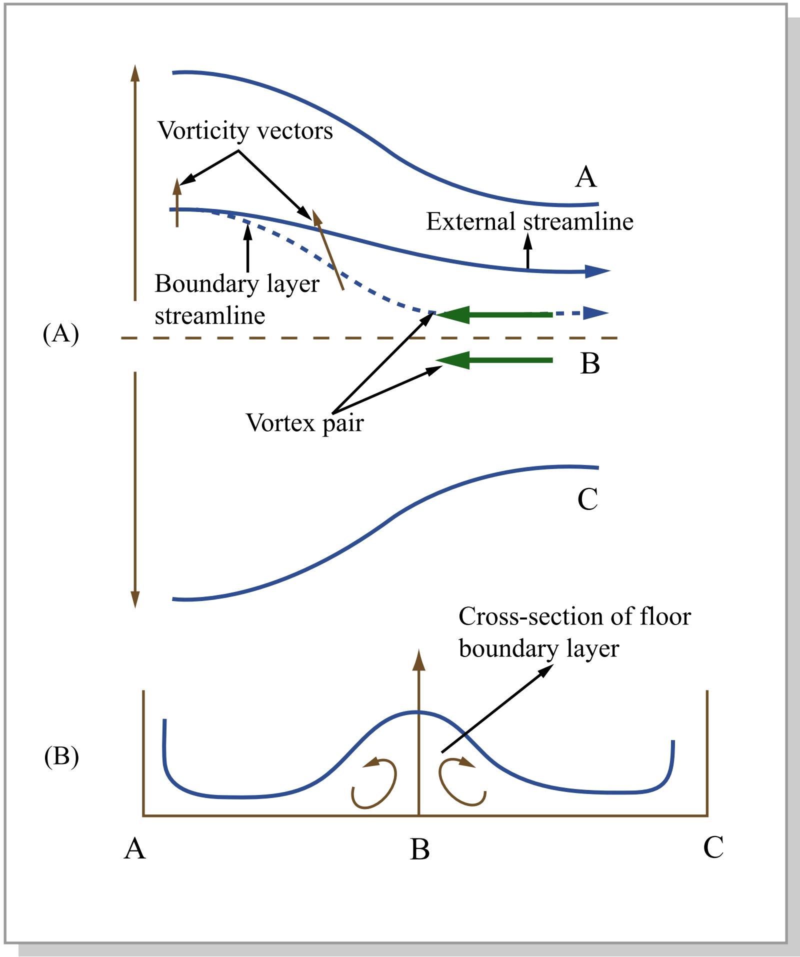


Figure by MIT OCW.

MIXING ENHANCEMENT DUE TO STREAMWISE VORTICITY

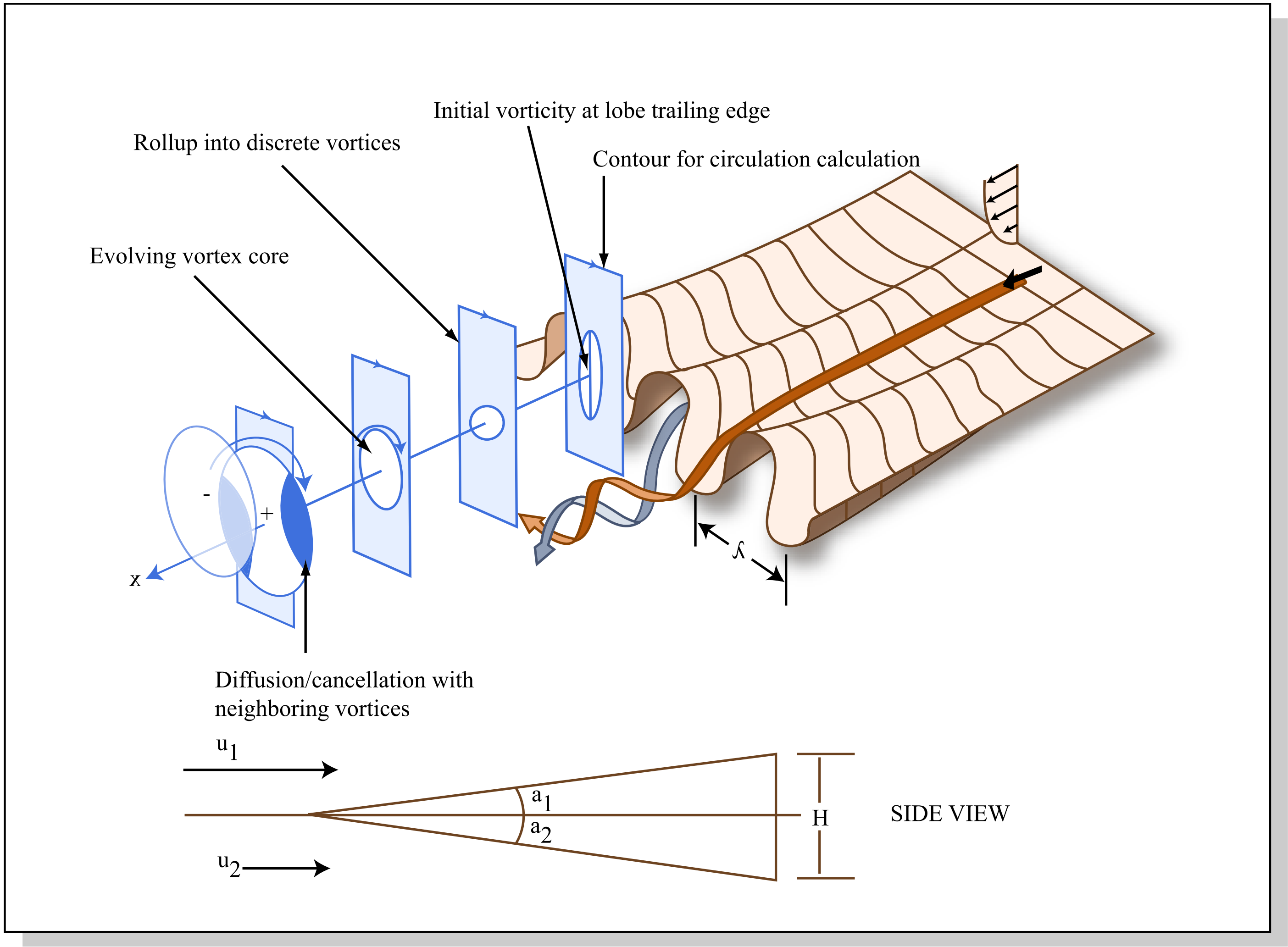
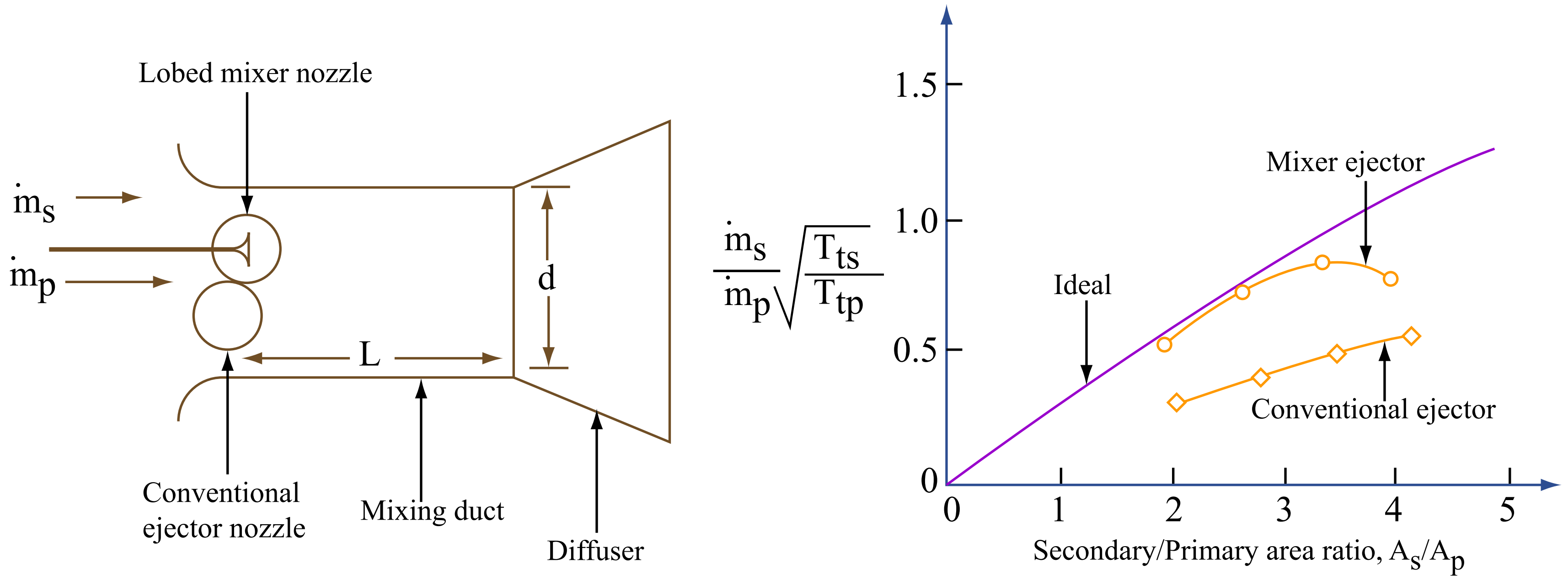


Figure by MIT OCW.

EJECTOR SHOWING (a) DIFFERENT NOZZLE CONFIGURATIONS AND (b) DIFFERENT PERFORMANCE [Tillman et al. 1992]



EJECTOR CONFIGURATION

EJECTOR PERFORMANCE

Figure by MIT OCW.

EVOLUTION OF THE INTERFACE DOWNSTREAM OF THE LOBED MIXER (*Qualitative*)

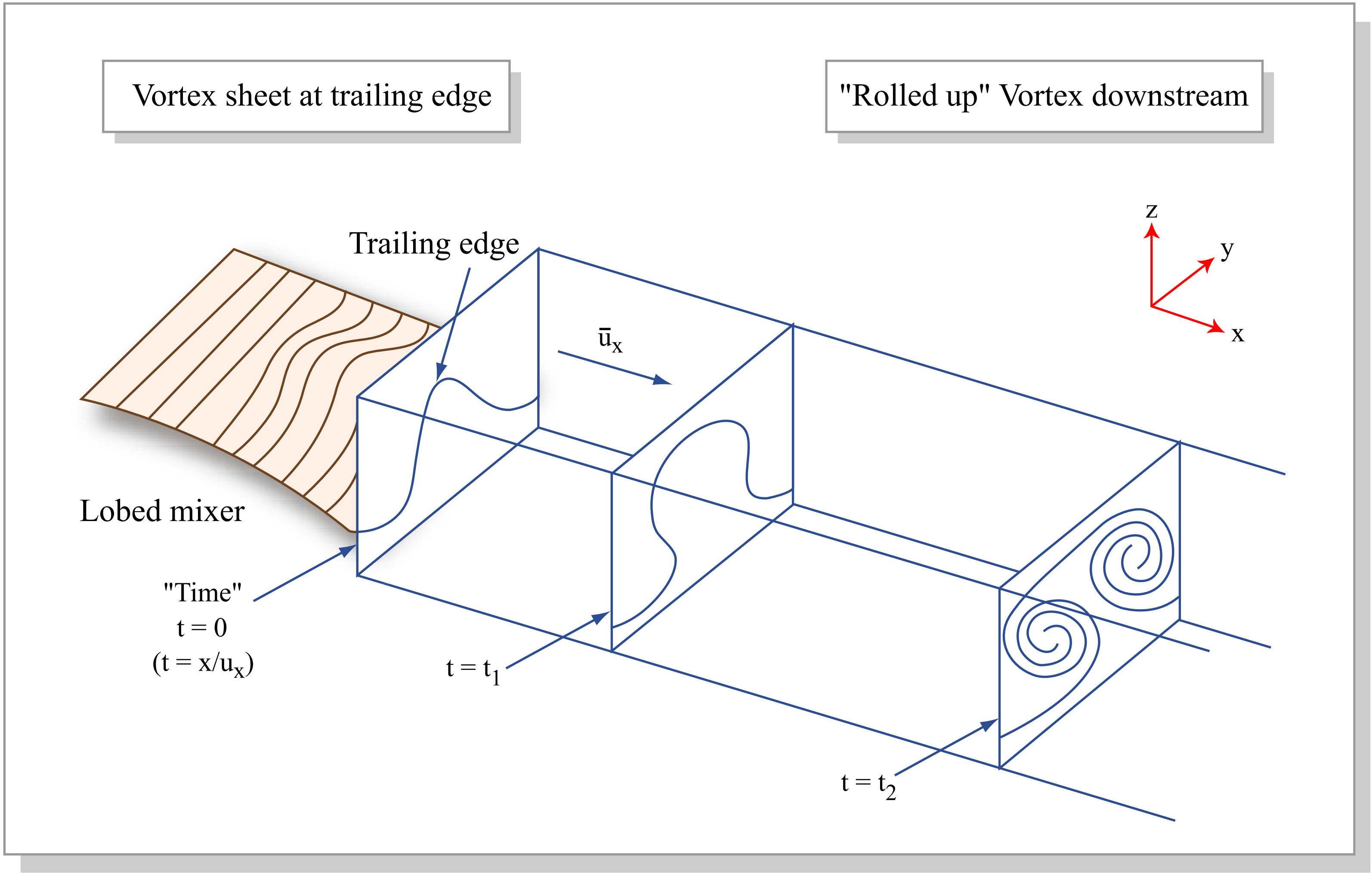


Figure by MIT OCW.

EVOLUTION OF THE INTERFACE DOWNSTREAM OF THE LOBED MIXER (*Quantitative*)

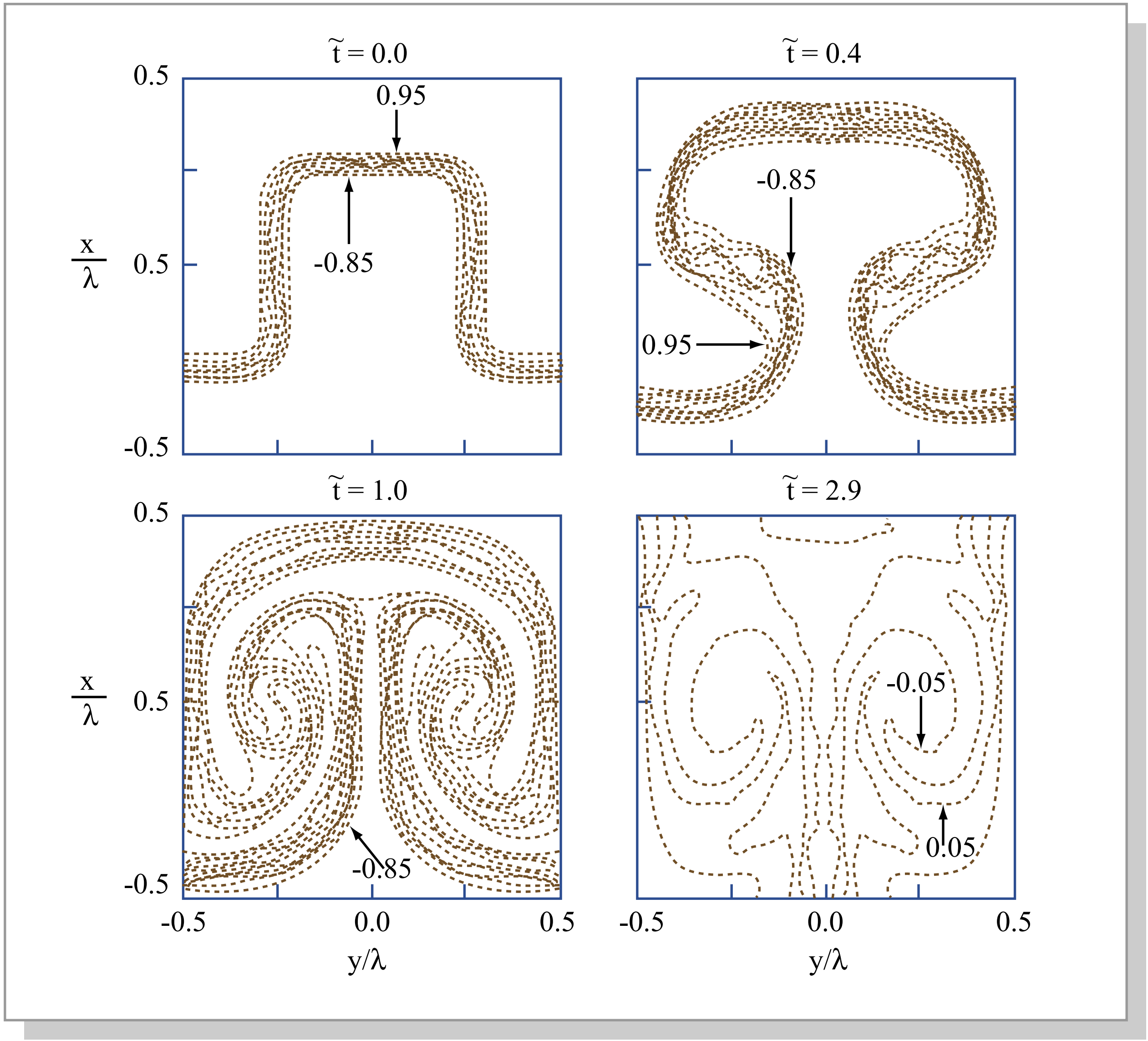


Figure by MIT OCW.

EFFECT OF STREAMWISE VORTICITY VS. EFFECT OF TRAILING EDGE AREA INCREASE

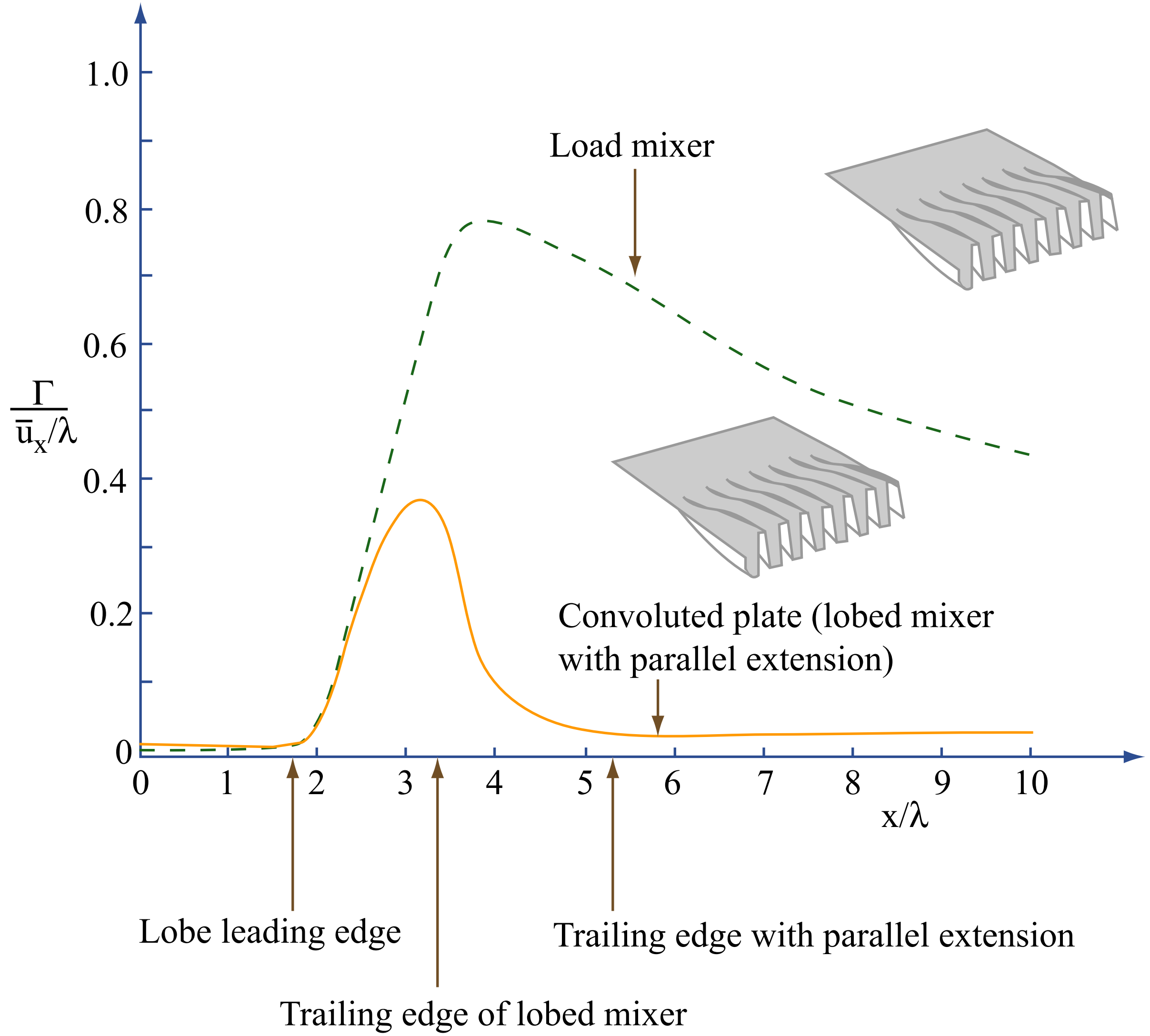


Figure by MIT OCW.

TWO-STREAM CONSTANT AREA MIXING

- **Mixing of two streams with different velocities in a constant area duct (incompressible)**

- **Momentum exchange between streams**
  - **Inititially different momentum ==> approaches uniform state**
  - **Amount of momentum mixing determines momentum flux at exit**
  - **Smallest exit momentum flux is when exit velocities are uniform**
- **Constant area control volume means that pressure change is equal to difference in momentum flux**

- **Static pressure rise thus a direct measure of amount of mixing**
  - **Control volume value is ideal value, fully mixed, no wall shear stresses**

CONSTANT AREA MIXING OF TWO STREAMS WITH DIFFERENT VELOCITIES: MIXER LOBE, CONVOLUTED PLATE, FLAT SPLITTER

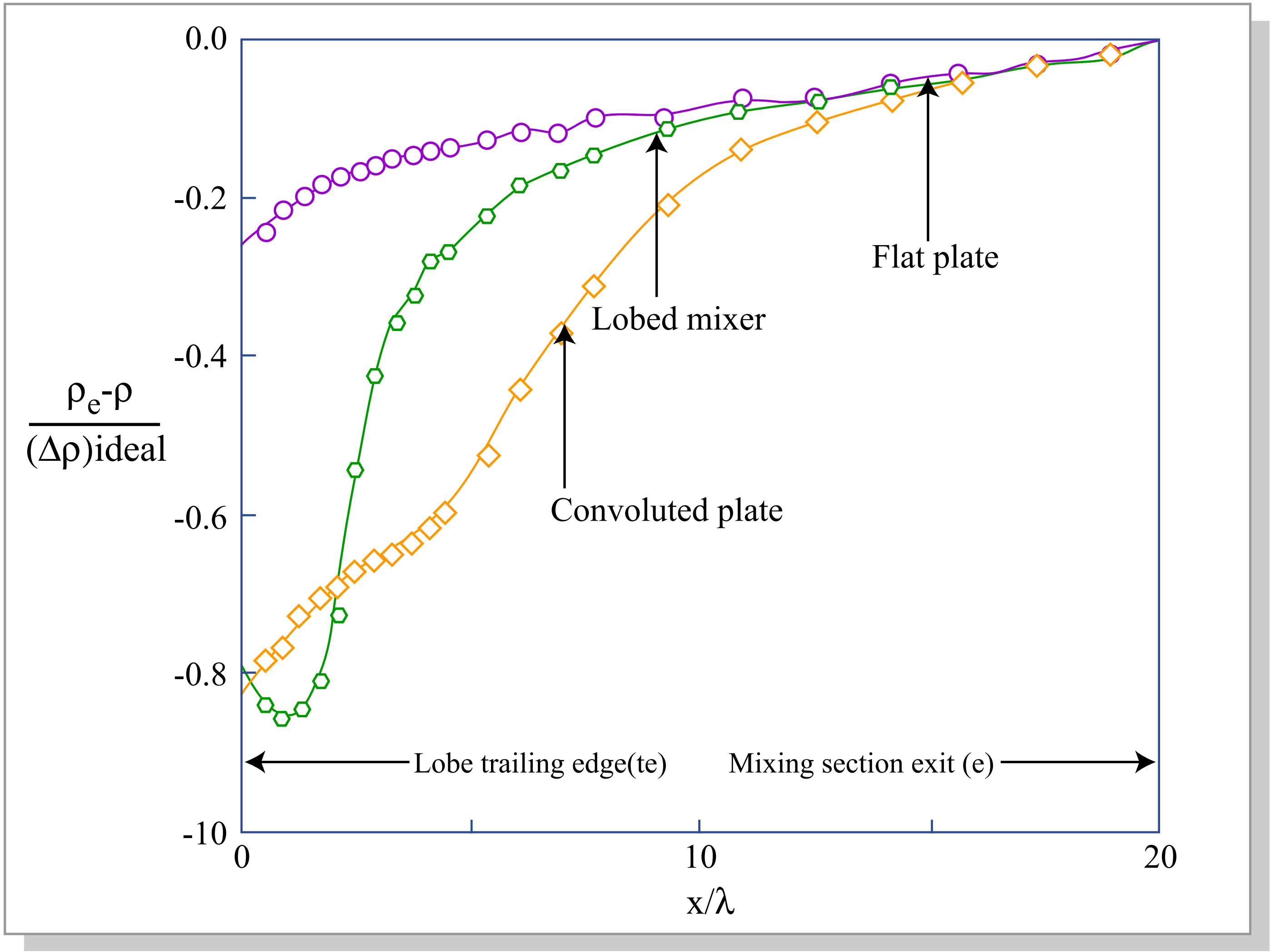


Figure by MIT OCW.

TIP CLEARANCE VORTEX KINEMATICS

- **Examine behavior of tip clearance flows in axial compressors**

- **Leakage of flow over the tip of the blade results in a vortex (as in a finite wing)**

- **Can predict trajectory using vortex methods (numerical calculations based on convecting the vorticity and then computing the associated velocity)**

CCOMPUTATION OF TRAJECTORY OF PARTICLES (i.e., streamlines) LEAKING OVER COMPRESSOR TIP [Furukawa et al.]

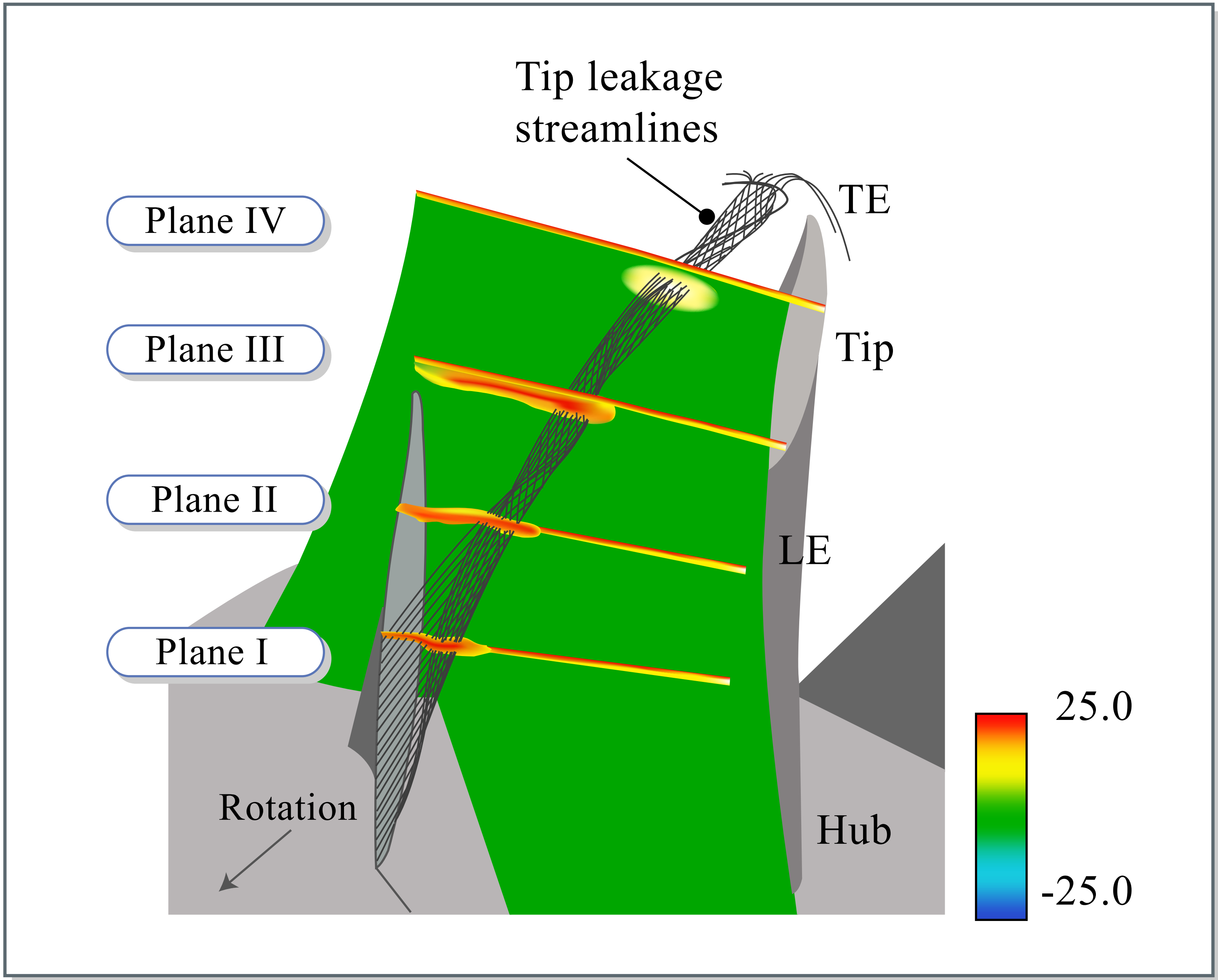


Figure by MIT OCW.

STAGNATION PRESSURE CONTOURS AND STREAMLINES IN LEAKAGE VORTEX (CLEARANCE VORTEX) [Furukawa et al.]

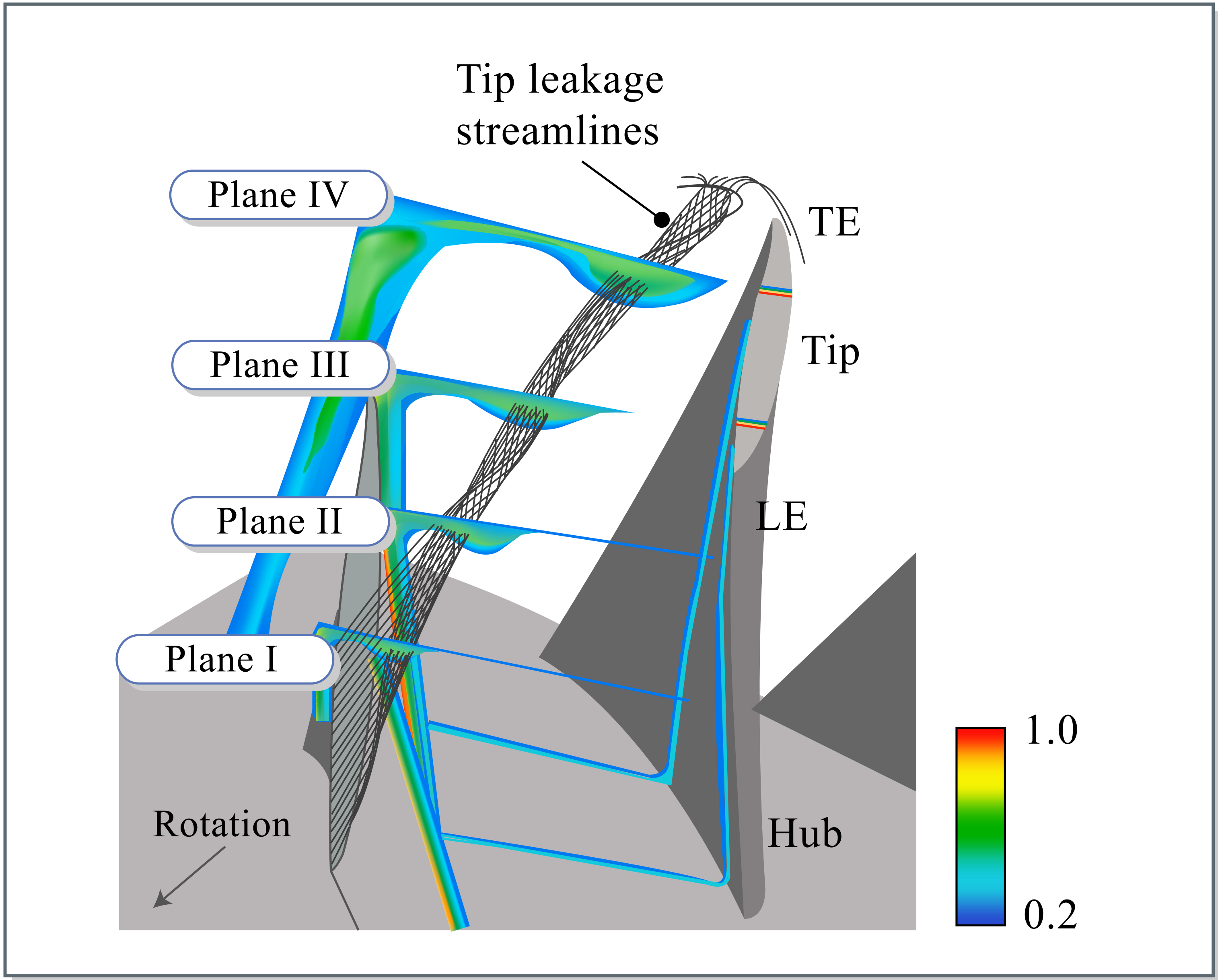
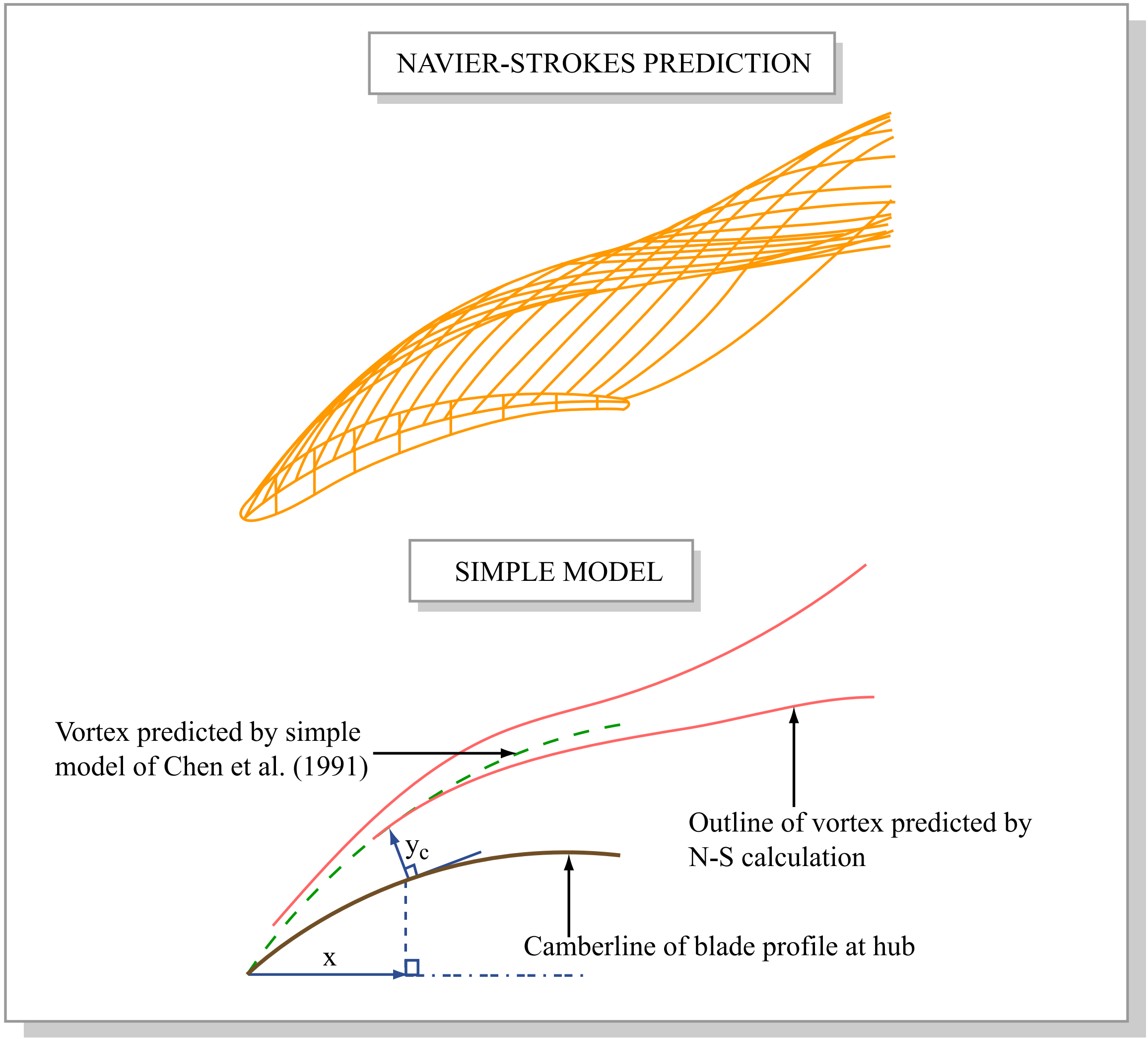


Figure by MIT OCW.

PREDICTIONS OF VORTEX TRAJECTORY IN A STATOR HUB [Storer and Cumpsty]


NAVIER-STROKES PREDICTION

SIMPLE MODEL

Vortex predicted by simple model of Chen et al. (1991)

Outline of vortex predicted by N-S calculation

Camberline of blade profile at hub

Figure by MIT OCW.

GENERALIZED TIP CLEARANCE VORTEX CORE TRAJECTORY [Chen et al.]

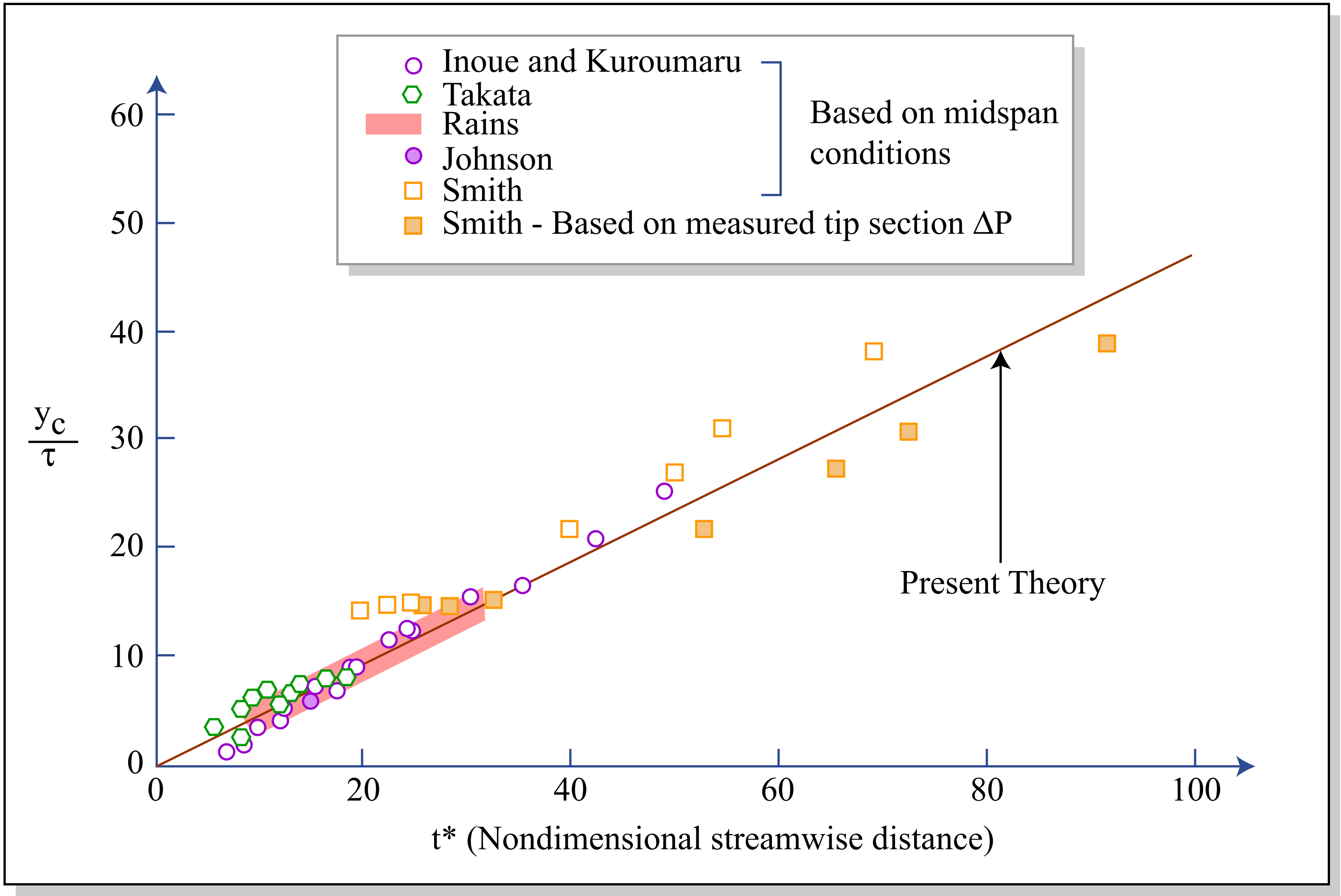


Figure by MIT OCW.

KINEMATICS OF VORTEX TRAJECTORY

- **Change in slope of vortex trajectory seen near trailing edge**
  - **Downstream trajectory has more inclination to axial (Vortex moves across the passage "faster")**

- **Analyze in terms of vortex/image vortex system**
  - **What is the velocity field associated with a vortex in the corner formed by the blade and endwall?**
  - **What is the velocity field associated with a vortex next to the endwall**

VORTEX AND IMAGE SYSTEM IN PASSAGE AND DOWNSTREAM

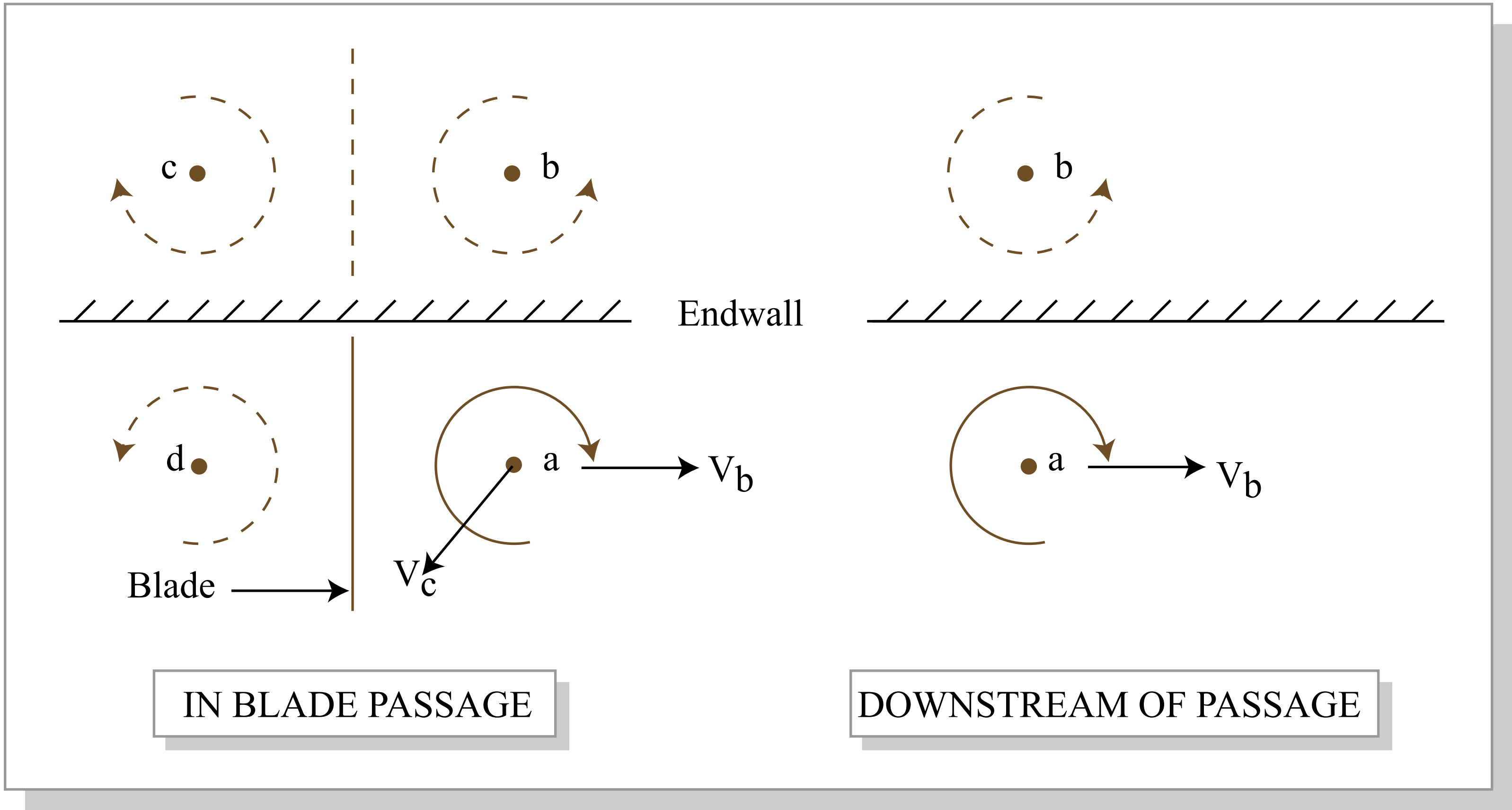


Figure by MIT OCW.

VORTEX TRAJECTORY IN PASSAGE AND DOWNSTREAM

[Data of inoue et al.]

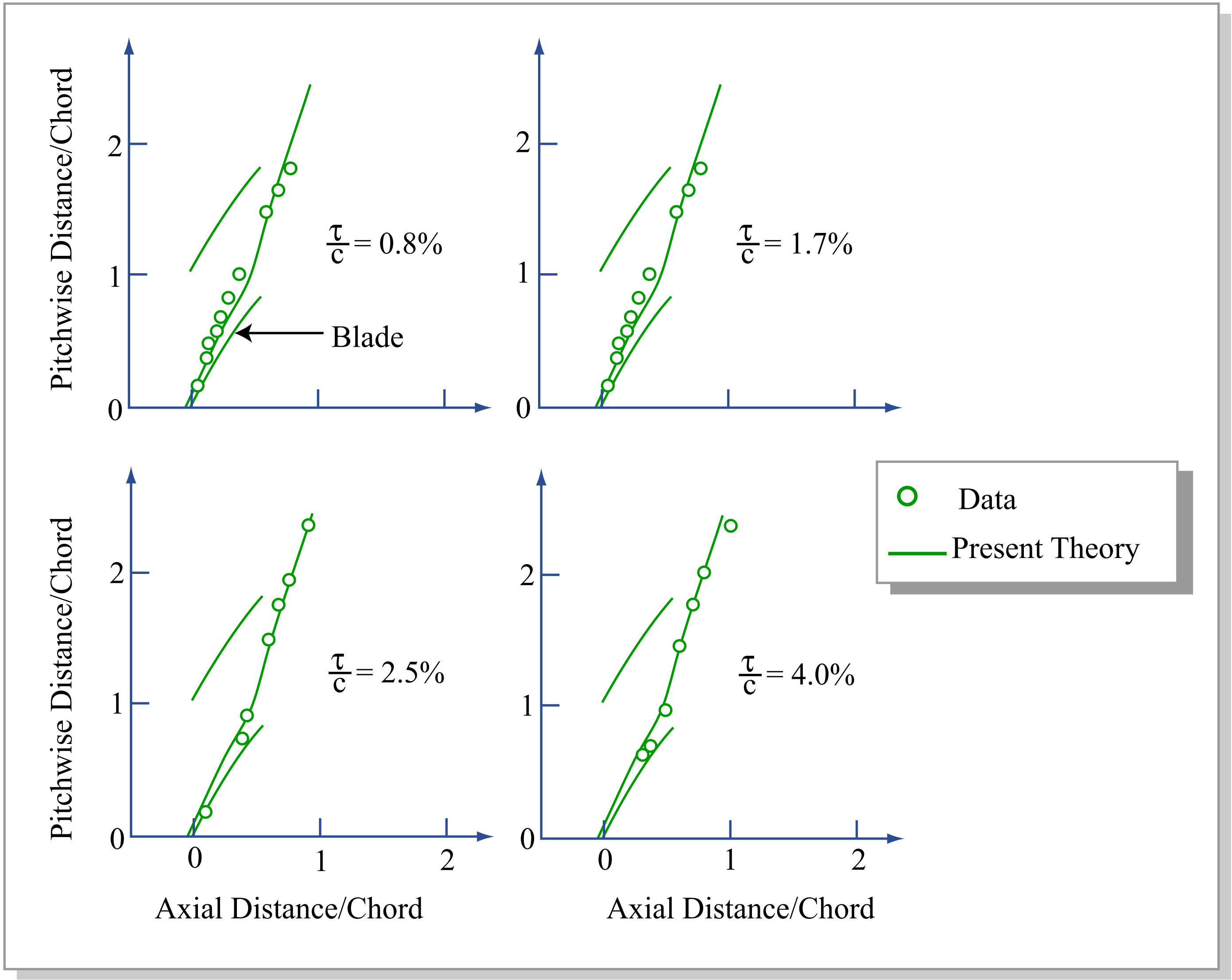


Figure by MIT OCW.