MUDDY POINTS

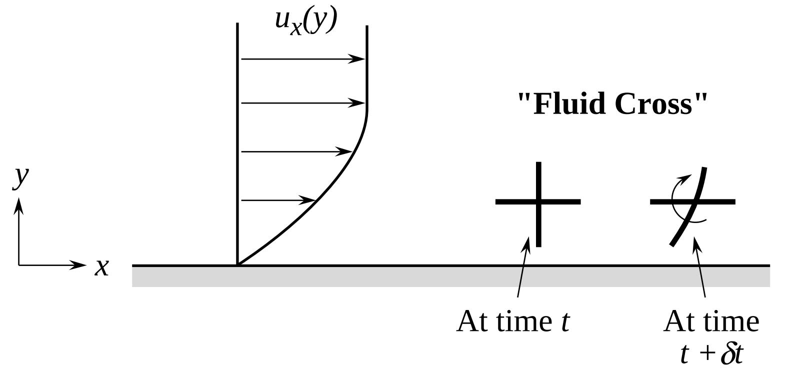
- Disappearance of vorticity?
- Vortices *versus* vorticity?
- Linear velocity variation in a nozzle?

DOES VORTICITY EVER DISAPPEAR?

- Vortex lines are only "locked" to fluid particles for inviscid flow with conservative body forces and either uniform constant density or $\rho = \rho(p)$
  - Otherwise vorticity can diffuse out of, or into, fluid particles through the action of viscosity
  - Otherwise vorticity can be produced (or reduced) through the agency of baroclinic torque
- Consider the flow in a 2-D rotating cylinder
  - Eventual steady-state is constant vorticity
- Suppose we stop the cylinder?
  - Eventual steady-state is no motion
- Can we describe this in terms of diffusion of vorticity?
- Could we do the same for two infinite parallel plates, one moving but then brought to rest?

VORTICITY AND STREAMLINE CURVATURE

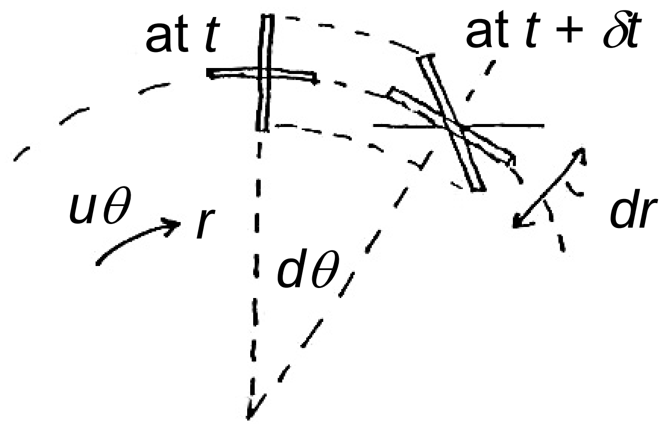
- Vorticity can be present *even if streamlines are straight*



- Have rotation rate of vertical fluid line of $-\dfrac{\partial u_y}{\partial x}$, no rotation horiz. line

- Total rotation of 2 perpendicular lines is $-\dfrac{\partial u_x}{\partial y} + 0 = \omega_z$

VORTICITY IS NOT NECESSARILY PRESENT IF STREAMLINES ARE CURVED



- Consider 2-D flow with $u_\theta \propto 1/r, \;\; u_r = 0$

Horizontal leg rotates dθ in clockwise direction

$$d\theta = \frac{u_\theta}{r}\delta t$$

Vertical leg rotates $\frac{\partial u_\theta}{\partial r}\delta t$ in anti-clockwise direction

Net rotation in anti-clockwise direction is $\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)\delta t$

If $u_\theta \propto 1/r$, $\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) = 0$, NO NET ROTATION OF FLUID PARTICLES => $\underline{\omega = 0}$

CIRCULATION AND VORTICITY

- Circulation around a contour C is equal to the flux of vorticity through area A bounded by C
- If the circulation is non-zero there must be vortex lines that thread through the area enclosed by the contour

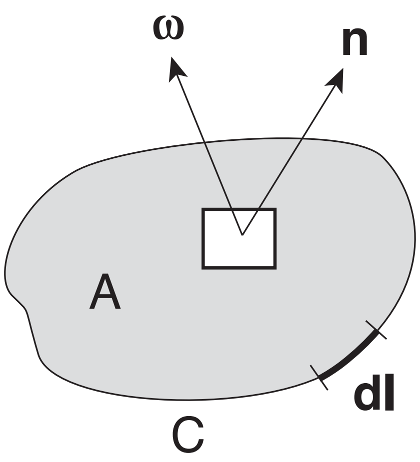
DEFINITION OF CIRCULATION

$$\Gamma_C = \oint_C \mathbf{u} \cdot \mathbf{dl}$$

Closed contour, C

$$= \iint_A \nabla \times \mathbf{u} \cdot \mathbf{n} \, dA$$

(Stokes)



$$\Gamma = \iint_A (\mathrm{w} \cdot \mathbf{n}) \, dA$$

Flux of vorticity through area A, bounded by C

A VORTEX *versus* VORTICITY

1) If we have an identifiable "vortex" in a flow (what does that mean?) we have vorticity somewhere

- Vortex: a definite structure which one could look at and say "there is a vortex"? (Eddying motion? But what is an "eddy"?)

- HOWEVER, vorticity is also present in parallel flows--don't need a vortex to exist

2) If we have circulation in a flow there must be vorticity within the contour

- Consider potential flow round an airfoil
  - There is circulation
  - There must be vorticity somewhere
  - Where is the vorticity?
  - Is there any fluid that one could say was "vortical"
- Have vortex sheets that bound the fluid (between the fluid and the surface) and no regions of fluid with vorticity

LINEAR VELOCITY PROFILE IN A NOZZLE?

- On the quiz we had a 2-D flow with a linear (in y) velocity profile at inlet
- It was claimed that a linear velocity profile, with the same slope, must exist at a far downstream station--this was motivated by use of

$$\frac{D\omega}{Dt} = 0$$

- Some disbelief existed about this result and there was a desire to see if it was directly consistent with Bernoulli and continuity
- Start from Bernoulli
  - Stagnation pressure is constant along a streamline
  - Stagnation pressure differs from streamline to streamline

WORK IN TERMS OF A STREAM FUNCTION

- Continuity is brought in by the definition of a stream function, ψ, related to the velocity components by

$$u_x = \frac{\partial \psi}{\partial y}$$

$$u_y = -\frac{\partial \psi}{\partial x}$$

- Using the stream function the equation of continuity is **identically** satisfied
- Bernoulli says that p_t is a function of ψ only [$p_t = p_t(\psi)$]
- The variation in stagnation pressure can be written as

$$\frac{\partial p_t}{\partial y} = \frac{dp_t}{d\psi}\frac{\partial \psi}{\partial y} = H(\psi)\frac{\partial \psi}{\partial y}$$

EXAMINE VELOCITY GRADIENT

- From the definition of stagnation pressure, at the upstream station,

$$\frac{\partial p_t}{\partial y} = \frac{\partial p}{\partial y} + \rho u_x \frac{\partial u_x}{\partial y} = \rho u_x K; \;\; K \text{ is a const}.$$

- Equating the two descriptions of the derivative of the stagnation pressure

$$\rho u_x K = H(\psi) u_x$$

so $H(\psi)$ is a constant.

- But since $H(\psi)$ only depends on the stream function it has the same value at any x location
  - The velocity gradient $\partial u_x/\partial y$ is thus constant downstream as well
  - Any location at which the gradient of static pressure is zero will have a velocity gradient that is constant, i. e., a linear velocity
- This is what we said last time using vorticity arguments **AND IT WAS MUCH MORE DIRECT!**