WEATHER SYSTEMS ON A ROTATING EARTH

[Adapted from Kleppner and Kolenkow]

- Coriolis forces turn straight line motion on a rotating sphere into circular motion

- Consider a particle of mass *m* moving with velocity *w* at latitude λ on the surface of a sphere

- Coriolis force is

$$\mathbf{F}_{Coriolis} = -2m\left(\Omega_V \times \mathbf{w} + \Omega_h \times \mathbf{w}\right)$$

- Horizontal Coriolis force arises solely from the term $\Omega_v \times \mathbf{w}$

$$\mathbf{F}_H = -2m\,\Omega \sin\lambda\; \mathbf{e}_r \times \mathbf{w}$$

$\mathbf{e}_r$ is unit vector in radial direction

$$\left|\mathbf{F}_H\right| = 2mw\,\Omega \sin\lambda$$

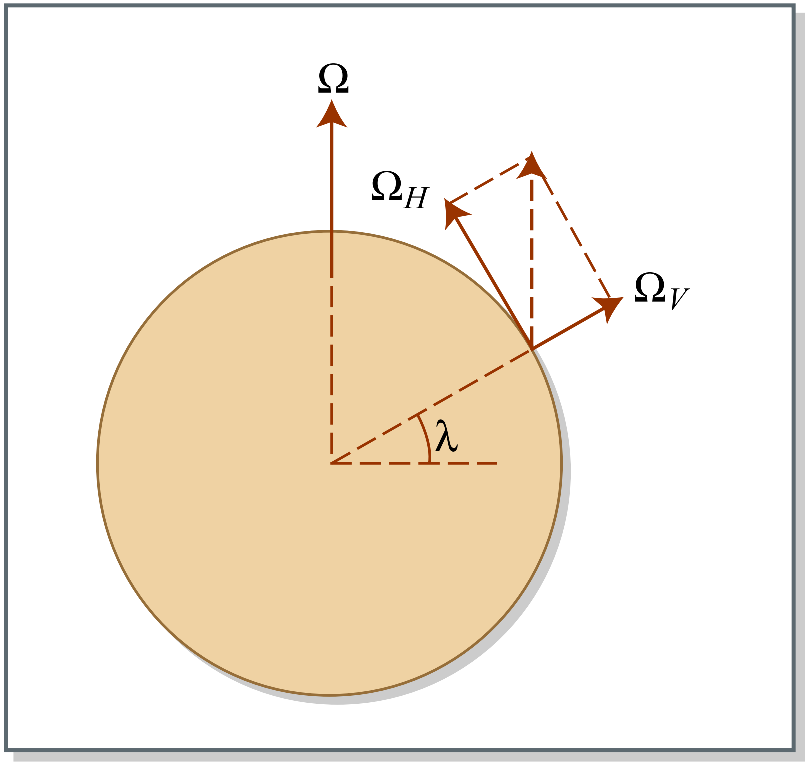


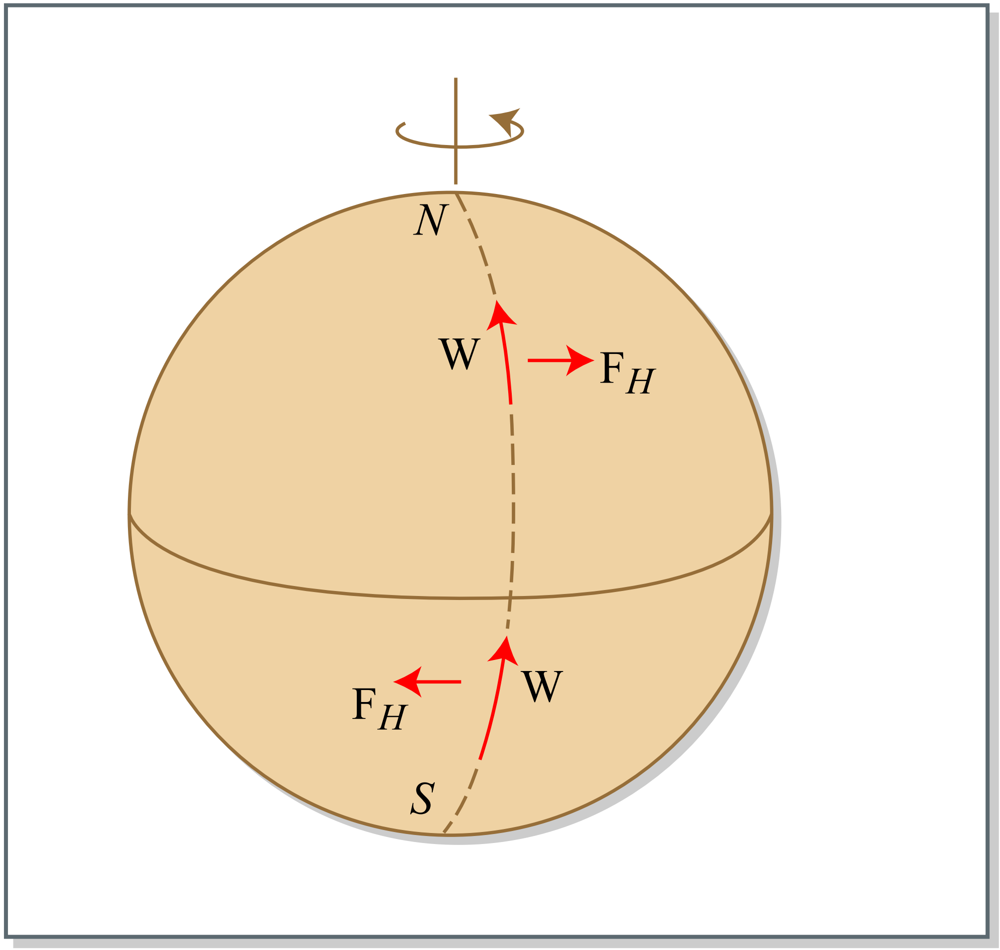
Figure by MIT OCW.

Components of the Earth's rotation [Kleppner and Kolenkow].

Coriolis Forces in the Northern and Southern Hemispheres

[Kleppner and Kolenkow].

- Coriolis force is maximum at poles, zero at equator



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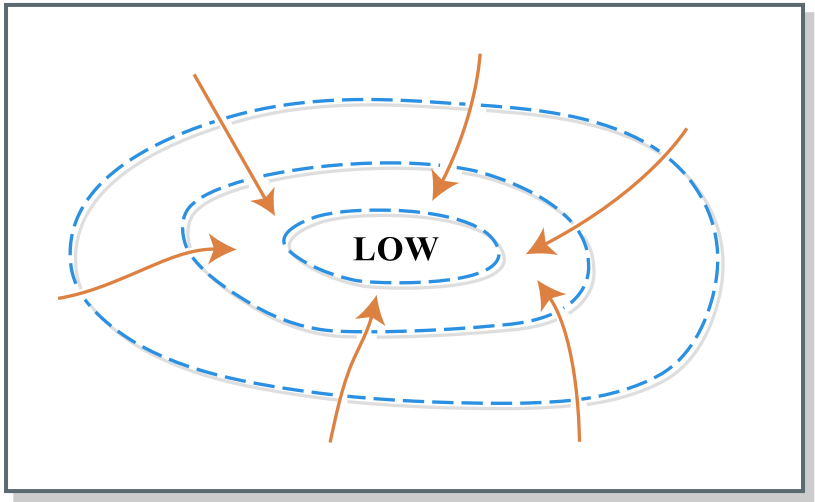
FLOW TOWARDS A LOW PRESSURE REGION

- Suppose we have a low pressure region as sketched below



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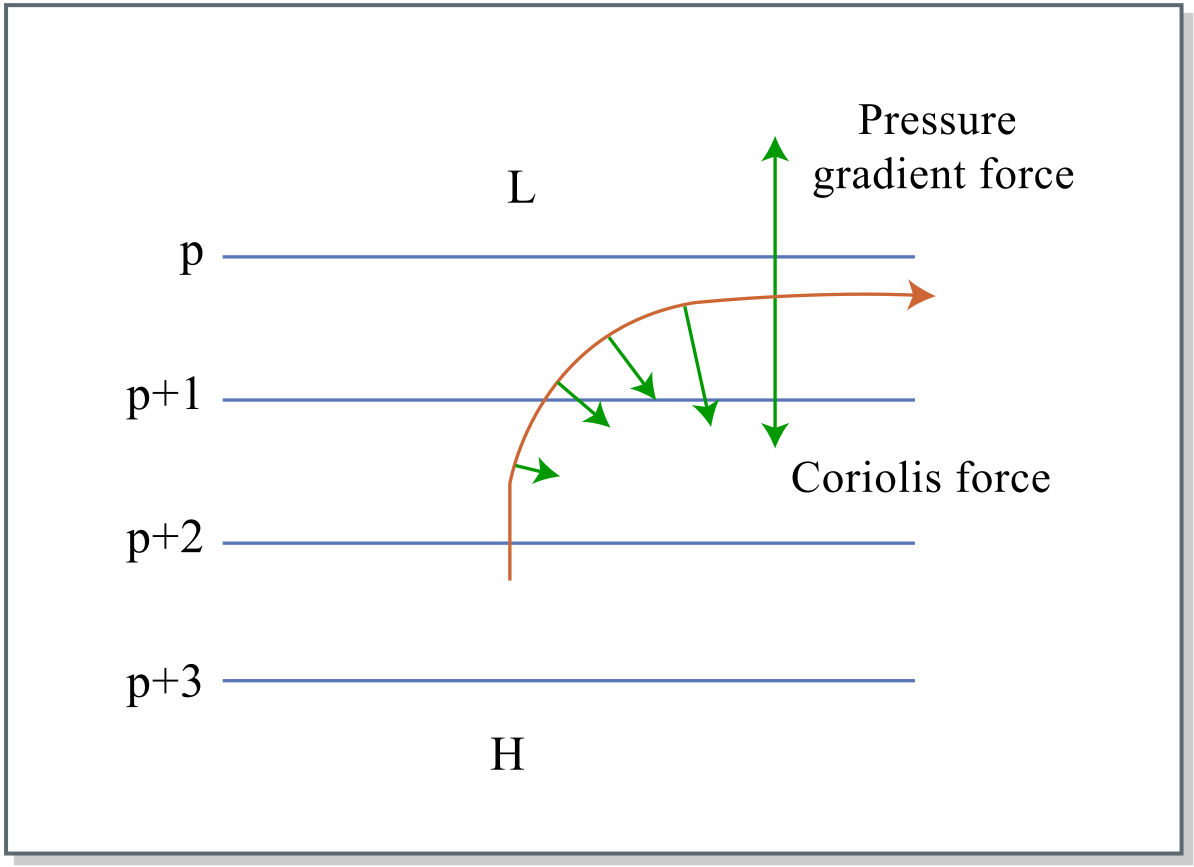
Flow towards a low pressure region [Kleppner and Kolenkow].

- The forces considered are pressure and Coriolis
  - Q: What about centrifugal forces?
- The momentum equation is

$$\frac{D\mathbf{w}}{Dt} = -\frac{\nabla p_{red}}{\rho} + \mathbf{F}_{Coriolis}$$

Path of a Fluid Particle on the Rotating Earth [Hess].

- A fluid particle is deflected as it moves from high pressure (H) to low pressure (L)



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- The trajectories thus look as sketched below

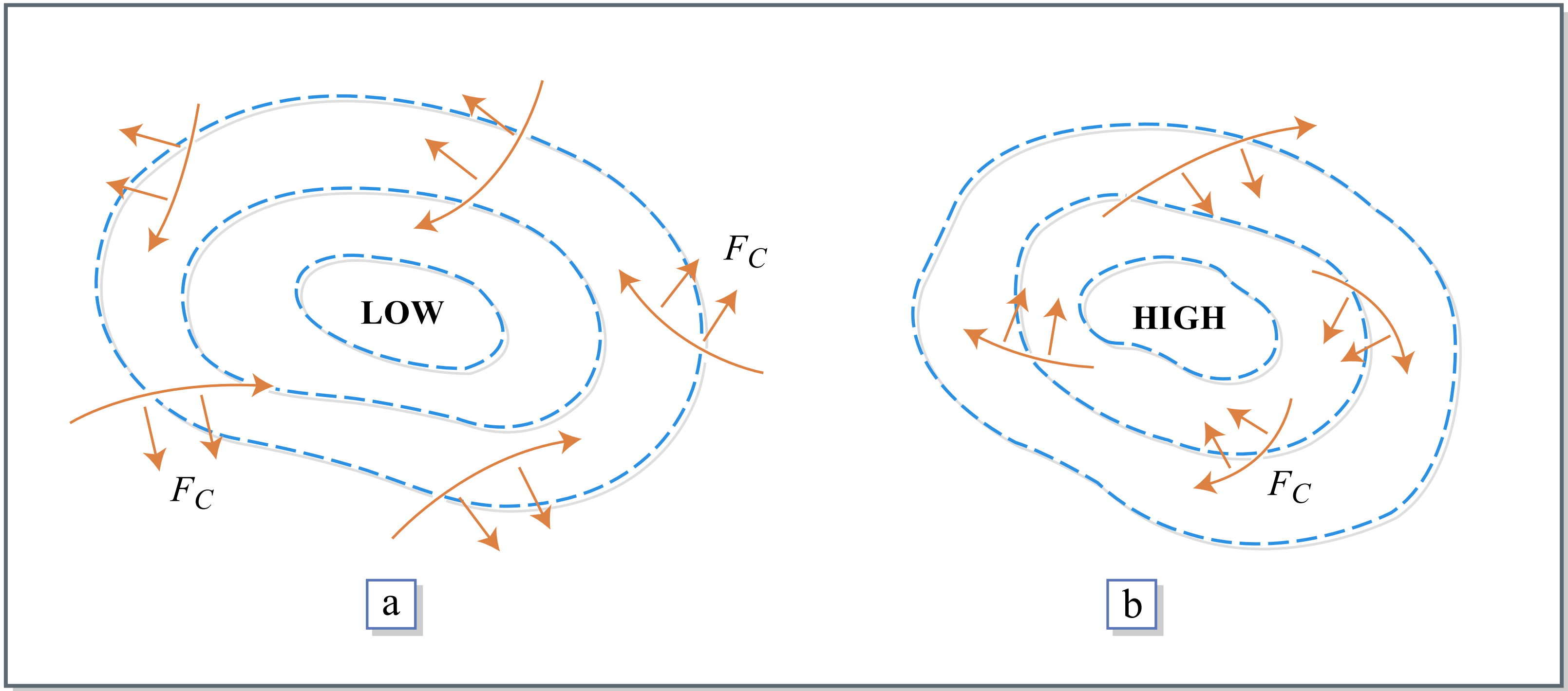
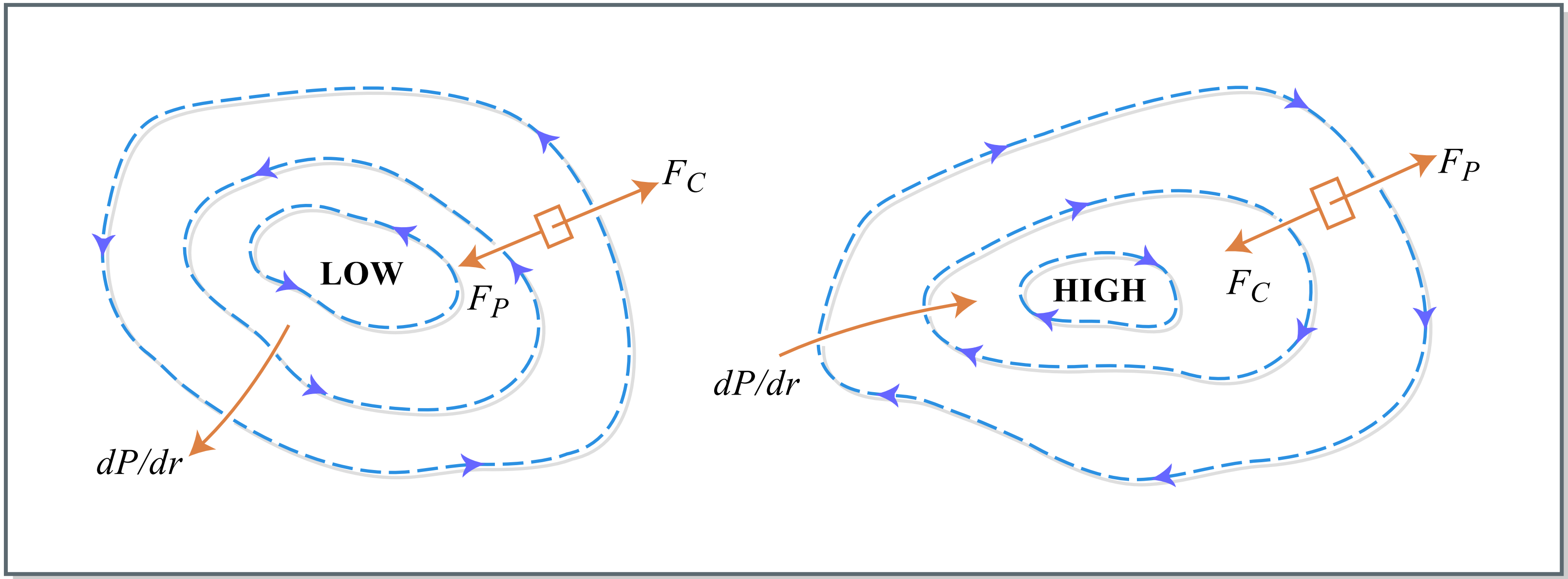


Figure by MIT OCW.

Flow around (a) low pressure region; (b) high pressure region [Kleppner and Kolenkow].

- "….On the rotating Earth a particle subject to a constant force does not move parallel to the force with constant acceleration….but ultimately will move perpendicular to the force with constant speed." [Hess]

FORCE BALANCES FOR HIGH AND LOW PRESSURE REGIONS



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Flow around a low and a high pressure region on the earth [Kleppner and Kolenkow].

- If we adopt a coordinate system locally approximating the Earth as flat, with a coordinate η measured from the center of a circular low or high pressure region, the "equilibrium" flow is given by

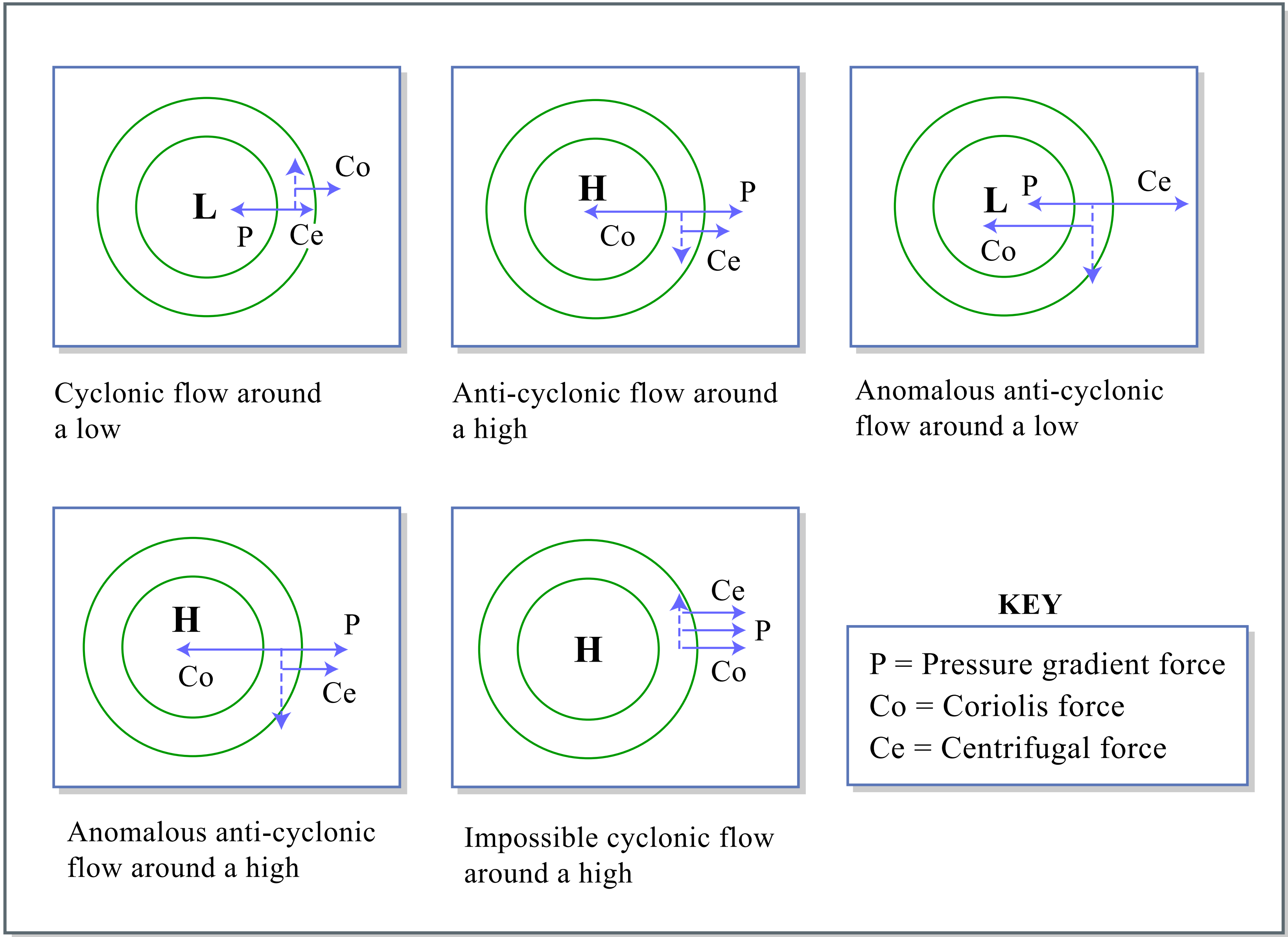
$$\frac{w_\theta^2}{\eta} = \frac{1}{\rho}\frac{dp}{d\eta} - 2w_\theta\,\Omega\sin\lambda$$

- If the pressure gradient is taken as given, this is a quadratic equation for the magnitude of the relative circumferential velocity, w_{θ},

$$w_{\theta} = -\eta \Omega \sin \lambda \pm \sqrt{(\eta \Omega \sin \lambda)^2 + \frac{\eta}{\rho} \frac{dp}{d\eta}}$$

- There is a difference between low and high pressure weather systems as shown from the above
- For strong pressure gradients a circular flow does not form around a high pressure zone
  - The Coriolis force is too weak to allow a balance
- Storms like hurricanes are thus always low pressure systems

DIFFERENT CASES OF GRADIENT BALANCE [Hess]



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