Formation Flight Control

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Laila Mireille Elias
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

Outline

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Formation Flight Motivation

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- Recent Trend
  - Few, large spacecraft -> several smaller spacecraft
  - "Don't put all your eggs in one basket."
  - Example: Mars Exploration Program ($4 billion Viking -> several "faster, better, cheaper" missions, each at <10% Viking cost)
- *Extend* this idea to formation flight (FF)
  - Several small spacecraft all used to accomplish a single mission
  - Fly satellites in formation to form a large "virtual" satellite
  - Allows for modularity, replacement of individual failed satellites, reconfiguration of cluster in case of failed satellite. Single failure won't kill entire mission!
  - Also enables new technology… Space Interferometry!
- Traditional Telescopes vs. Interferometers
  - Monolithic telescopes such as Hubble have improved angular resolution with increased aperture size.
  - Apertures (e.g. 2.4 m Hubble aperture) are approaching an upper limit on size, due to launch vehicle constraints
  - Can form a very large "virtual" aperture by combining light from several spacecraft separated at large distances
  - Angular resolution of interferometers increases with aperture spacing.
  - Formation flight used for "course" control, while optical instruments provide "fine" control

Formation Flight Applications

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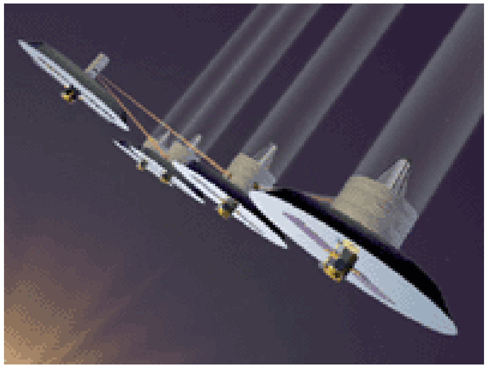
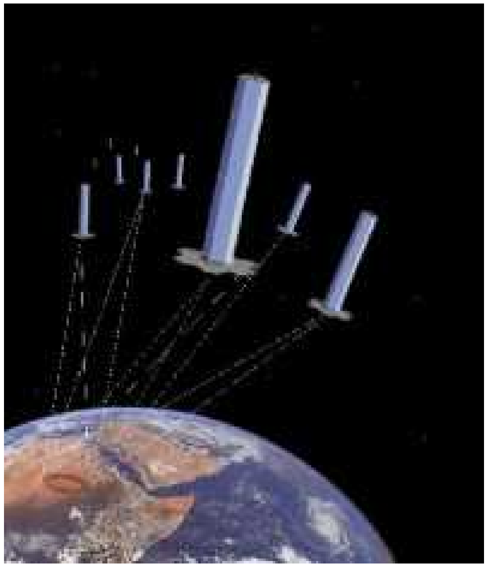
- Starlight: http://starlight.jpl.nasa.gov
  - First ever formation-flying space interferometer
  - Will develop and validate FF and space interferometry technologies
  - Scheduled to launch in June 2006
  - Two spacecraft with 30-125 m baseline achieve the angular resolution of an equivalent-diameter telescope
  - Tolerances: Distance +/- 5 cm, Angle +/- 1 arcmin
- Terrestrial Planet Finder (TPF): http://tpf.jpl.nasa.gov
  - Will use nulling interferometry to study planets as small as Earth in extrasolar systems
  - Scheduled to launch in 2012
  - Five spacecraft with 75 meter baseline for planetfinding
  - Angular resolution is .75 marcsec
  - Detailed reference mission in "TPF Book" on website
- TechSat 21: http://www.vs.afrl.af.mil/Factsheets/techsat21.html
  - Will demonstrate distributed satellite formation flying as a platform for sparse-aperture space-based radar
  - 35 clusters of 8 spacecraft for full global coverage
  - 3-spacecraft experiment scheduled to launch in 2003



NASA's TPF

Image is taken from NASA's Web site: http://www.nasa.gov.



AFRL's TechSat 21

Image courtesy of Air Force Research Laboratory (AFRL): http://www.vs.afrl.af.mil/.

Formation Flight Maneuvers

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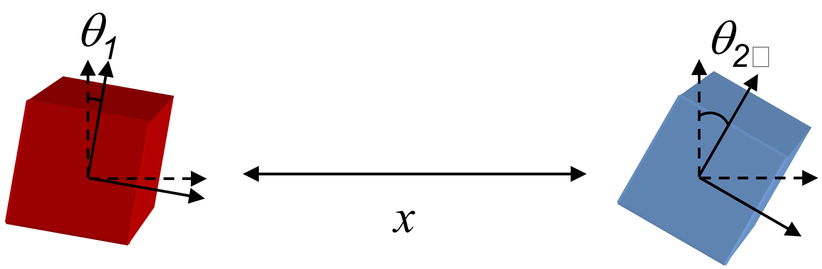
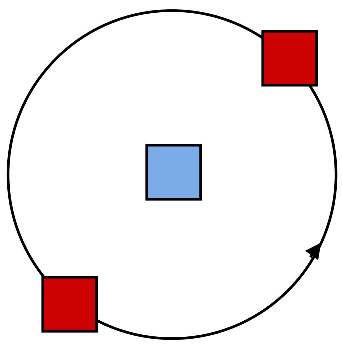
- Position/Attitude Hold



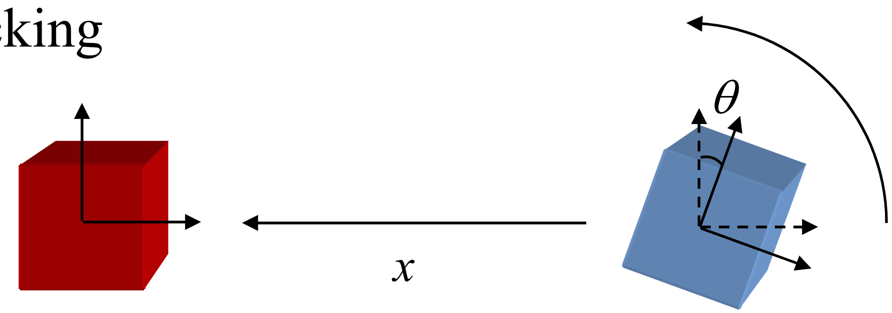
  - *Relative* vs. *absolute* position
  - Absolute attitude
  - Disturbance rejection (e.g. due to atmospheric drag, residual magnetic torques, etc.)

- Spin-Up/Down and Steady-State Spin



  - Spin-Up for Scientific Observation
  - Steady-State Spin during Observation
  - Spin-Down for Orbit Reconfiguration

- Docking



  - Docking of two bodies
  - Precise control of relative distance and relative attitude

Disturbance Sources

- Aerodynamic Drag
  - Due to an offset between the CM and the drag center of Pressure (CP).
  - Only a factor in LEO.
- Magnetic Torques
  - Induced by residual magnetic moment.
  - Model the spacecraft as a magnetic dipole.
  - Only within magnetosphere.
- Solar Radiation
  - Torques induced by CM and solar CP offset.
  - Can compensate with differential reflectivity or reaction wheels.
- Mass Expulsion
  - Torques induced by leaks or jettisoned objects.
- On-board Disturbances
  - On-board equipment (machinery, wheels, cryocoolers, pumps etc…).
  - No net effect, but internal momentum exchange affects attitude.
- 1-g Torque
  - Torque due to misalignment of CG with net reaction force

Magnetic Torque

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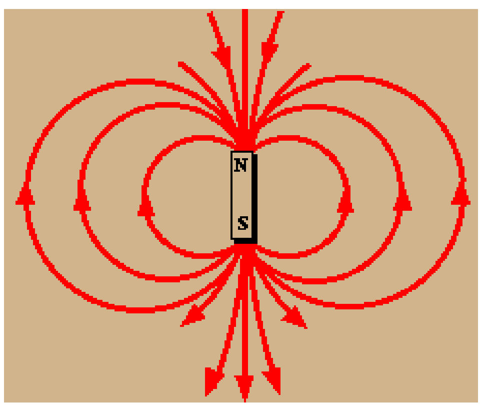
$$\underline{T} = \underline{M} \times \underline{B}$$

$\underline{M}$ = Spacecraft residual dipole moment in AMP-TURN-m^2 (Atm^2)

$\underline{M}$ is due to current loops and residual magnetization, and will be ~0.1 Atm^2 or more for small spacecraft *without* electromagnetic actuators.



Typical Values:
$B = 3 \times 10^{-5}$ TESLA
$M = 0.1\ Atm^2$
$T = 3 \times 10^{-6}$ Nm

$\underline{B}$ = Earth magnetic field vector in spacecraft coordinates (BODY FRAME) in TESLA.

$\underline{B}$ varies as $1/r^3$, with its direction along local magnetic field lines.

Disturbance Torque for CDIO

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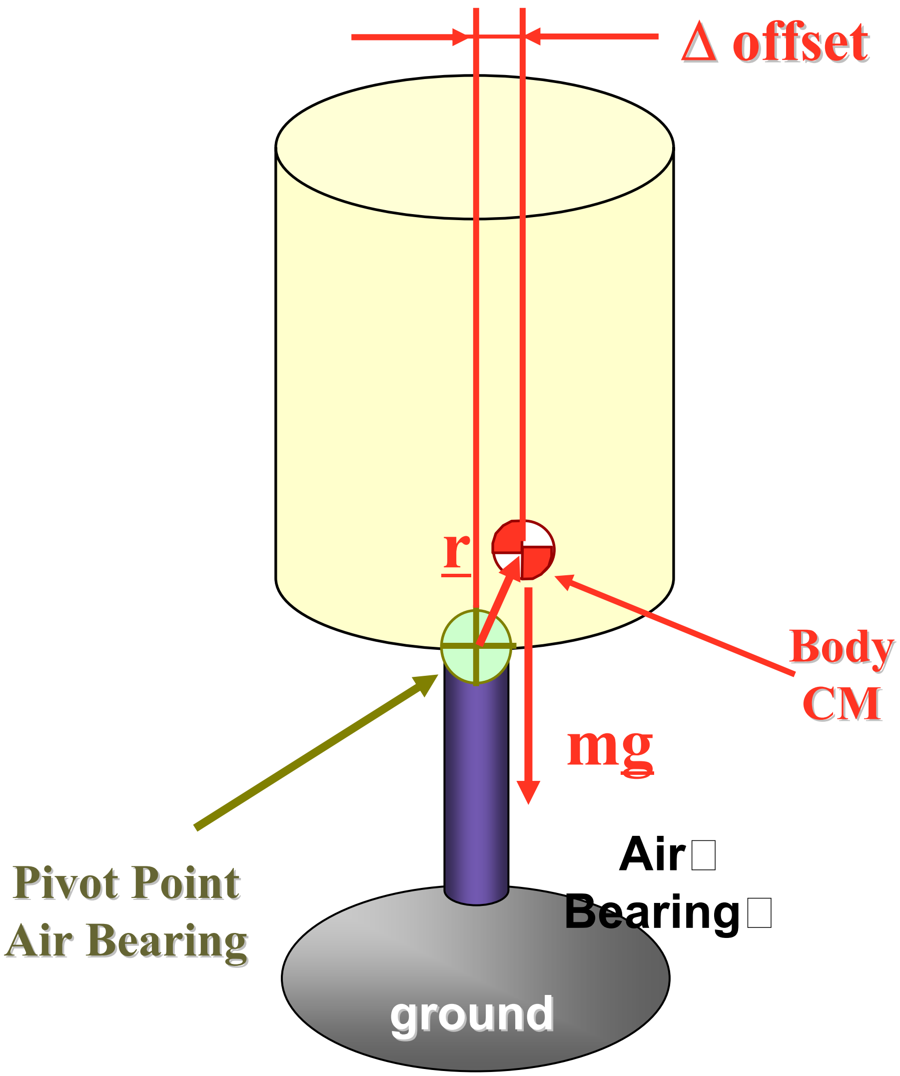

Expect residual gravity torque to be a significant disturbance.

Important to balance!

Example:

$$T = \left| \underline{r} \times m\underline{g} \right| \cong 0.01 \cdot 10 \cdot 9.81 \cong 1 \text{ Nm}$$

Modeling the "Plant"

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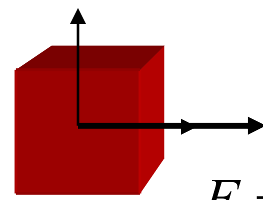
- Our "plant" is the satellite to be controlled.
- Modeling decisions:
  - Rigid body?
  - Flexible body?
- "Multi-stage Control" for formation flight:
  - "Course" rigid-body control (m to cm level authority)
    - Thrusters
    - Electromagnets!
  - "Fine" optics control
    - Optical delay lines (cm to µm)
    - Fast-steering mirrors (µm to nm)
- Formation flight control: Assume rigid body dynamics!

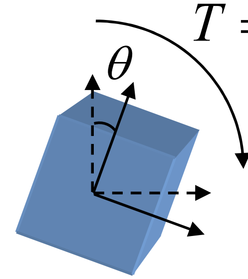
- With loads applied at at the CG, the EOM reduce to:
  - Linear position:



$$F = m\ddot{x}$$

  - Angular position:

$$T = I\ddot{\theta}$$



  - Need good mass and inertia estimates of the plant!
  - When loads are applied away from the CG and not along principle inertia axes, EOM are more complicated, but mass and inertia estimates are still important.

Euler Angles for Attitude Representation

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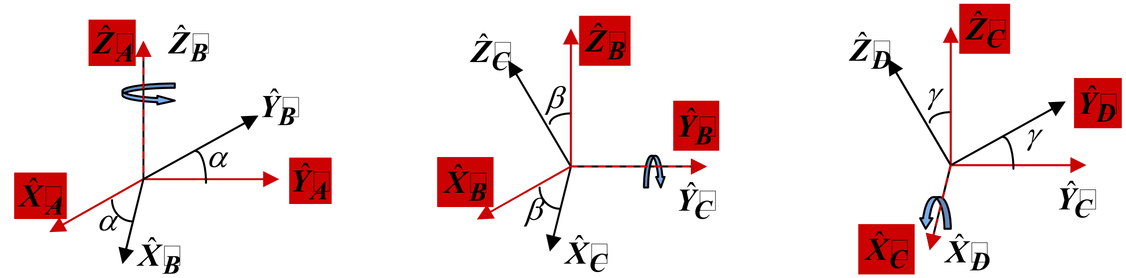
- **Euler angles** describe a *sequence* of three rotations about different axes in order to describe body orientation with respect to a reference coordinate frame.
- Can be defined as a transformation matrix
- Non-unique -> exact sequence is critical!



Rotate about $\hat{Z}_A$ by α

$$R_B^A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

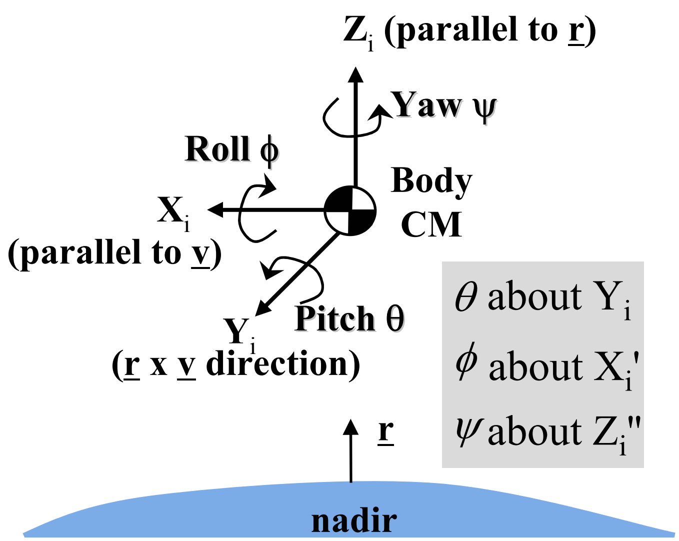
Rotate about $\hat{Y}_B$ by β

$$R_C^B = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

Rotate about $\hat{X}_C$ by γ

$$R_D^C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

$$R_D^A = R_B^A \, R_C^B \, R_D^C$$



Goal: Describe kinematics of body-fixed frame with respect to rotating local vertical

Transformation from Body to "Inertial" frame:

$$T_{B/I} = \underbrace{\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{YAW}} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}}_{\text{ROLL}} \cdot \underbrace{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}}_{\text{PITCH}}$$

Note: $T_{B/I}^{-1} = T_{I/B} = T_{B/I}^{T}$

Quaternions for Attitude Representation

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EULER'S THEOREM

The Orientation of a body is uniquely specified by a vector giving the direction of a body axis and a scalar specifying a rotation angle about the axis.

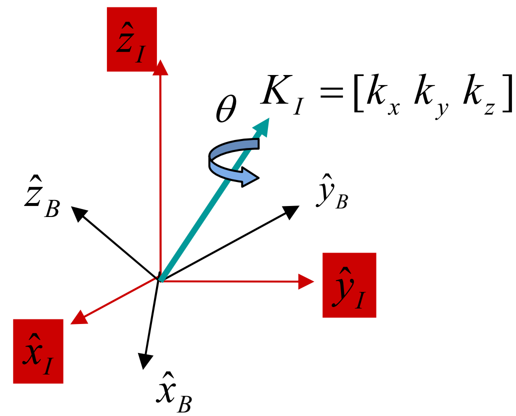
$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \vec{q} \\ q_4 \end{bmatrix}$$

$\vec{q}$ = A vector describes the axis of rotation.

q_4 = A scalar describes the amount of rotation.

- Eliminates the singularity caused by Euler angles.
- Definition introduces a redundant fourth element, which eliminates the singularity.
- This is the **"quaternion"** concept.
  - Has no intuitively interpretable meaning to the human mind
  - Is computationally convenient, robust.
  - Ideal for digital control implementation.

I: Inertial
B: Body



CONSTRAINTS:

$$q_1 = k_x \sin\left(\frac{\theta}{2}\right) \qquad q_2 = k_y \sin\left(\frac{\theta}{2}\right) \qquad K_I = \begin{bmatrix} k_x & k_y & k_z \end{bmatrix}^T$$

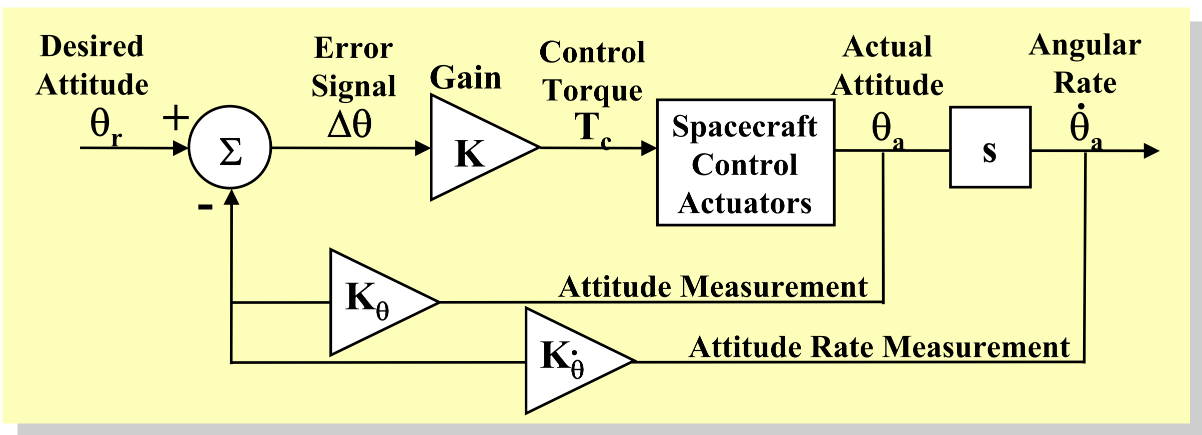
$$q_3 = k_z \sin\left(\frac{\theta}{2}\right) \qquad q_4 = \cos\left(\frac{\theta}{2}\right) \qquad |K_I| = 1 \longrightarrow |Q| = 1$$

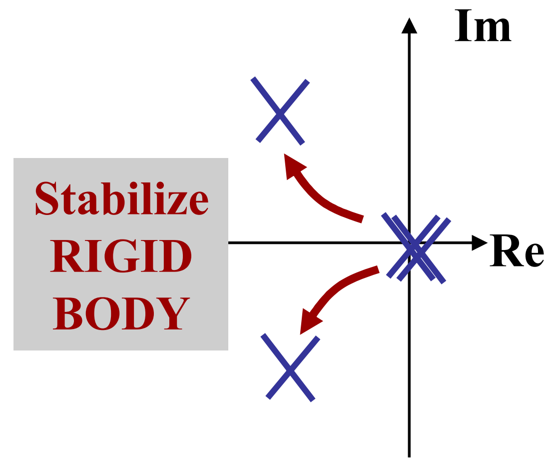
Feedback Control

- With no internal or external torques, the rigid-body rotational EOM reduces to:

$$T = I\ddot{\theta} \xrightarrow{\;L\;} Is^2\theta = 0 \longrightarrow \text{Poles } s_{1,2} = 0$$

- Now add **feedback control** to sense the spacecraft "state" and supply commands to change the system dynamics (i.e. to move the poles where we want them).
- **Example**: angular position and rate feedback for attitude control:





Regulator: θ_r=0
Feedback Control Torque:

$$T_c = I\ddot{\theta} = -K\,(K_{\dot{\theta}}\,\dot{\theta} + K_\theta\,\theta)$$

$$\xrightarrow{\;L\;} Is^2\theta + K\;K_{\dot{\theta}}\,s\,\theta + K\,K_\theta\,\theta = 0$$

Torque ~ θ, $\dot{\theta}$ -> rotational "spring-mass-damper!"

$$\text{Poles } s_{1,2} = -\frac{K\;K_{\dot{\theta}}}{2I} \pm \sqrt{\left(\frac{K\;K_{\dot{\theta}}}{I}\right)^2 - \frac{4K\,K_\theta}{I}}$$

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Gain and Bandwidth

- Compare to spring-mass-damper 2nd order characteristic equation:

$$s^2 + \frac{K K_{\dot{\theta}}}{I} s + \frac{K K_{\theta}}{I} = 0 \quad \longleftrightarrow \quad s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\sqrt{\frac{K K_{\theta}}{I}} \longleftrightarrow \omega_n \quad , \quad \frac{K K_{\dot{\theta}}}{I} \longleftrightarrow 2\zeta\omega_n$$

Natural Frequency:

$$\omega_n = \sqrt{\frac{K K_{\theta}}{I}}$$

Damping Ratio:

$$\zeta = \frac{K_{\dot{\theta}}}{2}\sqrt{\frac{K}{I K_{\theta}}}$$

- This natural frequency is ~ equal to the system bandwidth.

$$f = \frac{\omega_n}{2\pi} \quad \Rightarrow \quad \tau = \frac{1}{f} = \frac{2\pi}{\omega}$$

- τ is the system time constant.

EXAMPLE:

$$I = 1000 kgm^2$$
$$K = 100 Nm/rad$$
$$K_{\theta} = 500 Nm/rad$$
$$K_{\dot{\theta}} = 100 Nm\,s/rad$$

$$\omega_n = 7.1\, rad/s$$
$$\zeta = 0.71$$
$$f = 1.1\, Hz$$
$$\tau = 0.89\, s$$

Feedback Control: Sensors

- Used to "sense" the system or *measure* its current "state"
  - Linear or angular positions, velocities, and accelerations
  - *Absolute/inertial* vs. *relative* measurements
- Examples discussed here:
  - GPS
  - Magnetometers *
  - Star Trackers
  - Sun Sensors
  - Limb Sensors
  - Rate Gyros*
  - Inertial Measurement Units (IMUs)*
  - Ultrasound/IR Distance Sensor*

* Useful in an indoor EMFF testbed environment

Sensors: GPS and Magnetometers

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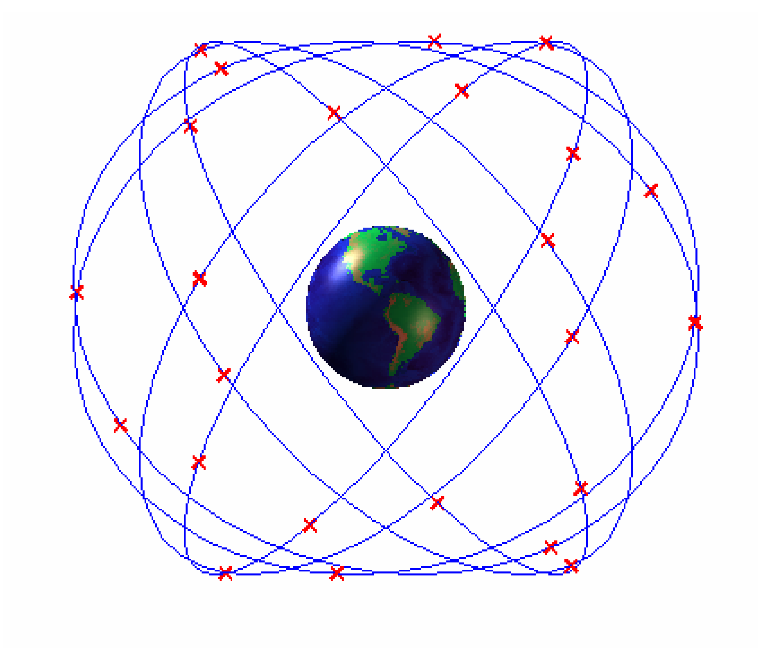
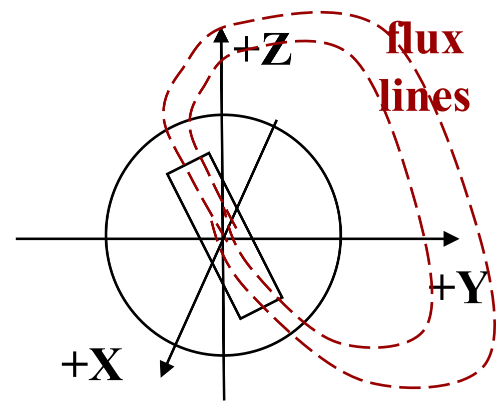
- Global Positioning System (GPS)
  - Currently 27 Satellites
  - 12hr Orbits
  - Need 4 satellites to transmit to ground receiver
  - Accurate Timing
    - Selective Availability 100 m
    - Stand-Alone 10-20 m
    - Carrier-smoothed DGPS 1-2m



- Magnetometers
  - Measure components B_x, B_y, B_z of ambient magnetic field B
  - Sensitive to field from spacecraft (electronics), mounted on boom
  - Get attitude information by comparing measured B to modeled B
  - Typical accuracy: 1 degree
  - Economical, orbit-dependent, LEO only
  - Tilted dipole model of earth's field:

$$\begin{bmatrix} B_{north} \\ B_{east} \\ B_{down} \end{bmatrix} = \left(\frac{6378}{r_{km}}\right)^3 \begin{bmatrix} -C_\varphi & S_\varphi C_\lambda & S_\varphi S_\lambda \\ 0 & S_\lambda & -C_\lambda \\ -2S_\varphi & -2C_\varphi C_\lambda & -2C_\varphi S \end{bmatrix} \begin{bmatrix} -29900 \\ -1900 \\ 5530 \end{bmatrix}$$

where: C=cos , S=sin, ϕ=latitude, λ=longitude
Units: nTesla



ACS Sensors: Star Trackers

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- Star Trackers with Star Catalogue
  - Images stars on a CCD, identifies stars and their celestial positions using catalogue, calculates S/C angles, not trivial
  - Single star capability gives two axes requiring two star trackers for three axes
  - Multiple star capability gives three axes using one device
  - Arc-second accuracy, BUT: high cost, power, weight, requires substantial processing
  - Sensitive to sun, earth, moon stray objects (dust) in FOV
  - Prefer to use bright stars to avoid catalogue confusion problems
  - Typical accuracy: 0.001 degrees
  - Max update rate about 0.2 - 1 Hz
  - Heavy, complex, expensive, most accurate
  - Gimbaled Trackers: points at star, angles from gimbal, moving parts, wide effective FOV
  - Fixed Head Sensors: electronic scan, or by S/C motion, no moving parts, narrow FOV

ACS Sensors: Sun and Limb Sensors

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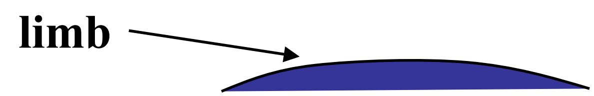
- Sun Sensors measure two-axis direction to SUN
  - Does not measure S/C roll about the S/C-sun line
  - Exist as analog or digital sensors
  - Single axis or two-axis
  - Can be used spinning or despun
  - Typical accuracy: 1 min
  - Simple, reliable, & low cost, but not always visible

- Limb Measures Edge of Earth or other Body
  - Good for scanning instruments
  - Orbit-dependent
  - Most sensitive in IR at 12-16 mm (“CO2-band”); good day/night sensitivity, insensitive to cloud coverage
  - 15 mm horizon varies +/- 20 km -> angle accuracy ~ 0.05 degrees
  - Normally scan in cone, around pitch axis; can get both pitch and roll
  - Relatively expensive, but less expensive sensors with no moving parts exist
  - Forms basis of common LEO ACS design



Sensors: Rate Gyros and IMUs

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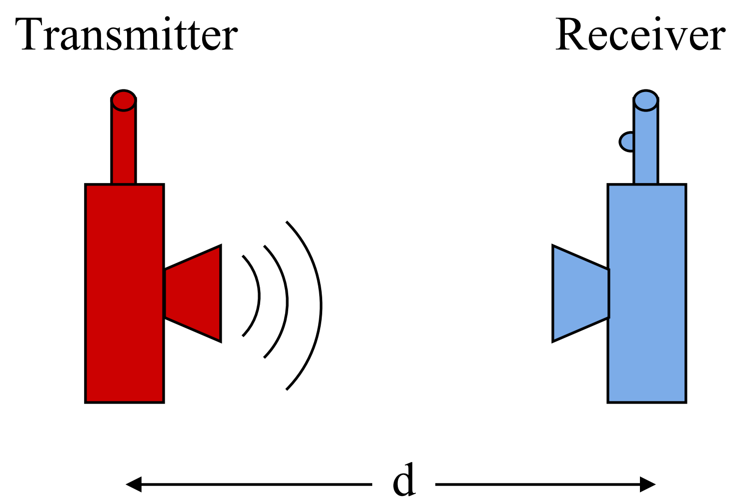
- Rate Gyros (Gyroscopes)
  - Measure the angular rate of a spacecraft relative to inertial space
  - Need at least three. Usually use more for redundancy.
  - Failing gyros are critical (e.g. HST)
  - Can integrate angular rate to determine angle.
  - However…
    - DC bias errors in electronics cause the output of the integrator to ramp and eventually saturate (drift)
    - Thus, need inertial update!

- Inertial Measurement Unit (IMU)
  - Integrated unit with sensors, mounting hardware,electronics and software
  - measure translation of spacecraft with accelerometers
  - measure rotation of spacecraft with rate gyros
  - often mounted on gimbaled platform (fixed in inertial space)
  - Typical gyro drift rate: 0 .003 to 1 deg/hr
  - Frequently updated with external measurement (Star Trackers, Sun sensors, etc.)

Sensors: IR and US Ranging System

- Used by the SPHERES program
  - For telemetry
  - To update drifted IMU sensors
- Sends simultaneous Infrared and Ultrasound pulses
  - IR pulse travels at the speed of light, almost "instantaneously" compared to the US pulse.
  - US pulse travels at the speed of sound.
  - Time delay of US pulse determines range.
  - Triangulation is used to measure absolute position and angle of body with respect to inertial coordinates.
- Use of thrusters interferes with this system
  - High speed gas creates ultrasonic white noise
  - Noise triggers receivers prematurely



$$d = c_{US}(t_{US} - \cancelto{0}{t_0}) - c_{IR}(\cancelto{\sim 0}{t_{IR}} - \cancelto{0}{t_0})$$

$$\boxed{d = c_{US} t_{US}}$$

Feedback Control: Actuators

- Used to “actuate” the system or *change* its current “state”
- Examples discussed here:
  - Reaction Wheel Assemblies (RWAs)*
  - Control Moment Gyros (CMGs)
  - Magnetic Torque Rods*
  - Thrusters
  - Electromagnets!*

* Useful in an indoor EMFF testbed environment

Actuators: Reaction Wheel Assemblies (RWAs)

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- Most common attitude actuators
- Operate on the principle of conservation of angular momentum: flywheel on motor accelerates in one direction, causing spacecraft to rotate in opposite direction about same axis.
- Fast -> continuous feedback control
- Moving parts; relatively high power, weight, and cost
- Internal torque only; “external” torque required for “momentum dumping”
- Control logic simple for independent axes (can get complicated with redundancy).
- For three-axes of torque, three wheels are necessary. (Four for redundancy.)
- Wheels accelerate to counteract external torques, eventually reaching an RPM limit (~3000-6000 RPM), or “saturation.”
- Static & dynamic imbalances can induce vibrations
  - RWAs need to be carefully balanced.
  - Can be mounted on isolators.
- Usually operate around some nominal spin rate to avoid stiction.

Typical RWA Data:

Operating Range: 0 +/- 6000 RPM
Angular Momentum @ 2000 RPM: 1.3 Nms
Angular Momentum @ 6000 RPM: 4.0 Nms
Reaction Torque: 0.020 - 0.3 Nm

Actuators: Control Moment Gyros

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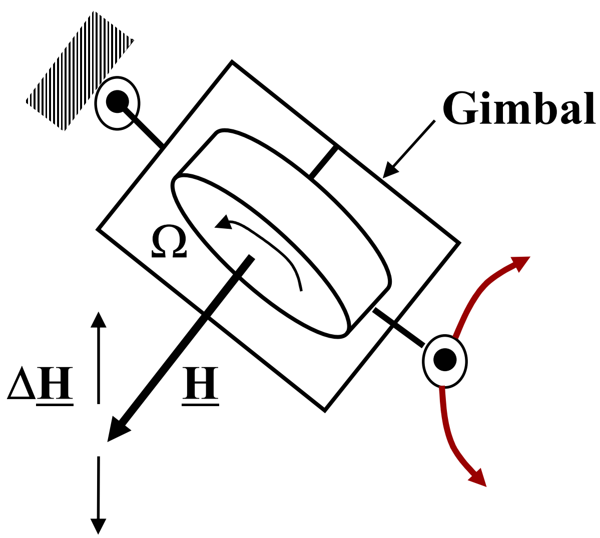
- Like a gimbaled momentum wheel. Torque applied at gimbal produces change in cross-axis momentum, hence reaction torque on BODY.
- Heavy, but can give very high control authority, exceeding large RWA by a factor of 100 or more.



Honeywell CMG's (content.honeywell.com/space /products/mom_controls.htm)

- **Control Moment Gyros** are single wheels that spin at a constant rate
- Torques are created by gimbaling its angular momentum vector (spin axis)
  - Generate greater torques than RWA
  - Constant spin rate means that vibrations are at known frequencies

Actuators: Magnetic Torquers

- Often used for Low Earth Orbit (LEO) satellites
- Useful for initial acquisition maneuvers
- Commonly use for momentum desaturation ("dumping") in reaction wheel systems
- May cause harmful influence on star trackers
- Slow, but low weight and cost
- Typical accuracy: 0.01 deg

- Can be used:
  - For attitude control
  - To de-saturate reaction wheels
- Torque Rods
  - Torque rods are long helical coils
  - Use current to generate magnetic field
  - This field will try to align with the Earth's magnetic field, thereby creating a torque on the spacecraft
  - Can also be used to sense attitude as well as orbital location

Actuators: Thrusters/Jets

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- Thrust can be used to control attitude but at the cost of consuming fuel
- Fuel supply often limits the satellite lifetime
- Calculate required fuel using "Rocket Equation"
- Advances in micro-propulsion make this approach more feasible. Typically want $I_{sp} > 1000$ sec
- Use consumables such as Cold Gas (Freon, N_2) or Hydrazine (N_2H_4)
- Must be ON/OFF operated; proportional control usually not feasible: pulse width modulation (PWM)
- Redundancy usually required, makes the system more complex and expensive
- Fast, powerful, but high cost
- Often introduces attitude/translation coupling
- Standard equipment on manned spacecraft
- May be used to "unload" accumulated angular momentum on reaction-wheel controlled spacecraft
- Typical accuracy: 0.1 degrees

Introduction to State Space Representation

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- An alternative representation to classical control
  - Useful for representing MIMO systems.
  - Classical control useful only for SISO systems.
- However, limited to *linear* systems. Can treat nonlinear systems by *linearizing* equations of motion.
- CONCEPT: Any system's equations of motion (linear or nonlinear) can be written in the form:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t)$$

where:

- the "states" (usually positions and velocities) are $\underline{x} = [x_1 \quad x_2 \quad \ldots \quad x_n]^T$
- the control variables are $\underline{u} = [u_1 \quad u_2 \quad \ldots \quad u_m]^T$
- f is a set of p linear or nonlinear equations
- The EOM can be *linearized* about a nominal point (x_0, u_0) and reduced to the form:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

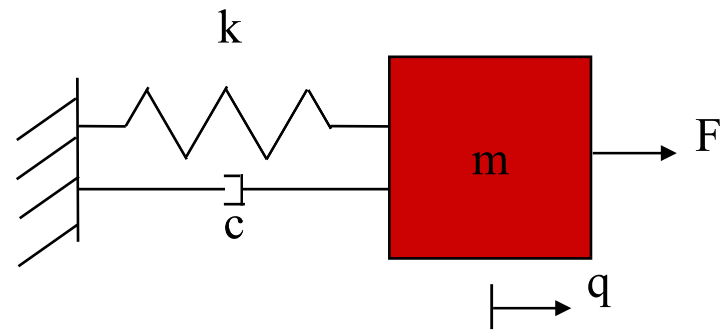
where: $$A = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}_0, \underline{u}=\underline{u}_0} \qquad B = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\underline{x}=\underline{x}_0, \underline{u}=\underline{u}_0}$$

- For LTI systems, A and B are constants.

State Space Representation Example

- Example: Linear Spring-Mass-Damper
  - Already an LTI system
  - EOM is:

$$m\ddot{q} + c\dot{q} + kq = F$$



  - Define state as vector of position and velocity:

$$x = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T$$

- The linear EOM can be written as:

$$\underline{\dot{x}} = A\underline{x} + B\underline{u} \longleftrightarrow \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

where:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \qquad \underline{u} = F$$

- Note: the first equation is trivial ($\dot{q}=\dot{q}$) so that the EOM can be written in state space form.

State Space Linearization Example

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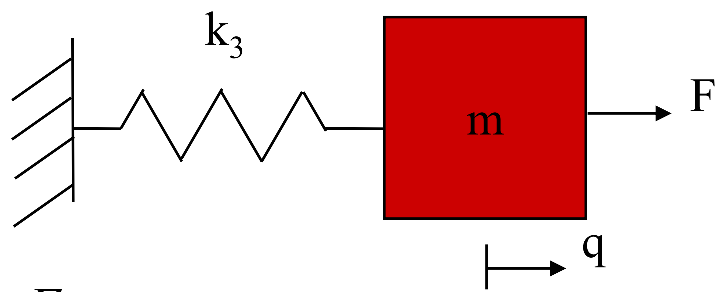
- Another Example: Nonlinear Spring-Mass
  - Already a time-invariant system
  - Must *linearize* to achieve LTI
  - Nonlinear EOM is:



$$m\ddot{q} + k_3 q^3 = F$$

$$\Rightarrow \ddot{q} = f_2(q, \dot{q}, F) \text{ where } f_2 = -\frac{k_3 q^3}{m} + \frac{F}{m}$$

  - Trivial EOM is:

$$\dot{q} = f_1(q, \dot{q}, F) = \dot{q} \qquad \underline{f} = \begin{bmatrix} f_1 & f_2 \end{bmatrix}^T$$

  - Linearizing about nominal values $x_0 = [q_0 \ 0]^T$ and $u_0 = 0$, we find the linearized EOM in state-space form:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \longleftrightarrow \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{3k_3 q_0^2}{m} & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

where:

$$x = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T \qquad \underline{u} = F$$

$$A = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\underline{x}=\underline{x}_0, \underline{u}=\underline{u}_0} = \begin{bmatrix} 0 & 1 \\ -\frac{3k_3 q_0^2}{m} & 0 \end{bmatrix} \qquad B = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\underline{x}=\underline{x}_0, \underline{u}=\underline{u}_0} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

State Space Representation Tools

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- "Full" State Space representation consists of *dynamic* and *measurement* equations:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad y = C\underline{x} + D\underline{u}$$

  where y is the sensor measurement
  - Usually a linear combination of the states, x, determined by matrix C
  - Usually D=0. This term represents "feedthrough" of control signal to sensor measurement -> messy!
- Two useful analysis tools:
  - Controllability Matrix: $\mathcal{C} = [B \;\; AB \;\; A^2B \cdots A^{n-1}B]$
  - Observability Matrix: $\mathcal{O} = [C\,;\, CA\,;\, CA^2\,;\cdots;\, CA^{n-1}]^T$
- By simply calculating these matrices, we can determine whether a system is "controllable" and/or "observable" using the following criteria:

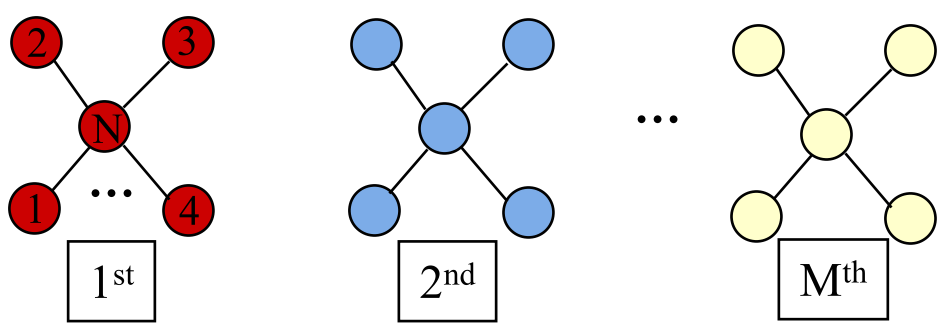
Observable iff rank($\mathcal{O}$) = n
Controllable iff rank($\mathcal{C}$) = n

where n is the number of states.

Dipole Geometry and Configuration

EMFF CDIO

- Tool developed by Dr. Edmund Kong in his Ph.D. thesis
  - Based on State Space representation of an EMFF satellite configuration
  - Developed in Matlab
  - User inputs dipole geometry
    - Number of EM poles
    - Pole strengths
    - Pole positions
  - States are:
    - Positions and velocities of poles *relative* to the M^{th} spacecraft
    - *Absolute* angles via quaternions ($\underline{Q}$) and *absolute* angular rates ($\underline{\omega}$) of poles
    - Example: M spacecraft, each with N poles



  - Automated code
    - Linearizes the highly nonlinear equations of motion
    - Generates A and B matrices for the configuration
    - Tests for controllability by checking rank of C

Dipole Modeling Tool: Sample Results

EMFF
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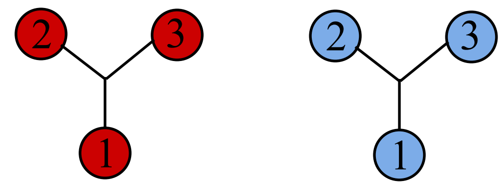
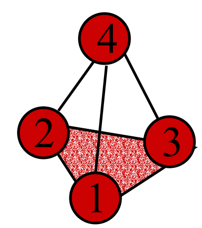
- Sample results of small-perturbation control (no attitude control) from Edmund Kong’s Ph.D. thesis:
  - Two dipoles

    - At rest, two dipoles with poles aligned in a 2-D plane
    - Result: rank(C)=2 -> Can control only position and velocity in 1-DOF, as expected.
  - Two Y-poles
    - 120 degree angles and 75 meter baseline
    - Result: rank(C)=4 -> Can control only 2-D positions and velocities.



  - TPF 2-D Case
    - Five-spacecraft results:
    - Dipoles: rank(C)=8 -> not fully controllable.
    - Y-poles: rank(C)=26 -> Fully controllable in 2-D.
  - TPF 3-D Case
    - Same five-spacecraft configuration
    - Need rank(C)=54 -> only achievable with a four-pole configuration, with poles not all in the same plane.



Summary

- Formation Flight Applications
- Formation Flight Maneuvers
- Formation Flight Control
  - Modeling the "Plant"
  - Euler Angles vs. Quaternions
  - Feedback Control
    - Sensors
    - Actuators
- State Space Representation
  - Concept and Examples
  - Tools
- Dipole Geometry and Configuration
  - Framework
  - Sample Results

References

- Professor Olivier de Weck and Ms. Alice Liu, "Attitude Determination and Control," 16.684 Lecture, 2001. (On CDIO2 Website.)
- James R. Wertz and Wiley J. Larson: "Space Mission Analysis and Design", Second Edition, Space Technology Series, Space Technology Library, Microcosm Inc, Kluwer Academic Publishers
- Dr. Edmund Kong, Ph.D. Thesis Chapter 5, 2002. (Soon to be on ssl.mit.edu website.)
- Alvar Saenz-Otero, "The SPHERES Satellite Formation Flight Testbed: Design and Initial Control," SM Thesis. (On ssl.mit.edu website.)