Welcome back to recitation. What I'd like us to do in these two problems is to understand how to compute the flux in three dimensions-- the flux of a vector in three dimensions-- across a surface, without maybe doing a lot of calculations. So we're going to see if we can figure out how to do these problems without doing a lot of computation.

So the first one is to find the flux of the vector k through the infinite cylinder x squared plus y squared equals 1. So notice this doesn't depend on z, but in fact, at every height it is a unit circle. So it's an infinite cylinder. And then the second problem I'd like you to think about and to try is to find the flux of the vector j through one square that has side length 1 in the xz-plane. So you pick any square, in the xz-plane, of side length 1 and find the flux of j through that square. And so I think that that's enough information. So why don't you try both of those problems, pause the video, and then when you're ready to see my explanation of how they work, bring the video back up.

OK, welcome back. Again, what we're trying to do is to understand the flux of a vector field across a surface. And we're hoping to do it with the least amount of calculation possible, for these particular problems. So obviously it's going to be helpful if you can draw a picture, to draw a picture. So I'm going to draw the surface that is the infinite cylinder first and then I'm going to look at the vector field k.

So let me draw my picture first. If these are my coordinate axes, I get-- I think you have something usually like this. This is x, this is y, and this is z. This is how you do them in your class. And so it's not going to be a great picture but I'm going to try and make it look like a cylinder up here coming down. So this is in fact going to be infinitely long, going down forever, but I'll stop at somewhere down here. And then so every slice in the z, at a fixed height for z, is going to be a circle. I should think about these are intersecting the x- and y- axes at (1, 0) and (0, 1). So there's actually a sort of unit circle down here as well.

Now that's the surface. Now what does the vector field look like? Well, in general what does the vector field look like? k is just a constant vector field that points in this direction. This is k. So k at every point on the surface is just the vector that's pointing straight up in this direction. And the normal to the surface, if you think about it, the normal to the surface is independent of z. It doesn't depend on z at all. It is always going to be a vector that is in the x-y direction only. It's going to be-- essentially at every point, it's going to be sitting in the plane z equals a constant.

Because if you think about what you have, you have a unit circle at every height. And it doesn't vary in the z-direction at all, the bending of that unit circle. So in fact the normal is always going to point straight out from the unit circle. There's going to be no z-component. Or I should say the z-component's 0. Maybe that's the best way to say it. So that means that the normal dotted with k is going to be 0. And so the answer to the first question is the flux of k through the infinite cylinder is actually 0. So the answer to part a is 0. Been getting a lot of zeroes in my video so far.

The next one is to find the flux of the vector j through one square in the xz-plane where the squares have side length 1. So I can draw any square I want. That seems to imply that maybe it'll be the same answer everywhere. So let's see what we get. Let me draw a picture for part b. Label this first maybe. Let me label my axes. And I'm going to draw the simplest one I can. That does not look like a square but I'm not great at this. There we go.

So this is my surface sitting in the x-z plane. This side length is 1 and this side length is 1. So what is the normal to that surface? Well, we have two choices and so we will actually have a possibility of two answers. So let me point out that the normal to the surface-- well, what direction does it point in? Because this plane is in the xz-plane, the normal to the surface is either j or it's minus j.

And so if I'm integrating j dotted with the normal over the surface-- I'll just call this surface capital R-- if I'm integrating over R j dotted with the normal dS, j dotted with the normal is going to be either 1 or minus 1. And hopefully that makes sense because j-- let me draw this-- j is pointing exactly in the y-direction. And the normal is either in this direction or in the opposite direction. Up to how I choose to orient the surface.

And so j dotted with n is either plus or minus 1, and so I just get the area of R with a plus or minus-- ooh, that doesn't look like a plus or minus-- with a plus or minus in front. Depending on whether j dotted with n is 1 or whether j dotted with n is minus 1. So the solution for this computation is just the area of R or minus the area of R. Well, what's the area of the region? The area, it's a square of side length 1, so it has area 1. So the final answer is just plus or minus 1.

So again, let me remind you what we're trying to do. We're trying to determine these fluxes of vector fields across surfaces without doing a lot of calculation. And in the first case we had a vector field that pointed in the z-direction only, and the normal was only in the x and y direction. And so the flux was 0, even though it was a vector field on an infinite cylinder, the flux was still 0.

And in the other case, I had actually here, I had a surface that was exactly in the xz-plane. And so its normal was exactly either in the same direction as j or 180 degrees around from j. So j dotted with the normal was either plus or minus 1. And so I only had to know the area of the region. Which is why it didn't matter where I moved this unit square. I didn't tell you where the unit square had to sit, so that's where you can see why it didn't matter. Because it's just the area. OK, I think that's where I'll stop.