PaK

11/18/05

Permutation patterns

$w \in S_k$

Conj. # w-avoiding $\sigma \in S_n < c^n$ for some constant c depending on w $\ell(w) \geq 3$

Stanley-Wilf

Prop: # w-avoiding ~~$\ell(w) \geq 3$~~ is $\geq c^n$ for some c $\ell(w) \geq 3$

pf: contains length 3-avoiding, which we already said is that big

Prop (Arratia)

Assume $|\Sigma_n(w)| < c^n$. Then $\exists \lim_{n \to \infty} \frac{\log|\Sigma_n(w)|}{n} < \infty$

Pf: Lemma^{WWB} $|\Sigma_{m+n}(w)| \geq |\Sigma_m(w)| \cdot |\Sigma_n(w)|$

look at $\sigma_1 \in S_m$ next to $\sigma_2 \in S_n + m$, that makes avoiding perm when σ_1, σ_2 avoid w + n is left of 1, o/w symmetry ✓

Def'n A 0-1 matrix in $\mathbb{R}^{n^2}$, B another in $\mathbb{R}^{k^2}$

A contains B if $\exists$ k rows + k columns ~~[illegible]~~ s.t. $1 \in B \Rightarrow 1 \in$ corresponding minor

A avoids B otherwise

Conj (Füredi - Hajnal) $\in S_k$ $\in \mathbb{R}^{k^2}$

$\forall$ permutation matrix P, A P-avoiding $\Rightarrow$ $|A| = \#\, 1\text{'s} = O(n) = \alpha(k) \cdot n$

Lemma: FH $\Rightarrow$ SW

$T_n(P) = \#\, P$-avoiding $n \times n$ 0-1 matrices

$|T_{2n}(P)| \le |T_n(P)| \cdot 15^{O(n)}$

divvy up $2n \times 2n$ into $n\times n$ $\boxplus$ blocks, change to $n \times n$ which has 1 iff $\exists$ 1 in the block, this avoids iff orig does (since $P \in S_n$, so $\exists!$ 1 per row, column) Also this happens $15^{\max \# 1\text{'s in } P\text{-avoiding}} = 15^{O(n)}$ by F-H $\Rightarrow$ telescoping to get $\le 15^{2\alpha(n)}$ ✓

Claim: $f(n,P) = \max\limits_{A \in T_n(P)} |A|$, then $\cancel{f(n,P) \le (k-1)^2}$

$$f(n,P) \le (k-1)^2 f\left(\tfrac{n}{k^2}, P\right) + 2k^3 \binom{k^2}{k} \cdot n$$

iterate gives $\le \alpha(k)\cdot n$, ~~which is enough~~

pf: Divide into $k^2 \times k^2$ blocks, as before let new matrix have 1 iff $\exists$ at least one 1 in the block. As before, new matrix is P-avoiding, he'll finish next time