Massachusetts Institute of Technology

Department of Ocean Engineering

Department of Civil and Environmental Engineering

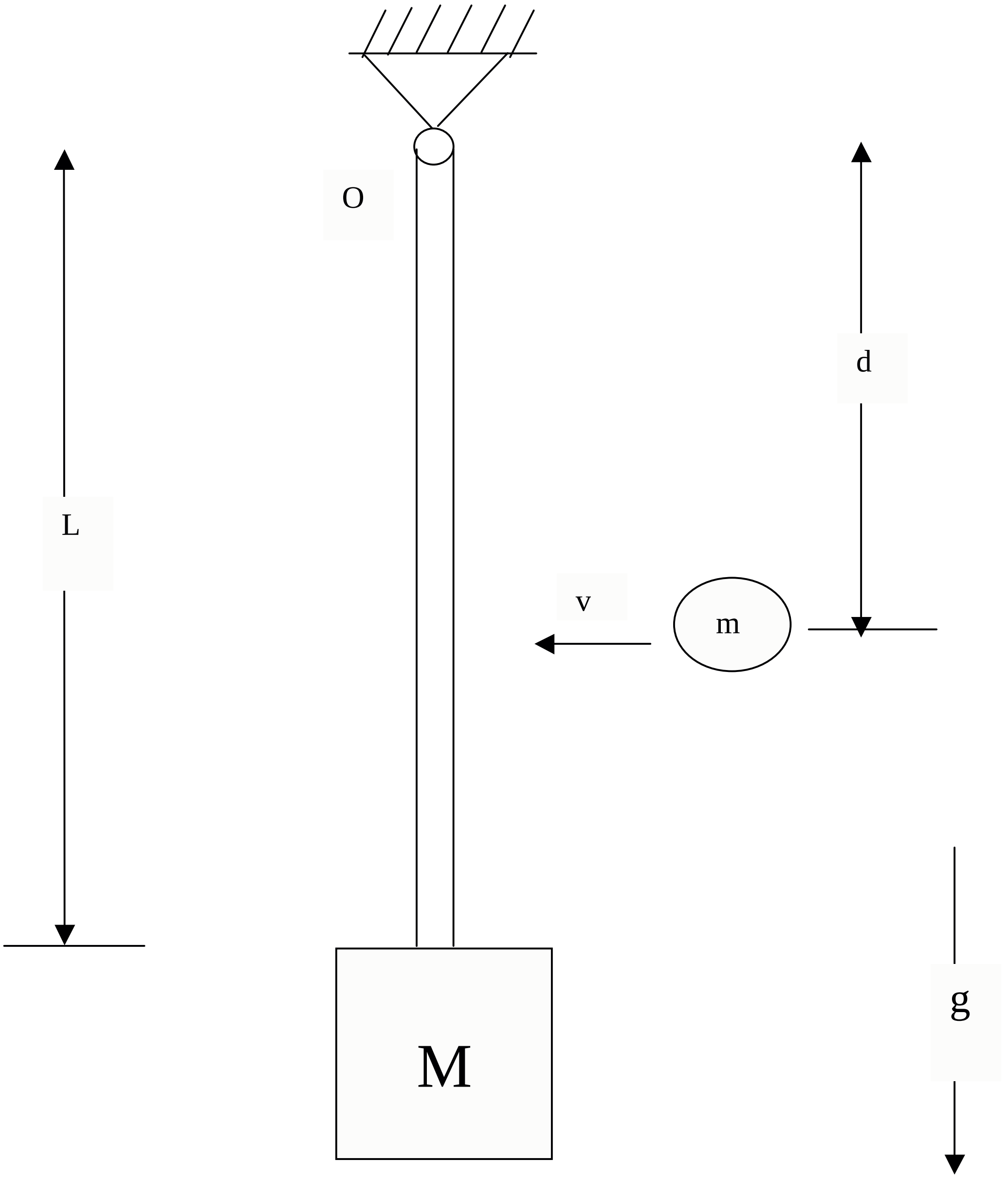
13.013J/1.053J Dynamics and Vibration

Fall 2002

Quiz I

- “Closed book and notes”, one sheet of formulas allowed.
- Individual effort.
- Read all problems first.
- This quiz contains 6 printed pages.

Figure for Problem 1:



Problem 1: **(50 Points)**

A pendulum of mass *M* with a massless rigid rod of length *L* hanging from a fixed frictionless pin *O* is initially at rest in a field with acceleration due to gravity *g.* The mass *M* can be considered as a point mass (particle with negligible dimensions). A particle of mass *m* and horizontal velocity *v* flying at a vertical distance *d* from the pin collides with the pendulum rod and sticks to it.

a) a i. Explain why kinetic energy and linear momentum are not in general conserved during the collision.

   a ii. Determine the angular velocity and kinetic energy of the system immediately after the collision, and the kinetic energy loss during the collision, as a fraction of the initial kinetic energy of the mass *m*.

   a iii. Determine the impulse from the pin *O* on the rod during the time of the collision.

   Hint: Impulse is the integral of the force $\overrightarrow{F(t)}$ over the time of collision $\vec{J} = \int_{0-}^{0+} \overrightarrow{F(t)}\, dt$, assuming the time interval of collision is very small.

(2 + 10 + 5 = 17 Points)

b) Determine the second order (nonlinear) ordinary differential equation of motion of the system after the collision. The motion can be described in terms of the angle $\theta(t)$ the rod makes with the vertical line through *O*.

(10 Points)

c) c i. Determine the maximum angle θ_{max} the pendulum will make with the vertical after the collision.

   c ii. Determine under what condition for the initial speed *v* will the maximum angle of the pendulum after collision be equal to π *radians*.

(5 + 3 = 8 Points)

d) Determine an expression for the force from the rod on the pin after the collision as a function of $\theta(t)$.

   ***Hint*:** Eliminate any dependence on $\dot{\theta}(t)$ and leave only the dependence on $\theta(t)$ using an appropriate conservation law valid after the collision.

(10 Points)

e) Determine an *integral expression* for the time that will elapse between the collision and the maximum angle θ_{max} of question (c i).

   ***Hint*:** Using an appropriate conservation law valid after the collision (just as in *question d*), express $\frac{d\theta}{dt} = f(\theta)$, where $f(\theta)$ is a function of θ.

(5 Points)

Figure for Problem 2:

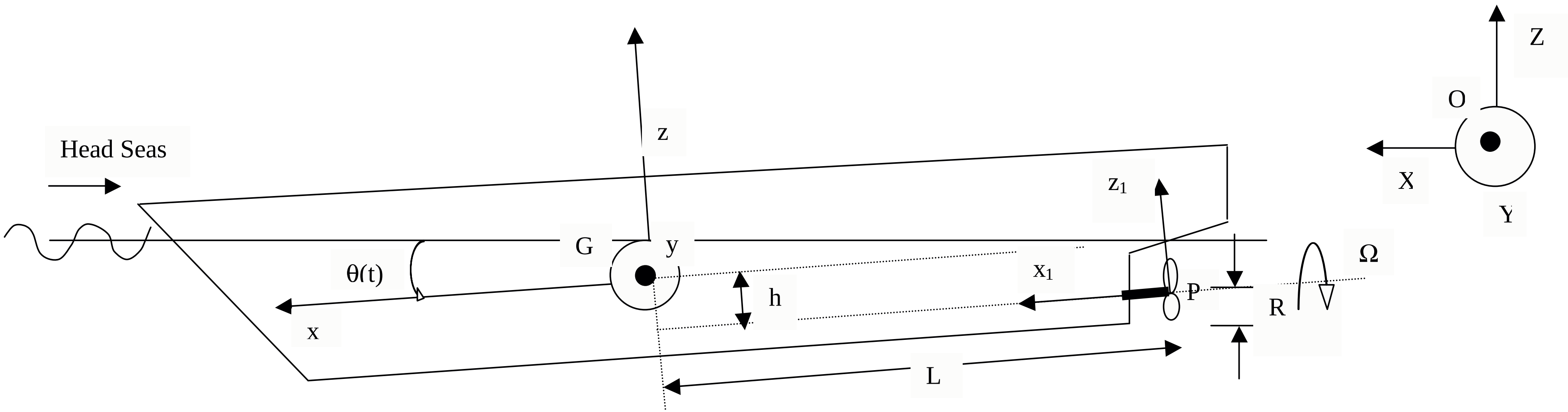
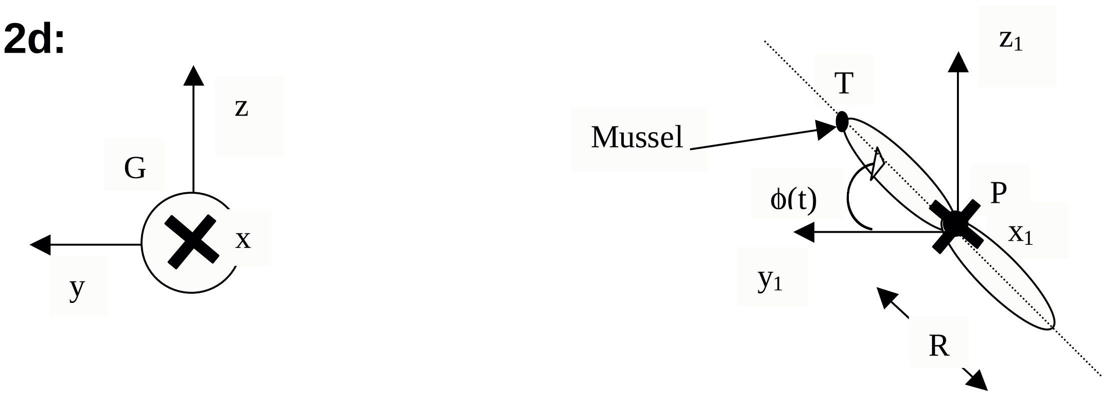


Figure for Problem 2c, 2d:



Problem 2: (50 Points)

A ship is traveling at a constant velocity V_{mean} with respect to the earth, along the axis *OX*, where *OXYZ* is an inertial system (with unit vectors $\vec{I}, \vec{J}, \vec{K}$) attached to the earth. A moving system *Gxyz* (with unit vectors $\vec{i}, \vec{j}, \vec{k}$) is attached to the ship at its center of gravity *G*, so that the axis *Gx* makes an angle $\theta(t)$ to the horizontal axis *OX*. Due to head seas (a storm with waves whose crests are parallel to *OY* axis and propagating along *OX* axis) the ship oscillates in the vertical plane heaving and surging, and rotates about an axis orthogonal to its center plane (ie. pitching). The velocity of *G* with respect to the inertial frame is

$$\overrightarrow{V_G} = V_{mean}\vec{I} + u(t)\vec{i} + w(t)\vec{k}$$

where *u(t)* and *w(t)* are the surge and heave velocities of the ship. The angular velocity of the ship with respect to *OXYZ* is $\vec{\omega} = \omega_2(t)\vec{J}$, where $\omega_2(t) = \dot{\theta}(t)$, also known as pitch angular velocity.

At the same time the ship's propeller of radius *R* is rotating with its shaft at a constant angular velocity Ω with respect to the ship, ie.

$$\vec{\omega}_{propeller,ship} = \Omega\vec{i}$$

a) Determine the inertial velocity $\overrightarrow{v_p}$ and acceleration $\overrightarrow{a_p}$ of the center of the propeller, *P* using the ship fixed system *Gxyz* for your calculations. For simplicity of algebra express $\overrightarrow{v_p} = V_{mean}\vec{I} + \alpha\vec{i} + \beta\vec{k}$ and $\overrightarrow{a_p} = \gamma\vec{i} + \delta\vec{k}$, and determine α, β, γ and δ. **(15 Points)**

b) Determine the angular velocity of the propeller with respect to *OXYZ* and its time rate of change with respect to *OXYZ.* **(5 Points)**

c) The line *PT* makes an angle $\phi(t)$ with the *Y* direction (same as direction y_1) and assume, for simplicity that the line *PT* is on a plane Py_1z_1 parallel to the plane *Gyz*. Note that $\dot{\phi}(t) = \Omega$. Find the inertial velocity $\overrightarrow{v_T}$ and the acceleration $\overrightarrow{a_T}$ of the tip *T* for an arbitary $\phi(t)$ and in particular when $\phi = \frac{3\pi}{2}$, ie. when the tip T is at the lowest point.

Hint: Use a system $Px_1y_1z_1$ fixed to the ship with origin at the propeller center *P* and with the axes parallel to *Gxyz*, respectively. **(20 Points)**

d) Imagine that a mussel of mass *m* happens to be attached to the tip *T,* of the propeller. For the condition of *question (c) and* $\phi = \frac{3\pi}{2}$ find the contact force from the blade on the mussel assuming the (static and dynamic) fluid force on the mussel is known and equal to $\overrightarrow{F_f(t)}$. Note that the acceleration of gravity is *g*. Assume that the propeller does not emerge from the ocean surface during the oscillations of the ship. **(10 Points)**