PROFESSOR: I'm very interested in teaching in ways that really help you learn. And one of the most difficult things to learn is when you have to unlearn something you believe. So if you have learned a concept wrong in your previous education and learnings, your mind is really reluctant to let go of it. OK?

And I have a wife who is a middle school science teacher. She also has a PhD in biology. But she really studies to understand kids and how they learn. And MIT students aren't much different from 6th graders, to tell you the truth, when it comes to learning. None of us are. But they're just a little more transparent.

What you can do to help a person unlearn something they've learned wrong, the best time is when you run into something called a "discrepant event." So when you think you know the answer, and I'm asking you these questions, and you're wrong, really try to figure out what it is you believe that led you to that, and see if you can undo that in getting to the answer.

So some of these concept questions, if you'd like to talk about them further, I think they're great for discussions in your recitation section. Look at Professor Gossard here. So talk about these things, and really try to get to the bottom of why you perhaps had a concept that wasn't quite up to what you needed.

OK. We're going to move on. OK. Now we're going to get on to talking about velocities in rotating frames. And by velocities, I also mean derivatives of vectors in rotating frames.

Just to give you a quick reminder of where we started this conversation, this is concept if you have three points-- A, B, and O in this fixed frame. This is r A with respect to O, B with respect to A, and B with respect to O, all vectors.

And we're interested in computing the velocities of, say, point B. So we were talking about the derivative with respect to time of this vector. And we can make it up as a sum of the derivatives with respect to time of these other two vectors.

But now there's a really important point. When you're taking time derivatives, you have to be explicit about what it's with respect to-- what frame of reference you're talking about. In this case we want to know the velocity with respect to this reference frame. So both of these derivatives have to be taken with respect to that frame.

Well this one's easy. This is purely translational. It has nothing to do with rotations, or won't. We're going to think in terms of having an object out here. So maybe a rigid body to which we have attached a rotating frame at A. And I'll call this the rotating reference frame A x prime y prime. And it might be rotating. But this point is just the point with which we describe the translation of the body.

So this one's pretty easy. This one's just the velocity of A with respect to O. But this one has complications. And I'm going to do it first, the answer to this, just intuitively-- appeal to your intuition about why this should be.

So this is my merry-go-round. And I've set up my little coordinate system. This is my A, x prime y prime coordinate system. I've stuck my dog here on the x-axis.

And if he's running some direction, he has a velocity with respect to this reference frame. If you're here at A, watch it, and you'll see him moving. And it will have nothing to do with the rotation. You will just see the dog moving.

And if this is rotating, and you're sitting there rotating with it watching the dog, does the motion of the dog change? You just see it just the same way, right?

OK. So there's one part of the motion of the dog that's with respect to this frame that has nothing to do with rotation. OK.

So now let's say in an outside, and from an outside point of view, if we measure the velocity of the dog with respect to point A-- that's the one you can see-- and to observer outside says what's the velocity of dog with respect to A, will the answer differ if there's rotation?

OK. Now, what we're going to find out is that there's actually two contributions. So the dog running with respect to this merry-go-round is one contribution to the velocity. There's another contribution that comes from the fact that you're sitting out there watching, and you see definitely the dog move when the thing rotates, right?

OK. And if we only do that, if the only rotation-- and we'll do a case where the dog's not running, just sitting still-- what's the velocity of the dog with respect to point A due to the rotation?

I mean you've done lots of problems like this. This is just pure rotation problem. You know the angular rate-- theta dot, or omega. What's the velocity of point B if point A is fixed and this thing rotates?

r omega. You all know that. OK. And then we'll call the velocity of the dog relative to point A due to the fact that he is moving in this frame, we'll call that velocity to the dog. From the point of view of in the frame is a different quantity. They're both vectors, and you can add them together.

So in fact the answer-- I'm just going to give you the answer to this problem-- the answer to this is the velocity of A with respect to O plus the derivative of r B/A with respect to time as seen from within the A xyz prime frame, plus a term here due to rotation.

And we know that the magnitude of this answer is some r omega. We know it looks like r omega. So to interpret this as meaning the derivative of this vector-- this thing-- as if you were in the frame. And that accounts for the dog running. This is just the dog running.

And mathematically, I actually find it easier to say what this piece means is this is the velocity of the dog with respect to A with omega equal to 0. It's the same thing. This is what you see if there's no rotation. That piece.

And this is the piece with rotation. But I want you to think something through here. Come on. I'm still learning how to do this. OK.

So I have my little reference frames like this. I'm going to rotate it in this direction, which given xyz ought to be in that direction. So the rotation rate, if it's going like this, I'm going to give it a constant value, is that the vector omega is some omega in the k hat direction, right?

And I have on this body-- here's my wheel, and here's B, and here's A. And this is my little x-axis, my x-prime axis. And it has associated with it a lowercase i hat unit vector. And off in this direction then would be the y prime with a j hat unit vector. OK. So what's the vector r B/A? How would you write it in vector notation in components here?

So this is r B/A from here to here. And what's its direction?

AUDIENCE: i hat?

PROFESSOR: i hat. And is it positive or negative? So this is the vector. At some magnitude r B/A in the positive i hat direction, the arrow goes, when you say r B/A, this is B with respect to A, the arrow goes with B. So this, it's positive r B/A i hat.

And what is the velocity that we have discovered up here, the velocity of B with respect to A as measured inside of A xyz? That's this piece here. In vector terms? And we've figured out what its velocity was. It's some r omega. The r's r B/A-- the magnitude, the length of that r. Clearly got to be this length right here. And the omega is given. So what direction? You know that the magnitudes are omega. What's the direction?

AUDIENCE: j hat?

PROFESSOR: j hat. OK. So what we're saying is the velocity of B with respect to A as seen from inside this A frame is r B/A omega with respect to O in the j hat direction.

Now this has a k associated with it. That has an i associated with it. And this has a j associated with it. So as a product r omega, what kind of product is it? What you know about vectors? What does it have to be?

Is it a dot product between omega and r? Is it a cross product? I hear somebody saying cross product. But what's the order? So I want you to figure out-- just deduce what the vector notation is that gets you the right velocity-- the correct velocity-- correct magnitude and correct direction.

So the answer is positive r omega j.

AUDIENCE: Omega cross r?

PROFESSOR: Omega.

AUDIENCE: Cross to the r?

PROFESSOR: Omega cross r. So I hear one person, omega with respect to O cross r B/A. And these are now both vectors. Anybody else?

So if we just think about unit vector, actually, let's check it. This is k hat direction. This was i hat direction. And k cross i is positive j. So that works.

So in general-- this is, in fact, the correct answer in general-- is that this term here is omega with respect to O cross r B/A. You're going to use that a lot.

So to summarize, then-- while that's coming down, the velocity of this dog in a stationary frame where A is stationary for a moment is entirely due to the rotation, and it's omega cross r B/A. And now if the dog starts to run with respect to this reference frame, he has some velocity, which is the other piece is velocity of B with respect to A as seen from A-- or as you would compute, you'd just set momentarily, omega equal to zero.

And finally then, the total formula for velocity. So the velocity of B with respect to O-- some point on a rotating reference frame-- is the velocity of the frame plus-- I won't write it as a derivative now. Plus the velocity of the point, but as seen from the point of view of somebody in the frame, plus omega with respect to O cross r B/A.

So this is an important formula. You're going to use it a lot.

Couple of subtle points in here. This one-- this thing from as seen from inside the frame. It's also the partial derivative of r B/A with respect to t when omega with respect to O equals zero. That's another way to say this.

And this term is another subtlety. I was careful to write with respect to O. Yes?

AUDIENCE: To make sure I'm not confused here on this vector right there.

PROFESSOR: Yes.

AUDIENCE: That leftmost term is the middle one there? Or the right one?

PROFESSOR: So this was when we were working with just the velocity due to the motion of the dog due to rotation plus-- actually this was rotation-- what did I say?

AUDIENCE: [INAUDIBLE] experiments. That the term on the left is the motion that you can see from the frame. From the right is the rotation that you don't see in the [INAUDIBLE], but needs to be [INAUDIBLE].

PROFESSOR: And I can't remember how I got to this point. But I think I was making a couple of points, and rammed them together. So this is the correct. Don't focus on this. This is the summary.

This thing is now rotating, and translating, and the dog's running. And I want you to be able to write down the total velocity of the point B as seen from a fixed reference frame fixed on the ground. This is with respect to O.

It has three terms-- the velocity of A, the velocity of the dog relative to the body it's on here, and finally, the component of the velocity-- contribution of the velocity that you see that's due to the rotation.

Now this turns out to be a really powerful formula. It's a generalization. It's a special case of a very general formula. Let me catch up in my notes here. Because the following statement is true.

So the time derivative of any vector that's defined in a rotating frame is given by-- and we'll call it just some vector A. It might be, for example, angular momentum. In fact we'll use this formula a lot when talking about angular momentum.

The time derivative of this vector A-- whatever it represents-- position, velocity, angular momentum-- as seen from a reference frame xy-- O xyz, reference frame O-- is made up of the derivative of A from inside the frame plus--

So it's the same thing. This was the derivative of r B/A. Has two pieces-- the piece that came from the dog running and the piece that comes from rotation. Any vector, this piece is true. It's the derivative of the vector inside the frame in which it's defined plus omega with respect to the reference frame you want the derivative in cross A. Very powerful formula.

This now is covered in a couple of different places in the readings, and in the handout that's posted called "Kinematics." So you should read by now the portion of the kinematics thing, at least on velocities. The second half of that kinematics handout is about acceleration. So we're going to use this formula to go from velocity to acceleration. I saw a couple hands up. Yes?

AUDIENCE: So what is the difference between O sub xyz and A sub xyz? Where are they [INAUDIBLE]?

PROFESSOR: OK. So this is a merry-go-round on a train, and you're looking down on it. And the train can move. And we fixed to the ground, not moving, a reference frame we call O. And I want to know in this case the velocity of the dog running around on this merry-go-round with respect to O.

It's the sum of three vector contributions-- the velocity of the train, the velocity of the dog with respect to the merry-go-round-- this is velocity of the train. This is with respect to the merry-go-round. And this is the velocity that you see out there. So you're sitting in O. You are in this fixed frame.

Which brings up another subtle point about velocities and inertial frames. If you are at any fixed point in an inertial frame-- you don't have to be at O; you can be right where you are-- this term is always the same. The velocity of this moving object relative to the frame is the same to any observer in the frame-- any fixed observer in the frame. OK?

And finally, the u. But the u, the part of the contribution to the velocity due to the rotation of this thing, you can see it from out there, right? It moves. Well, the rotation contribution is this. The dog running around is this. And the movement of the train is the first term. OK?

All right. And this is the general formula for the derivative of a vector in a translating, rotating frame. Yeah?

AUDIENCE: [INAUDIBLE] vectors beyond the [INAUDIBLE] v of p with respect to the [INAUDIBLE].

PROFESSOR: I'm not following, here.

AUDIENCE: Are any of the v, v of A, for example, or v of B with respect to A, are they all vectors?

PROFESSOR: These are all vectors. And I've gotten a little careless about drawing my underlines when I haven't broken them down into components.

This is a vector. And we can do this, because we're relying on formulas for the sums of vectors. So it's the vector. There's a vector describing the velocity of the train. For example, if it's moving in the capital I hat direction, you have to say that.

And this may actually be the dog. At the moment you catch him, the frame's like this. And let's say he's running in the y direction. His direction at that instant in time is in the lowercase j hat direction relative to this frame, right? And that j hat direction has some angle with respect to this frame, which if you want to reduce these velocities down to the unit vectors in the inertial frame, you can do that. But you're going to have to account for the angles of this reference frame compared to that one. So cosine thetas and sine thetas and that kind of thing.

One important point about this general expression for the derivative of a vector in a rotating frame, which therefore applies to this case, because this is just the specific example of derivative of a position vector giving you velocity. This term and this term-- these account for the change in length of the vector.

So just to keep it real, in terms of velocities, you see a velocity that's due to the fact that the velocity vector itself is changing in length. Or in this case, the r vector is changing in length, so the velocity has something to do with the change in length.

This term is due only to the fact that it is rotating. If the dog's not running, is his position vector changing in length? No. So this term here comes from taking this derivative of the position vector within the rotating frame.

So this accounts for the change in length of that position vector. This accounts for the rotation. This term is the same as seen from in any frame. This is the subtlety.

This term, you get the same answer if you see it from here. This velocity, this contribution. The dog's speed with respect to this frame it's in is the same to you out there in the fixed frame as it is to somebody sitting here in the frame moving with him. Important point.

It'll become more important as you do problems. You need to remember that doesn't matter where you're calculating. You're given the velocity of the dog in this frame, and little i j k components, it is in magnitude exactly the same velocity as you would see it in the fixed frame. But then you just have to account for the right unit vectors and so forth. Yes.

AUDIENCE: [INAUDIBLE] is the same as the frame, but the direction is changing?

PROFESSOR: The direction is the same in any frame also. It's just that you have to decide which unit vectors to represent it in. And when you're in this frame, it's easiest to represent it in little i j k, right? But if you're in the fixed frame, you may want to actually know the velocity in the fixed frames unit vectors, and there's just geometric conversions that'll give you that. Yes.

AUDIENCE: But if theta is like a function of time [INAUDIBLE] how would you use those? Does it just work the same way?

PROFESSOR: So she says if theta is a function of time, if you were trying to convert-- this dog's running in the y direction, which is going in the master frame over there, is going to have a component in the negative capital I hat and positive capital J hat direction, right?

And you could break down-- let's say his velocity vector is like this-- you could break it down into a component that's parallel to the ground, and a component perpendicular to the ground. And those then would be in that reference frame.

You've all, I would think, have done some problems using polar coordinates. And the conversion between x and y and r and theta is you used cosine thetas and sine thetas to compute it, right?

So read that handout on kinematics, because it proves this formula by doing just that. It puts everything in terms of thetas that are functions of time. And it's kind of a long, painful process, but you can do that, grind it out, and you will then, when you assemble the terms, you end up with this.

And once you know this formula, it's far, far easier than to calculate all this in terms of thetas and theta dots, and sines and cosines. OK?

All right. That's good. That's important stuff. We're going to make great use of these formulas over the rest of the term.

So I'm going to just take a break from kinematics. It's time to talk a little bit about Newton's laws, because we really want to be able to do dynamics problems. And so this has been about describing the motion. And now we want to talk about the laws on which we depend in order to be able to calculate things, and to draw, write equations of motion and so forth. So any final questions about kinematics? Because I'm going to change topics. Yes.

AUDIENCE: So for that equation to the [INAUDIBLE], then the acceleration of [INAUDIBLE] with respect to the reference frame, that's [INAUDIBLE] is just zero, right?

PROFESSOR: You're asking about acceleration, now? Do you really mean? Is that what you mean? Because I haven't talked much about acceleration.

AUDIENCE: I was just wondering, because also [INAUDIBLE] x to the velocity change.

PROFESSOR: This one?

AUDIENCE: Yeah.

PROFESSOR: So this equation is true of any vector. This is basically vector calculus. It's true of any vector in a rotating frame. It's time derivative with respect to a fixed frame. It is the derivative you would see in the rotating frame, which is just the change of length of the vector, and the part due to the fact that you are making it rotate. And we'll do examples of this, I promise you, very soon. So you'll see how it's applied. OK?

OK. So let's do a quick review of Newton. So Newton's Laws. He had three of them. And we're all pretty familiar with the second.

The first, though, is called the law of inertia. And basically, the first law says, if an object is motionless in the absence of forces, what happens? Stays motionless. Or, if it's at some constant velocity in the absence of forces it stays at constant velocity, right?

So these have been stated many ways over the years. I'm just going to try to come up with short ones for the board. So in the absence of forces an object-- a particle-- moves with constant velocity.

In fact, Newton only talked about motions of particles, not as in little tiny things. Not about rigid bodies that have finite dimensions. He thought of the planets, appropriately, as particles.

Second law. Second law, I won't write it all out. But basically F equals ma. Their vectors equals the time derivative of the linear momentum. That's what we know as the second law.

The sum of all the external forces equals the mass times the acceleration. Another statement of the second law. OK? So you know that one. That's the one you're most familiar with.

Third one. What's the third one?

AUDIENCE: Every action has an equal and opposite reaction.

PROFESSOR: So every action has an equal and opposite reaction. And I'm going to draw a picture for this one.

So here's a particle. Here's another particle. This is particle 2. Particle 1. There's a force on this particle that's the force on 2 due to the presence of 1. They've each got a little gravity, little attraction from one another. And this one has a force on particle 1 due to particle 2. OK? They could be two planets.

And basically what he said is that for every action, there's an equal and opposite reaction. That means that f 21 equals minus f 12. And this is called the strong form of Newton's third law, which the forces are equal, opposite, and collinear. They actually point exactly opposite one another. And this is true of mechanical systems.

You get some rather interesting subtleties when you get into electromagnetic fields, and charged particles, and things like that. So you have to think really carefully. But for the mechanical systems, the strong form will suit us just fine.

OK. Those are the three laws. Newton made one major condition for those to be true. What is the assumption that must be satisfied for these laws to be true?

AUDIENCE: [INAUDIBLE] an inertial reference frame?

PROFESSOR: I hear "an inertial reference frame," right? And that's what he assumed. You have to be in an inertial frame for these statements to be true.

So then what I want to spend a few minutes talking about is basically what's an inertial frame. Because that's going to be really important in this subject. When you start getting on things that move and rotate, things sometimes are not inertial.

So I'm going to ask you a quick question. So a reference frame. So we had our fixed frame up there, sitting on the ground, here on the Earth. Is the Earth an inertial reference frame, standing here observing things?

I want a show of hands. I really want some participation here. How many of you think Earth's an inertial frame? Raise your hand. It's true. How many think it's not true? How many think it depends? On what?

AUDIENCE: What you are focusing on.

PROFESSOR: What the problem is. He says it depends on what you're focusing on. So it really depends on the sizes of the forces and the motion you're interested in. Can you give me an example in which the Earth cannot be assumed to be an inertial frame? Practical example?

AUDIENCE: Oh. Not practical.

PROFESSOR: Impractical.

AUDIENCE: The rotation of Earth is slowing? The rotation of the Earth is slowing?

PROFESSOR: Rotation of the earth is slowing. That's interesting. To account for that, you would definitely not be able to just to assume we're inertial. But what's an everyday example of where, if you're trying to solve a problem in this field, you couldn't make this inertial assumption?

AUDIENCE: Rotation of the planets?

PROFESSOR: Well, no, for the most part, you could get most of it. But, yeah.

AUDIENCE: I think the weather, really, because the wind currents, or like the ocean currents are affected by the fact that the Earth is round.

PROFESSOR: OK. That's a good one. In order to account for the circulation you have to take into account the Earth's motion.

Let me give you an example. Clocks. Pendulum clocks. Does the speed of a clock change? Is it different at high noon from midnight? What do you think? Yes or no?

How many think that the actual speed of a clock-- the length of a second-- would be different at noon on the Earth-- a pendulum clock-- from midnight? How many think that might be true? It's different?

How many don't believe that it would be true? How many just not raising their hands? Come on you guys. Let's get with it. OK.

So the effective acceleration of gravity that that pendulum feels is different at noon from midnight. And one reason is because at noon the sun's pulling away from the surface of the Earth, and at midnight the sun's gravity is pulling in the same direction as toward the center of the Earth. The total effective gravitational pull that the pendulum field changes due to the rotation of the Earth, what it feels. The sum of the forces on the pendulum includes the Earth's gravity and the sun's gravity. And the moon also does this. So there's daily variations in the speed of pendulum clocks just because of the Earth's rotation with respect to the sun and the moon. Yeah.

AUDIENCE: Is that even great enough to be measurable?

PROFESSOR: So she says, "Is that even great enough to be measurable?" So I have a friend, a guy named Hugh Hunt. He's a professor that teaches dynamics at Cambridge University. And he is the keeper of the Trinity College clock. Trinity College is where Newton was.

So he took me to his clock one day up in this tower. And it's got about a two-meter pendulum on it. And he has got that clock to run so that it gains no more than one second per month. He's really tuned it up carefully.

And the key to being able to do that is to do things on monthly averages. So he's got it tuned so that over a month it just barely gains a tiny bit. But if you measure it very carefully over the course of the day, it has amazingly large fluctuations. One of them is due to the thing I just described. So if you're really trying to keep close time, it matters.

OK. Another one is gunnery. You're shooting long range. You're trying to hit a target. The fact that the earth rotates, you will not hit the target if you don't account for the effects that are caused by the Earth's rotation. And it's one of the first reasons that people got into understanding the importance of whether or not it's an inertial frame, was gunnery in the old days, from naval ships and so forth.

OK. So we have three laws. I want to talk a little bit about the first and the third. We're going to use the second a lot.

So the first law, most people think of the first law as being a special case of the second, right? It's just when there's no forces, nothing changes. But I think the first law is useful in its own right. And one of the reasons why it's called the law of inertia, it's the law that allows you to do a test to discover whether or not you're actually in an inertial frame. Useful to be able to do that.

So I'm going to give you an example. And actually, I did want to ask you a question. So three possible answers to this question. An inertial frame can-- how to pose this?

If you're in an inertial frame, can it be accelerating? If you're in an inertial frame, can it be rotating? Or if you're in an inertial frame, it can neither accelerate or rotate. So which of those three answers is the best answer for conditions to be in an inertial frame? Non-accelerating, non-rotating, or both?

How many believe non-accelerating? Just non-accelerating? How many believe in just not rotating? How many believe both? Both is right. If you are in a frame which is rotating or accelerating it's not inertial. OK. So just rotation causes it not to be [INAUDIBLE].

So let's test that. I'm going to pick two cases quickly. Let's use this law of inertia to set up a test to see if a couple different frames are in fact inertial.

So I've got a cart here. And I'm sitting here-- or you are. We're sitting on the cart. And this cart is accelerating. Acceleration of A with respect to O is, I'll call it a naught I hat. This cart's accelerating in that way, that direction, the positive I hat direction.

OK.

Now I'm sitting here on the cart-- not very sensitive. And I want to test whether or not I'm in an inertial frame.

So let's pretend this cart, I've got an air table there, frictionless table. And I've got a hockey puck. And I set it down on the frictionless table and let it go. What happens? What do I observe?

AUDIENCE: [INAUDIBLE]

PROFESSOR: So the puck will accelerate, you're saying, towards me, right?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Hmm. It'll move. So she says the puck will move towards you if you let it go. So is that an indication of whether or not you're in an inertial frame?

So the test-- this is first law, now. So the test-- if you're in this frame, and you want to know whether or not this is an inertial frame, and I claim you can set this puck out there, if you're in an inertial frame, what should it do?

Not move. If it moves, there's something going on. Something fishy, right?

OK. So the acceleration-- and we'll call the puck at B. So the acceleration of B with respect to O-- and there's no rotation here. So the acceleration of B with respect to O, it can be written as the acceleration of A with respect to O, plus the acceleration of B with respect to A.

Now this is O here. This is an inertial frame. And if it's an inertial frame, and there are no forces in the x direction acting on this puck, then I'm going to say that the sum of the forces on that puck equal zero.

And therefore, what can I say about the acceleration of the puck? What's the acceleration of the puck as seen from O? It's gotta be zero. If no forces-- so the summation of the forces with respect to this O frame in the x direction, if there's zero, that implies the acceleration of B with respect to O is got to be zero.

Knowing that, I can now solve for the acceleration of B with respect to A. And that's going to be minus the acceleration of A with respect to O. And that's minus a naught I hat.

So you are correct in saying that it moves. But it actually accelerates. From your point of view, what you see sitting there in A, you were going to see this accelerate in the opposite direction. And that's a dead giveaway that you're not in an inertial frame. OK. Come on. OK.

So a little harder problem. Now we've got our merry-go-round. OK. And it's fixed. Not on the train. It's just sitting here. But it can spin.

And you're sitting at A. So you're up above this merry-go-round looking down on it. And it's rotating. Out there you're in an inertial frame. But now you come over to here, and you sit right here at the center.

And if you're right at the center, you might not actually feel a thing, right? And there's no windows. So this could be a spaceship out there, slowly rolling over.

So you're sitting inside of this system, no windows, right at the center, can't feel a thing. And I want you to construct a test-- be using the first law-- that'll tell you whether or not you're in an inertial frame. What might you do?

AUDIENCE: Set a ball down on the ground [INAUDIBLE] roll off to the edge.

PROFESSOR: All right. He says set a ball on the ground, and see if it rolls out to the edge. Right? See if it moves.

AUDIENCE: Just [INAUDIBLE] is this [INAUDIBLE] under the effect of gravity, or is there [INAUDIBLE]?

PROFESSOR: Let's really make it a merry-go-round. So there's gravity. So I want gravity to be useful here. It keeps the thing on the surface. Doesn't just go drifting off.

So yeah, let's say we have gravity. The axis is vertical. You're sitting here, but you can't see outside, and you want to do this test. So do you agree if you set the ball down, you might learn something? OK.

So you set the ball down. And you're here watching. And here's the ball. And let's say you've got a ball, and you actually have a string on the ball. Set it out there.

So initially you've got this ball out there. And you're sitting here at the center. And this merry-go-round's going round and around. What can you sense that tells you that you're not in an inertial frame, if you're holding onto this string? Does the ball move, first of all? Would the ball move if I'm sitting here hanging onto the string, and I set it out there and set it down? No, it won't move.

But what do you feel in the string? Tension in the string. OK. So now you've got indication number one that there's something fishy.

OK. Now you let go of the string. What should happen? So you [INAUDIBLE] go out. All right. But now I'm going to ask you a little harder question. What direction should the ball travel in once you release it?

AUDIENCE: Radially outwards.

PROFESSOR: Radially outwards. I have one shot at radially outwards. Any other thoughts?

AUDIENCE: From your point of view, it wouldn't seem to go in a straight line. It would seem to curve off to one side.

PROFESSOR: So I have one postulate that it will curve-- it will go away and curve off, right? Which way would it curve? So you're saying not radially. You say it's going to curve.

AUDIENCE: Opposite direction that you're spinning.

PROFESSOR: Opposite to the spin. He says it will curve opposite to the spin.

AUDIENCE: But only from your point of view.

PROFESSOR: This is from the point of view on the merry-go-round. Point of view off the merry-go-round, might be easier to reason this. So now you're up above the merry-go-round in an inertial frame, up above this merry-go-round, just looking down, like sitting up in a tree and looking down on it. What do you see?

AUDIENCE: [INAUDIBLE], but if you're on the merry-go-round then you [INAUDIBLE]

PROFESSOR: All right. So this is an argument for if you're on the merry-go-round, you'll see radial motion. If you're in the tree, you'll see the curve. So we're going to do--

AUDIENCE: I would like to retract my statement.

PROFESSOR: OK. All right. I think we'd better take a vote here. So the possible answers are from-- how do we frame this? From an inertial frame, looking down on it, answer A is it will go in a curved path. Answer B is it will go in a straight, radial line. Answer C is it does-- any other guesses? No other guesses. So it's the only two choices. It curves, or it goes in a straight, radial line.

So how many vote for-- and everybody has to participate. How many vote for it goes a straight radial line? Let's have it. Straight out radial line from the point of view of the fixed reference frame. OK. A goodly number. OK.

How many from the point of view of that fixed frame, looking down on it, will see it curve? How many vote for that?

And how many didn't vote? Those are don't knows, huh? All right.

So A and B are wrong. A and B are dead wrong. And you could have figured out what the answer is if you went right back to basics, back to Newton's Laws.

When the string is released, the sum of the forces on the object are what? In the direction that it can travel. Still got gravity pushing on it, but it can't go in that direction.

So if you did a free body diagram, what are the forces in that horizontal [? direct ?] plane that it's sitting on? I hear centrifugal force. I hear frictionless. Let's make the table frictionless so it can easy to move. No friction. I hear centrifugal force.

AUDIENCE: Zero.

PROFESSOR: He says zero.

AUDIENCE: There's the force that you get from tangent of [? to ?] less perpendicular to the radial direction.

PROFESSOR: What's perpendicular to the radial direction?

AUDIENCE: [INAUDIBLE] it had to [INAUDIBLE] B traveling [INAUDIBLE] at one point had to have [INAUDIBLE] force with tangential to the circle.

PROFESSOR: So you were the one that provided that for us. You're on the merry-go-round. You set it out there, and held the string. You set out there in some radius r. And so because when you set it down, you're already turning. So you just set it down with respect to the merry-go-round. It's not moving. You just set it down. OK?

What are the forces-- so just draw the free-body diagram. Here's this puck. And you've released the string. And you got mg downwards, and you have a normal force upwards. And you have no friction. And I hear centrifugal force, but--

AUDIENCE: [INAUDIBLE]

PROFESSOR: Well, that was a guess. Somebody said straight out, and the answer is no. And we're going to figure it out.

First of all, we've got to sort out the forces. Well, what are the forces on this thing? So I heard centrifugal force.

AUDIENCE: Yeah, I think the only force is the centrifugal force. But there was a velocity that's not caused by the force. So the velocity that you got was velocity that you told us about that [INAUDIBLE] that starts out--

PROFESSOR: You set it down. And once you set it down, it's there. And then?

AUDIENCE: Yeah, your initial velocity from setting it out there.

PROFESSOR: Yes.

AUDIENCE: The change in direction-- the force only change direction of the ball. But once the string cuts, and the direction no longer changes, and the ball only goes the direction that is tangential to the circle.

PROFESSOR: All right. The man has it right. You probably couldn't hear him. He says that when you let it go, it goes tangential to the circle. And that's true.

The forces on in this direction-- in the horizontal direction-- once you release the object, there are no forces.

Centrifugal force is a construction of convenience called a fictitious force. And we'll talk about that later. It is not a real force. It as a result of an acceleration. And it's the result of the acceleration when you are making it go in a circle. There is indeed-- that tension is what some people call the centrifugal force holding it there. But once you release it, that's no longer there.

And if there are no external forces acting on the object in the horizontal direction, what's the mass times the acceleration?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Therefore what's the acceleration? Zero. And its velocity. What's its velocity at that moment in time?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Omega. R omega. So you have an R omega velocity. It's tangential to the motion.

So let's make this my little x, and this my axes attached to the merry-go-round. This thing is moving in the j-hat direction at R B with respect to A in the j-hat direction. That is the velocity of B with respect to A at the moment you release it. OK?

So it had better run off tangential to the circle at the moment of release. So let's test it. Now I want you to be my quality control person. If I smack somebody with this, I'm going to hurt them.

AUDIENCE: No.

PROFESSOR: Squeeze it. Soft and harmless, right? OK. So when should I release it if I want to hit the MIT sweatshirt sitting up there? When it's out here, right? I'll see if I can do it.

All right. I'd better not try it again. I probably can't do it twice in a row.

AUDIENCE: But that was a straight line.

PROFESSOR: It was in a straight line.

AUDIENCE: Yeah. So one of the guesses was--

PROFESSOR: Radial straight line was one guess. This is?

AUDIENCE: Tangential straight line.

PROFESSOR: Got it. So this tangential straight line is the right answer.

AUDIENCE: [INAUDIBLE] the velocity is equal to the radius [INAUDIBLE].

PROFESSOR: I can't quite hear you.

AUDIENCE: Here it says that the velocity of [INAUDIBLE]

PROFESSOR: Ah, I left out the omega, didn't I? Sorry about that. So it is omega is the result of omega cross r B/A. So it's omega with respect to O, r B/A in the j direction. Yeah.

AUDIENCE: [INAUDIBLE] the person on the merry-go-round does not see that.

PROFESSOR: Ah. So actually I almost forgot this. She is talking about what does the person see on the merry-go-round? What do you actually see on the merry-go-round? Certainly doesn't go radial. We've proven that, right? But the person on the merry-go-round, is you're turning, and this thing is going off in a straight line with respect to the fixed frame, and you're turning away from it.

So looking down on it, I release it here. It goes off in that direction. Your point of observation, you see it here. But now a short time later, when you've rotated to this point, and you're just keeping your eye on it, you see this thing start to move. But this is the spot that it was sitting on, which is now moved to here. But its position is now there. You see it moving away from you.

And as you get up to, say, here, then it will have moved out a radius this far. Well, actually it's not-- you don't know how fast you're-- well, you're actually going exactly the same speed it is.

So you've gone the arc length here. It's gone a quarter of a circle out to about there. Same point right here. But now you see it as being going off like that. To you, it's hooking off in the direction opposite to the direction of rotation. So the young man up there who described that early on was exactly right. So from your point of view, it goes mrrmm, like that.

And then if you go all the way around, it'll appear to come back. It'll be further away, but you'll be back down to the point where you see it released. So it'll look like it goes from your point of view. OK. Good one.

OK, we've got a few minutes left. I want to do something really important with the third law.

So the first law's actually pretty important and very handy. The second law, we're going to make lots of use of. So I want to talk a minute about the third law.

The third law is responsible for a law that we use, or an application that we use all the time. And let's see how this works out.

So Newton's third law is the one that says F 21 is equal to F 12. So we have these two particles. And I called them 2 and 1. And I'll give this one mass 1, mass 2.

And I'm going to say second law tells me that the sum of the forces, vectors, external forces, on 2 is-- and this one, let's say it has things, active forces acting on it. F i's. And it has this little f 21.

So the forces here are the external forces that are vectors, plus f, [? some ?] of the forces on 2. So forces on 2 due to 1. That's external influence of the other particle. OK?

And this had better be equal to m 2 a 2, I'll call it. And with respect to some fixed, some inertial frame. So the sum of all these external forces on that particle 2 had better be equal to the mass times the acceleration of particle 2.

And the same thing can be said about particle 1. So I've really confused things here. There's 2, and there's 1. Boy. OK. Guys, help keep me honest here.

So now that we can say the sum of the forces on 1 is equal to these external applied forces, plus f 12-- the force caused by the other body. And those had better equal to m 1 a 1/0. OK.

And these are also equal to the time derivative of m 2 v 2/0. That's the momentum, the time derivative of the momentum. It's fixed mass, so it's just the derivative velocity gives the acceleration. So these are clearly the same formula.

And this is a time derivative of m 1 v 1 with respect to O dt. And we call that P 2 dot, P 1 dot. So these are statements that are applying second law to each of these particles.

So if I want to compute the total momentum of the system. So the linear momentum, the total momentum of the system is going to be P 1 plus P 2. They're vectors. And I want to take the time derivative of that total momentum. Just the time derivative of a sum is the sum of the time derivatives. And so I get a P 1 dot plus a P 2 dot.

But I know what those are. I have expressions for them. So this is F 1 external forces plus f 12 plus the F 2 external forces plus f 21. These two things. This one plus this one, basically, is the sum of the two time derivatives of the momentum. But what's the sum of this term and that term?

AUDIENCE: Zero.

PROFESSOR: Zero. And that then allows you to say that this is the sum of the external forces on a system is equal to the time rate of change of the linear momentum of the system. OK. And that's basically this statement.

And it has nothing to do with internal forces. And this allows you to say that the time rate of change of the linear momentum of a rigid body, for example.

So a rigid body is made up of a whole mess of different particles. You could separate this into a whole bunch of little chunks, and treat each one of them as a particle which has connection, has forces with particles next to it. And all those internal forces-- so these are the internal forces-- all of those internal forces are equal and opposite and cancel.

So it's really the third law that allows you to say if you have a system of particles that the time rate of change of the total momentum of the system is zero if there's no external forces acting on it. And that's where you get the conservation of momentum-- the law of conservation of momentum. OK. That's a consequence of the third law. OK.

So you have a homework problem that has just about this. You have a bunch of particles. m 1. m i. r 1 with respect to O. r i with respect to O. And you have all these particles out there. And I want to find the center of mass of this group of particles.

So the center of mass is a vector. And let's just say it's here. So I'm looking for r G with respect to O. So it's a quantity that I defined-- r G with respect to O-- times the summation of the m i's. So I'm postulating there's a place out there that if I multiply it by the sum of the m i's, I get the same answer as if I summed the m i r i/O's.

And I'm going to define the center of mass as r G with respect to O is the summation of each particle times its position vector divided by the sum of the masses, which is just the total mass of the system. OK? And that's a definition of center of mass. And this is a vector. And these are vectors. OK? So that's all there is to the center of mass.

And if I take a time derivative of r G/O, then it's equal to the summation of the m i times the r i/O's. But they're timed derivatives-- and I'm going to put a dot right there, so I don't have to write out d by dt-- over MT. But that's just the summation of the individual momenta of each of the particles over MT. Very handy little formula.

And if I take another time derivative, so I get an r G/O double dot, then it's just the summation of the time derivatives of the individual momenta again over this.

So this is a statement. We move this to the other side. M T r G/O double dot equals the summation of the timed derivatives of the individual momenta. And that's the total momentum of the system times its timed derivative.

So just from third law, you can come up with all of the linear momentum formulas.

This is a statement for rigid body. The mass times the acceleration of the center of mass. The total mass of a rigid body times the acceleration of the center of mass is equal to the time rate of change of the total momentum of that object. Very important formula. You've used it a lot, right?

So I'll tell you a quick story, and then we'll knock off. So two, three years ago, we have doctoral exams in mechanical engineering. And in the dynamics oral exam, we had eight students a few years ago. And they were asked to find the center of mass of an object. It was just a step in a harder problem.

And seven out of the eight students could not remember the definition of the center of mass. Now they could do Lagrange equations and nasty dynamics problems, but they'd kind of forgotten some of these really, really basic things.

So I have one demo to show you, just to illustrate center of mass. So center of mass, we know how to calculate it. And this is a rod. How can I simply find the center of mass of this thing?

AUDIENCE: Balance it.

PROFESSOR: Balance it. OK. Young lady here says move my fingers. Where do you think the center of mass is? Right in the middle. OK. Let's find it. Should this work, by the way? You ever done this? All right, the center of mass, I ought to be able to just about balance this thing there.

So the center of mass is in the middle. That's because I cheated. I put a piece of steel in this end. OK? All right.

But indeed, you might figure out why is it that I can actually do this and make it work. You can do it with a broomstick. You can do it with any object. The key is it'll work so long as the friction coefficients are the same on both fingers. It's really easy to find center of mass that way. So this got a little steel weight in the end.

OK. We're done for today. And see you on Thursday.