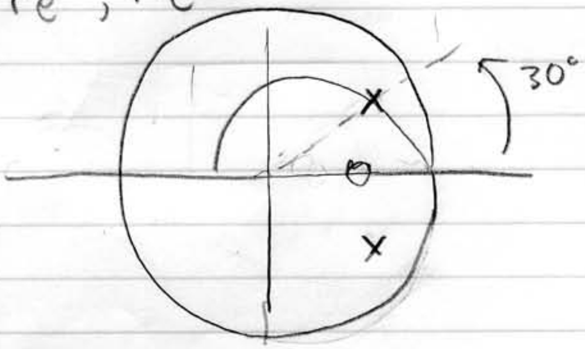
4.19 2nd order system

$\zeta = 0.5$

poles at $\theta = 30°$

$z = re^{i\theta}$, $r = e^{-\zeta \omega_n T}$



$$Y(z) = \frac{z - 0.6}{z^2 - 2r\cos\theta\, z + r^2}$$

Solving for parameters using table 4.2 in FPW...

$\theta = \omega_n T\sqrt{1-\zeta^2} = \frac{1}{6}\pi = \omega_n T\sqrt{1-.5^2}$

$\omega_n T = .6046$

$r = \exp(-\zeta\omega_n T) = \exp(-.5 \cdot .6046) = 0.739$

$G = 1/1.5036$ (for unity gain at DC)

The overshoot should be approximately 30-40% according to Figure 4.30 in FPW.

The discrete bode plot and the step response are attached.

4.28 $G(s) = \frac{10(s+1)}{s^2+s+10}$

$T = .01$ sec

$$\frac{G(s)}{s} = \frac{10(s+1)}{s(s^2+s+10)} = \frac{10}{s^2+s+10} + \frac{10}{s(s^2+s+10)}$$

$$= \frac{10}{s^2+s+10} + \frac{A}{s} + \frac{B}{s^2+s+10}$$

$$A(s^2+s+10) + Bs = 10$$
$$A = 1, B = -(s+1)$$

$$\frac{G(s)}{s} = \frac{10}{s^2+s+10} + \frac{1}{s} - \frac{s+1}{s^2+s+10}$$

$$= \frac{-s+9}{(s+.5)^2+9.75} + \frac{1}{s} = -\frac{s+.5}{(s+.5)^2+9.75} + \frac{9.5}{(s+.5)^2+9.75} + \frac{1}{s}$$

$$Z\left\{\frac{G(s)}{s}\right\} = \frac{z}{z-1} - \frac{z(z-e^{-.5T}\cos(\sqrt{9.75}T))}{z^2-2e^{-.5T}(\cos(\sqrt{9.75}T))z+e^{-T}}$$

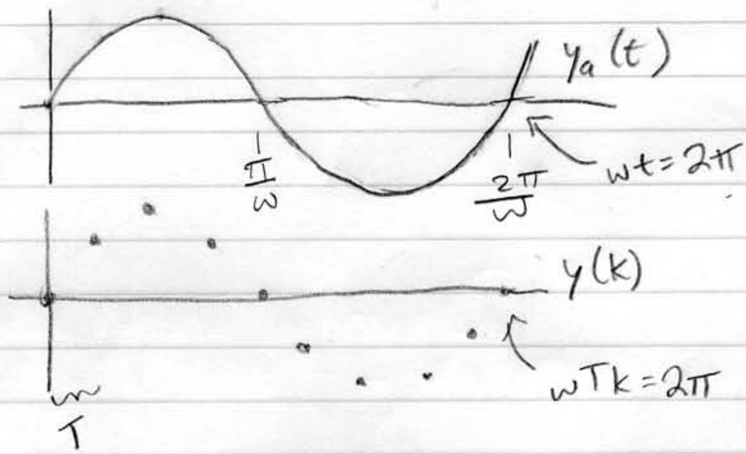
$$+ \frac{9.5}{\sqrt{9.75}} \frac{ze^{-.5T}\sin(\sqrt{9.75}\,T)}{z^2-2e^{-.5T}\cos(\sqrt{9.75}T)z+e^{-T}}$$

$$G(z) = 1-(z-1)\left(z-e^{-.5T}\cos(\sqrt{9.75}T) + \frac{9.5}{\sqrt{9.75}}e^{-.5T}\sin(\sqrt{9.75}T)\right)\frac{1}{z^2-2e^{-.5T}\cos(\sqrt{9.75}T)z+e^{-T}}$$

MATLAB Answer: $G(z) = \frac{0.09998z - 0.09899}{z^2-1.989z+0.99}$

$$G(z) = \frac{0.9998z - 0.09899}{z^2-1.989z+0.99}$$ given $T = 0.01$

Problem 4: (a)



$y(k) = \sin \Omega k, \ \Omega = \omega T$

(b) $\sin \Omega k = \frac{1}{2}[\sin \Omega k + \sin(\Omega - 2\pi)k]$

$\sin \Omega k = \frac{1}{2} \sin(ak) \sin(bk)$, where $a = \Omega$ and $b = \Omega - 2\pi$

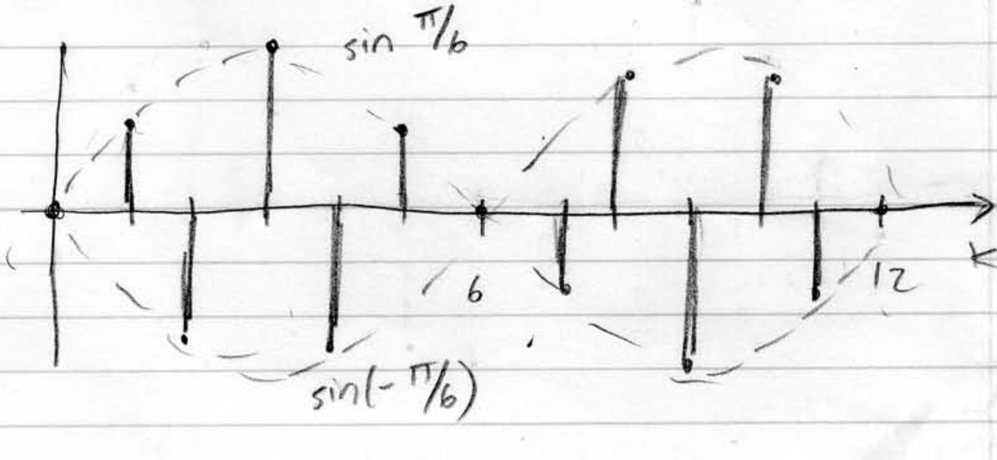
$= \frac{1}{2}[\sin(A+B)k + \sin(A-B)k]$ where $A = \frac{a+b}{2} = \Omega - \pi$, $B = \frac{a-b}{2} = \pi$

$= \sin(Ak)\cos(Bk)$

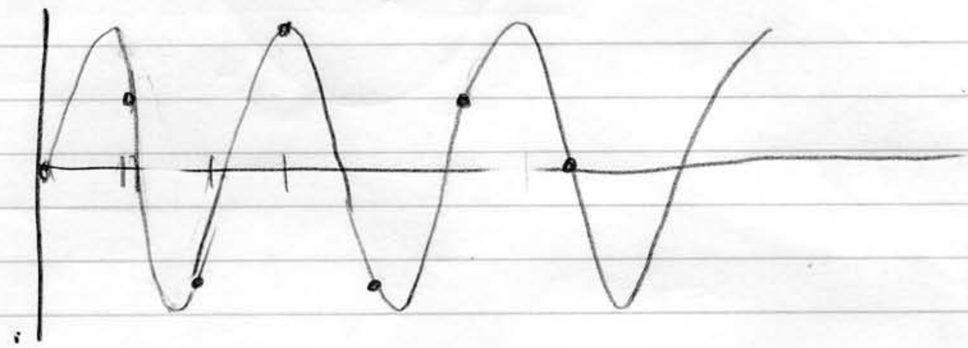
$= \sin((\Omega - \pi)k)\cos(\pi k)$

$= \sin(\Delta k)\cos(\pi k)$

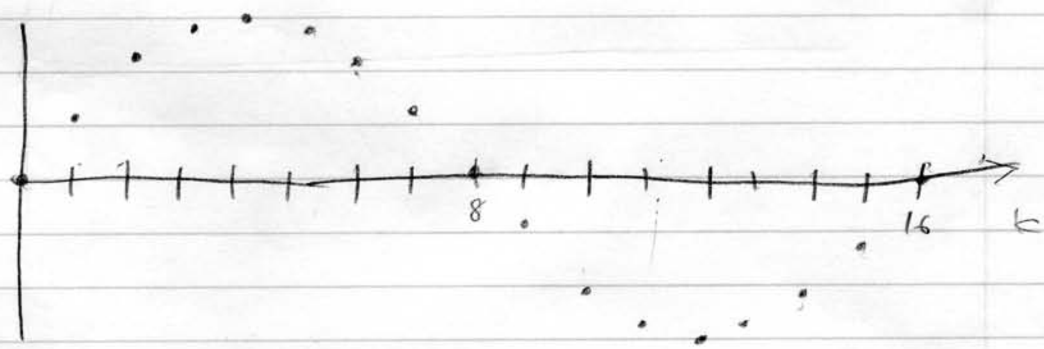
(c) $y(k) = \sin(\frac{5\pi}{6} k)$



(d) $y_a(t) = \sin\left(\frac{5\pi}{6} t\right)$

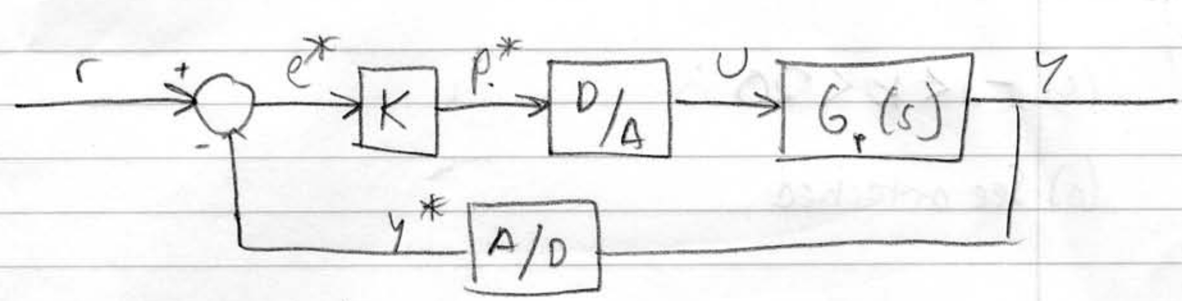


(e) Assume $y(k) = \sin \frac{\pi}{8} k$



The "beat" frequency is high so the beating envelope is not visible

Problem 5



$$G_p(s) = \frac{10(s+1)}{s^2+s+10}$$

$T = 10\,msec$

(a) $$G(z) = \frac{0.9998z - 0.09899}{z^2 - 1.989z + 0.99}$$

$$e^* = r^* - y^*$$
$$p^* = K e^*$$
$$U = p^*\left[\frac{1-e^{-Ts}}{s}\right]$$
$$y = G_p(s)\,U$$
$$y^* = [GU]^*$$
$$y^* = \left[G p^* \left[\frac{1-e^{-Ts}}{s}\right]\right]^* = (1-e^{-Ts})\, p^* \left(\frac{G}{s}\right)^*$$
$$y^* = (1-e^{-Ts})\, K e^* \left(\frac{G}{s}\right)^*$$
$$y^* = (1-e^{-Ts})\, K (r^* - y^*) \left(\frac{G}{s}\right)^*$$

$$(1-e^{-Ts}) K \left(\frac{G}{s}\right) = H^*$$

$$Y^* = \frac{H^*}{1+H^*} r^*$$

$$\frac{Y^*}{r^*} = \frac{Y(z)}{R(z)} = \frac{K(0.998z - 0.09899)}{z^2 - (1.989 - 0.9998K)z + 0.99 - 0.09899K}$$

(b) Zeros: $z = 0.0992$

Poles: $$z = \frac{(1.989 - 0.09998K) \pm \sqrt{(1.989 - 0.09998K)^2 - 4(.99 - 0.09899K)}}{2}$$

(c) $-.1 < K < 20$

(d) See attached

Problem 6 $H(z) = \dfrac{0.5z^5 - 0.5}{z^6 + 0.5z^5}$

(a) $H(z) = \dfrac{u(z)}{e(z)}$

$$(z^6 + 0.5z^5)\,u(z) = (0.5z^5 - 0.5)\,e(z)$$

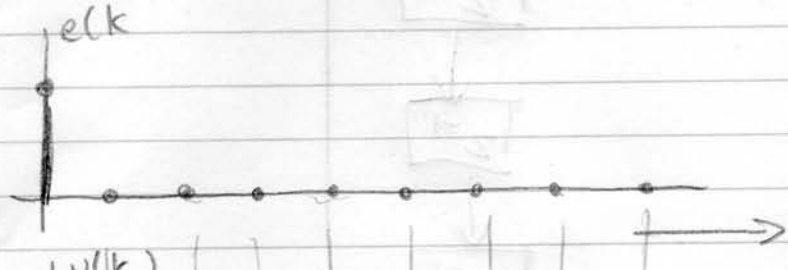
$$u(k+6) + 0.5u(k+5) = 0.5e(k+5) - 0.5e(k)$$

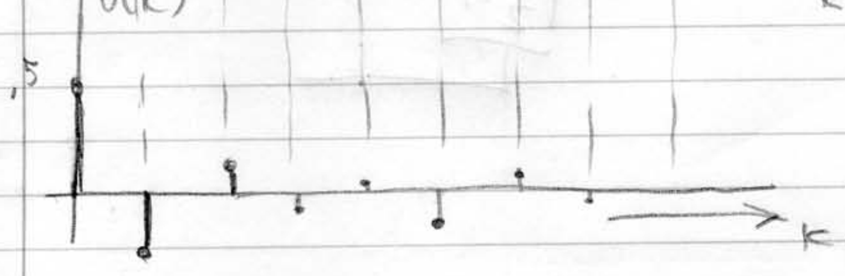
(b) The DC gain is found by setting $z = 1$

$$H(1) = \frac{0.5 - 0.5}{1 + 0.5} = 0$$

(c) $u(k) = 0.5e(k-1) - 0.5e(k-6) - 0.5u(k-1)$

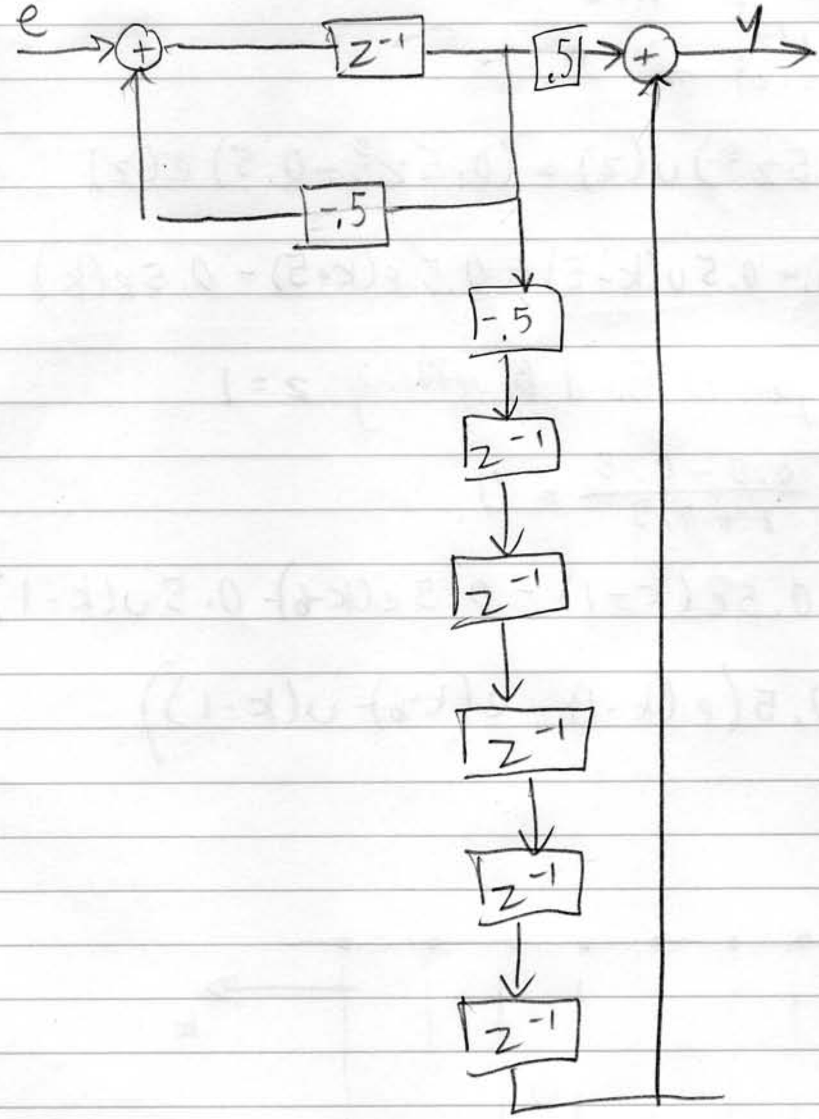
$$u(k) = 0.5\big(e(k-1) - e(k-6) - u(k-1)\big)$$



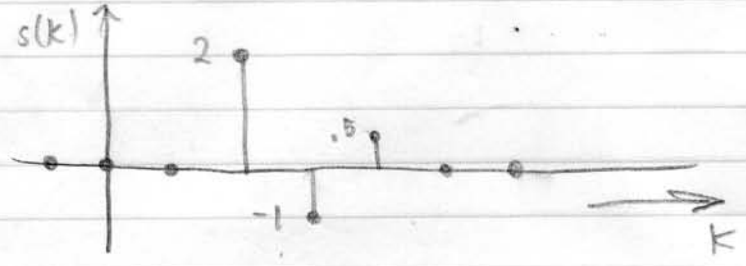


k	e(k)	u(k)	e(k-1)	e(k-6)	u(k-1)
0	1	0	0	0	0
1	0	.5	1	0	0
2	0	-.25	0	0	.5
3	0	.1.25	0	0	-.25
4	0	-.6125	0	0	.125
5	0	306125	0	0	-.6125
6	0	-.65	0	.1	.306125
7	0	.325	0	0	-.65
8	0	-.16125	0	0	.305

(b)



Problem 7.



(a) $s(k) = 2u(k-2) - 3u(k-3) + 1.5\,u(k-4) - .5u(k-5)$

(b) α_0

α_1

α_2

α_3

$y(0) = \alpha_0 = 2$
$y(1) = \alpha_1 = -3$
$y(2) = \alpha_2 = 1.5$
$y(3) = \alpha_3 = -.5$

(c) $S(z) = 2z^{-2} - z^{-3} + .5z^{-4} = \dfrac{2z^2 - z + .5}{z^4}$

$H(z) = 2z^{-2} - 3z^{-3} + 1.5z^{-4} - .5z^{-5}$

$= \dfrac{2z^3 - 3z^2 + 1.5z - .5}{z^5}$

$S(z) = H(z) \cdot \dfrac{z}{z-1}$

The step response is given by the impulse response multiplied by a unit step.