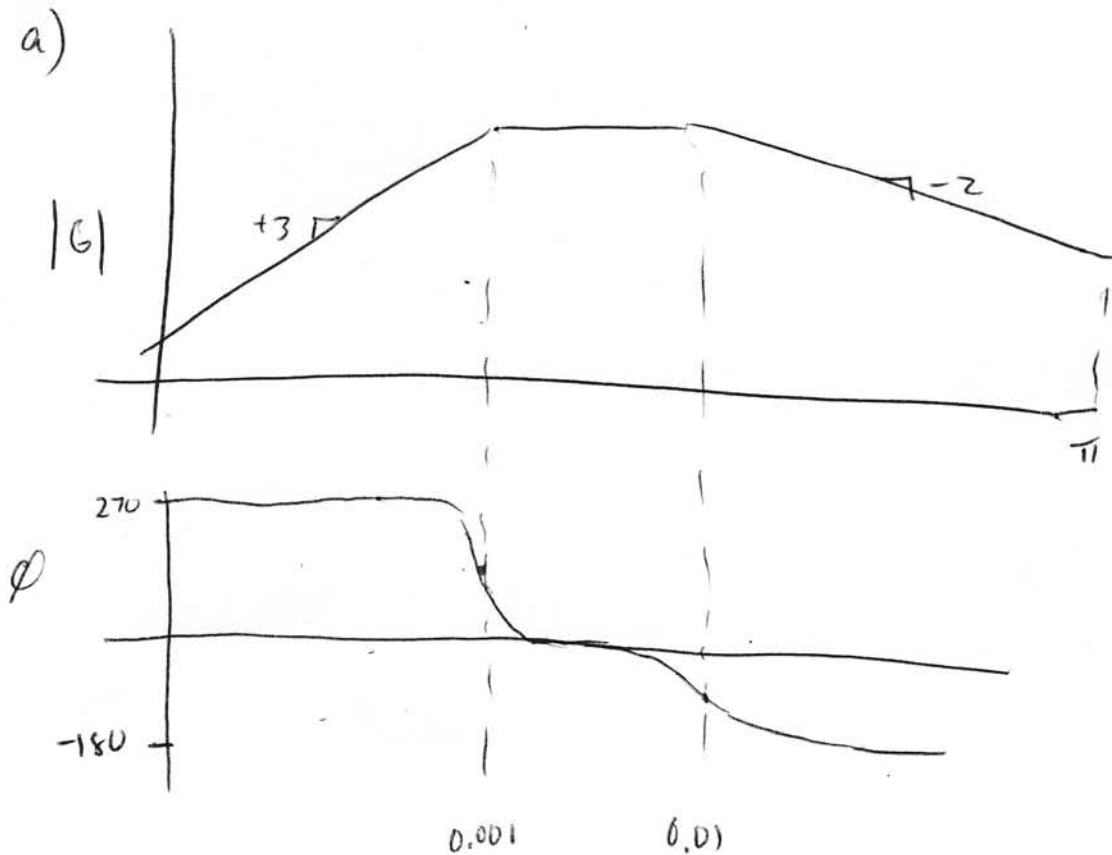2.171 Problem Set #6 Solutions

Problem 1

a)

b) The bandwidth limits are set by the high frequency
   gain and the sampling frequency.

   A lead, double-lag compensator was used.
   The lead compensator provides some extra
   high frequency phase margin, while the two
   lower frequency lag compensators reduce the
   phase at lower frequencies such that the phase
   is within acceptable limits.

$$G(z) = \underbrace{\frac{(z-1)^3}{(z-.999)^3 (z-.99)^2}}_{\text{Plant}} \underbrace{\frac{z-.2}{z-.05}}_{\text{lead}} \underbrace{\frac{(z-.9)^2}{(z-.94)^2}}_{\text{double lag}} \cdot \underbrace{1.0501}_{\text{gain}}$$

C.

MATLAB confirms the plot

Problem 2

a. $m\ddot{x} - k_1 x = k_2 i$

$$m s^2 X(s) - k_1 X(s) = k_2 I(s)$$

$$\frac{X(s)}{I(s)} = \frac{k_2}{m s^2 - k_1} = \frac{0.4}{0.025 s^2 - 20}$$

b. $G(z) = \frac{z-1}{z} Z\left\{\frac{G(s)}{s}\right\}$

$$\frac{G(s)}{s} = 0.4 \frac{1}{(0.025 s^2 - 20)} \frac{1}{s} = \frac{A}{s-31.6} + \frac{B}{s+31.6} + \frac{C}{s}$$

$$A = 0.01, \ B = 0.01, \ C = -0.02$$

Using the lookup tables in Appendix B of FPW

$$G(z) = \frac{0.004135 z + 0.00413 z}{z^2 - 2.414 z + 1}$$

c. $t_r \leq 0.1s$, $t_s \leq 0.4s$, overshoot $\leq 20\%$

$$\omega_n \geq \frac{1.8}{t_r} \Rightarrow \omega_n \geq 18 \text{ rad/se}$$

$$\zeta\omega_n \geq \frac{4.6}{t_s} \Rightarrow \zeta \geq .638$$

overshoot $\leq e^{(-\pi\zeta/\sqrt{1-\zeta^2})}$

$$0.2 \leq e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad \zeta \geq .46$$

The specifications were met using SISOTOOL

~~specs met~~
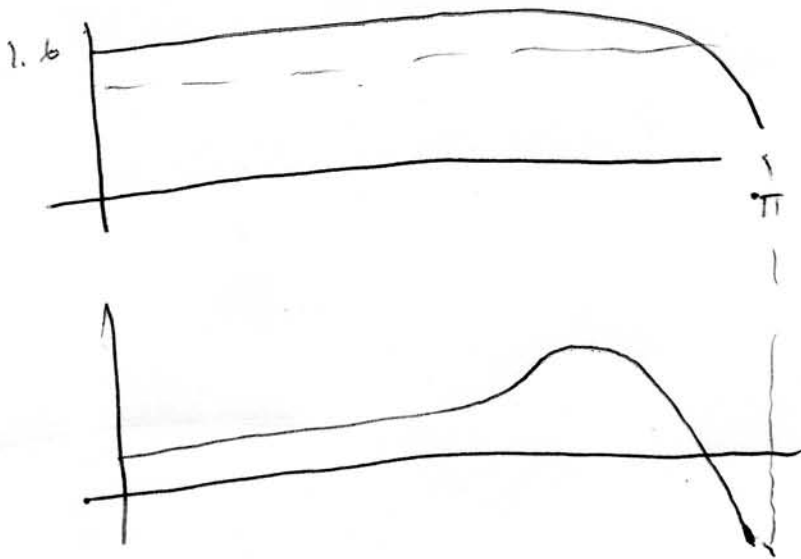
$$G_c(z) = \cancel{} 5600 \frac{z - .9983}{z - .986}$$

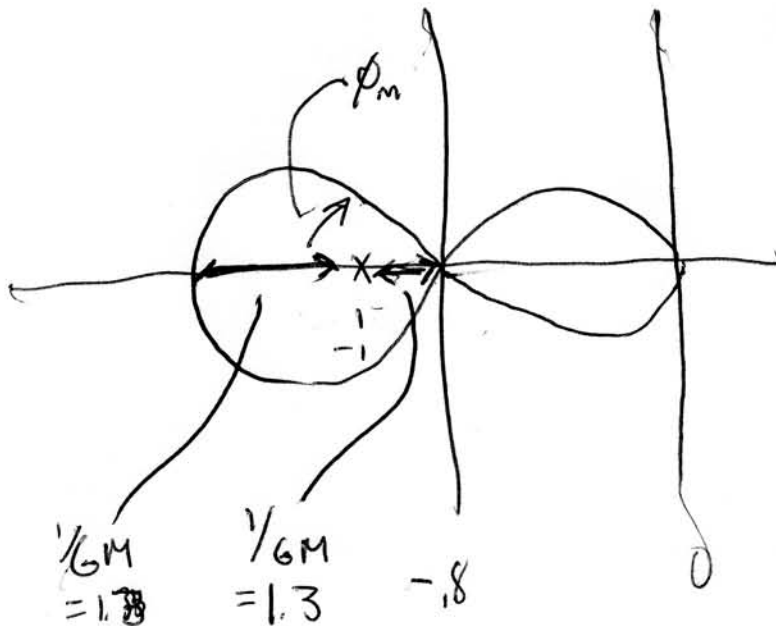The necessary compensator is a

lead compensator

Problem 3

(a) See attached, the design specs are evident in the error response

(b)

The phase margin is $26°$
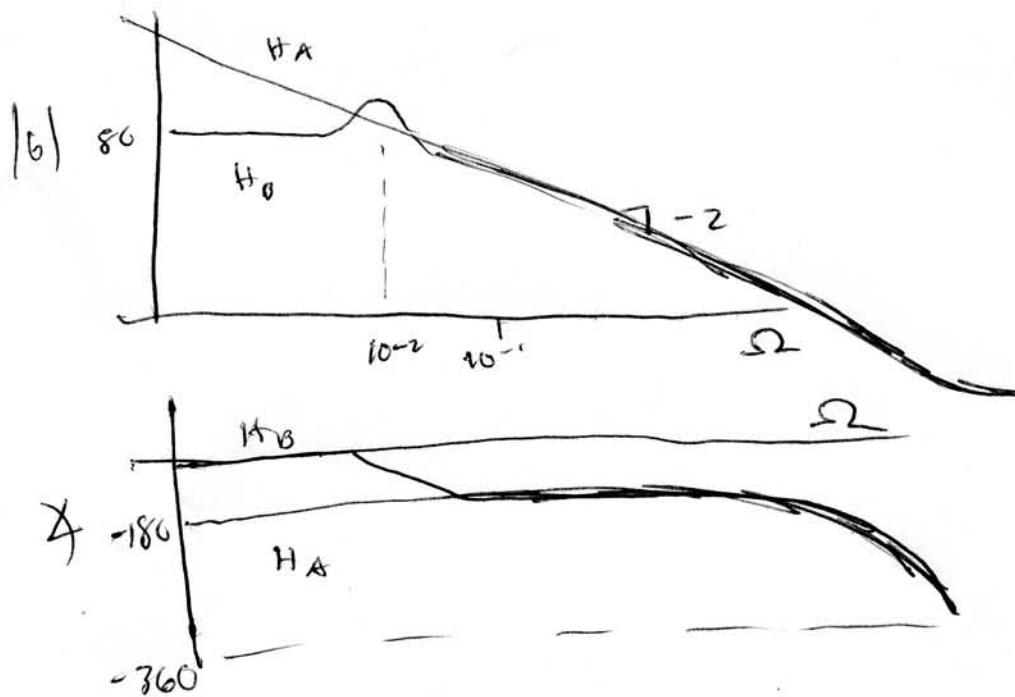
The gain margins can be represented on the Nyquist plot

$\frac{1}{GM}$
$= 1.3$

$\frac{1}{GM}$
$= 1.3$

$-.8$

0

(c)

$$G(z) = 4000 \frac{(z - .99)}{z - .94}$$

Problem 4

(a)

$|G|$ 80

H_A

H_0

-2

10^{-2} 10^{-1}

Ω

Ω

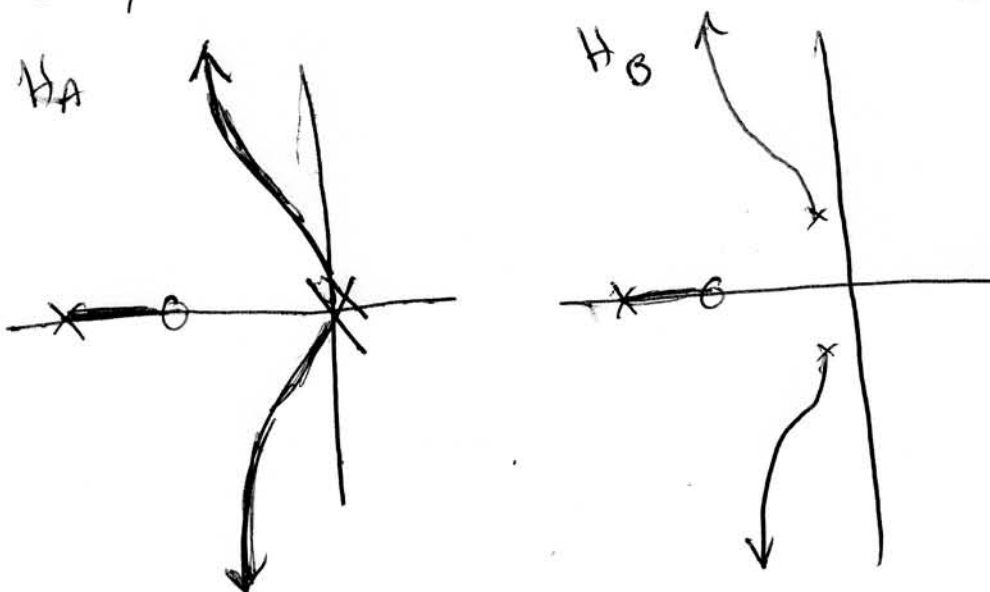$\measuredangle$ -180

H_B

H_A

-360

As seen graphically, H_A and H_B are nearly identical after $\Omega > 0.01$ rad/sec

(b) H_A H_B

(c) $G(z) = 0.02 \dfrac{z - 0.95}{z - 0.8}$

$\alpha = \dfrac{1 - 0.8}{1 - 0.95} = 4$

$\phi_{gain} = \sin^{-1}\left(\dfrac{\alpha - 1}{\alpha + 1}\right) = 36.9°$
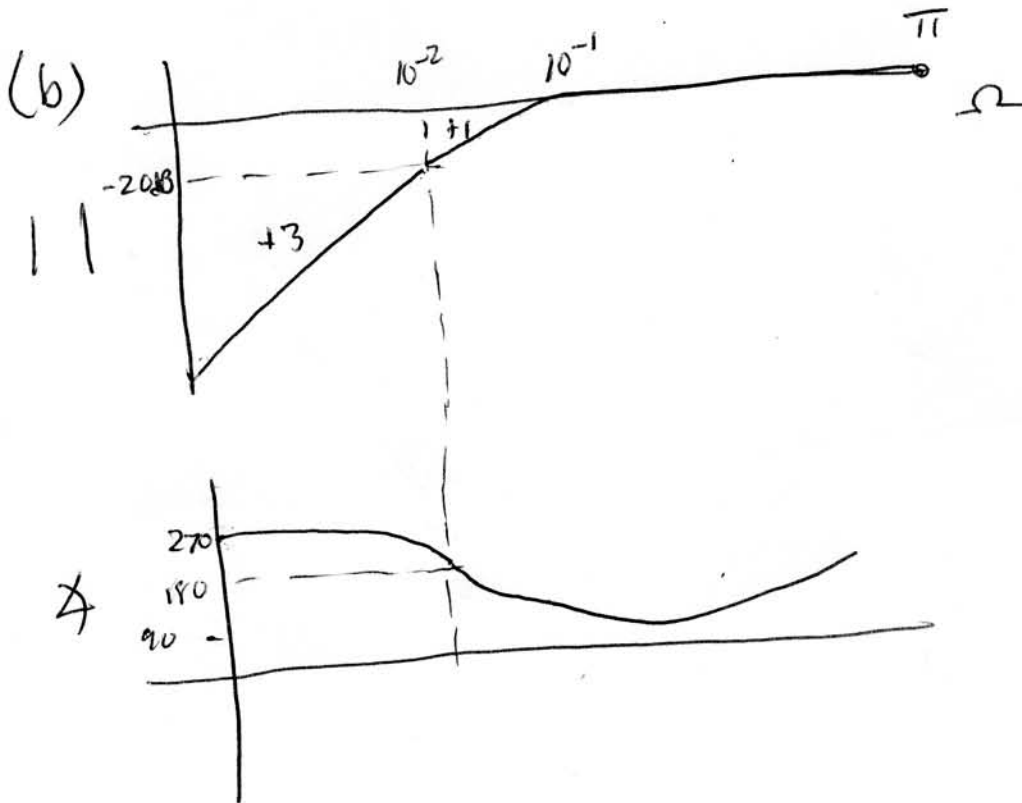
Problem 5

(a)

$$H_1(z) = \cfrac{K \frac{z-1}{z-a}}{1 + K \frac{z-1}{z-a} \frac{(z-1)^2}{(z-b)^2}} = \frac{K(z-1)(z-b)^2}{(z-a)(z-b)^2 + K(z-1)^3}$$
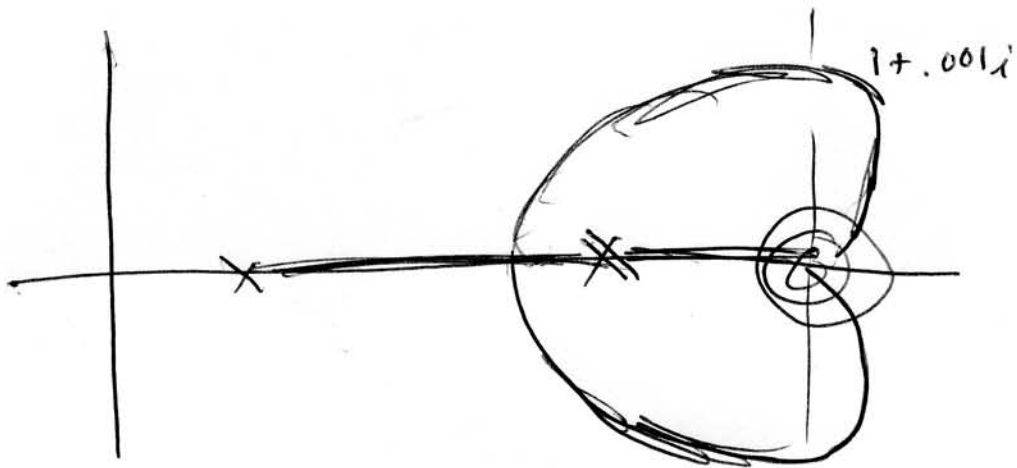
Zeros at $1, b, b$

$$H_2(z) = \cfrac{K}{1 + K \frac{z-1}{z-a} \frac{(z-1)^2}{(z-b)^2}} = \frac{(z-a)(z-b)^2}{(z-a)(z-b)^2 + K(z-1)^3}$$
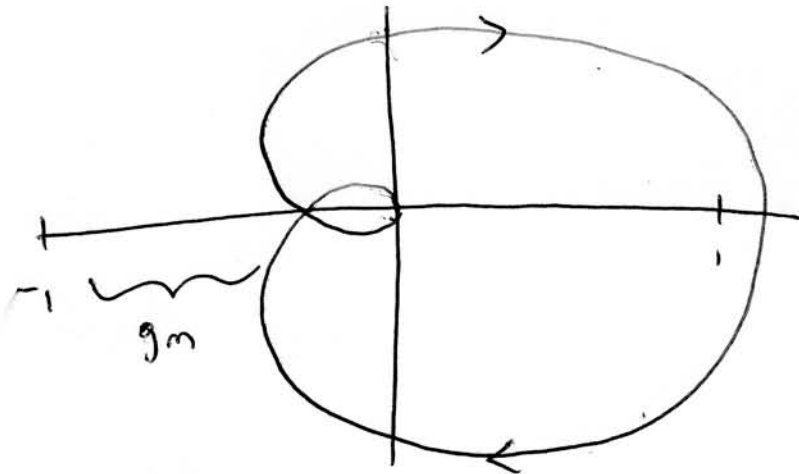
Zeros at a, b, b

(b)

c)

$1 + .001i$

d) At 10^{-1} rad/sample on the Bode plot, the phase is $180°$ and the gain is $-20dB$, thus $K \leq 10$

e)

-1 gm

(f) ~~⬚⬚⬚~~

Lag compensation can be used to meet the
phase margin design spec.

Using SISOtool,

$$G_c(z) = 30 \frac{(z-.6)(z-.95)}{(z-.12)(z-.99)}$$