2.23 PS #5 Soln

1) 2D NACA 66, $a = 0.8$ midline

$P_\infty = P_{atm} + \rho g h = 101\ kPa + 1000 \cdot 10 \cdot 2 = 121\ kPa$

$\sigma = \frac{P_\infty - P_{vap}}{\frac{1}{2}\rho V^2} = \frac{121000 - 2500}{1/2 \cdot 1000 \cdot (21)^2} = 0.54$

$\Delta\alpha_{req} = 2 \cdot \tan^{-1}\left(\frac{0.5\ m/s}{21\ m/s}\right) = 2.73°$

3 unknowns: c, $\frac{t_0}{c}$, $\frac{f_0}{c}$

3 constraints: $-c_p < \sigma$, $\Delta\alpha > \Delta\alpha_{req}$,
$L' = 10{,}000 = \rho U^2 c \frac{\pi}{2} 4 f_0/c$

valid for $\alpha = 0$

$c = \frac{L'}{\rho U^2 \frac{\pi}{2} \cdot 4 f_0/c} = \frac{3.97 \times 10^{-3}}{f_0/c}$

$\frac{t_0}{c}$	$\frac{f_0}{c}$	$-c_{p_{min}}$	$\Delta\alpha$	
0.08	0.05	~~0.75~~	3	
0.12	0.02	0.54	3.4	✓

$c = \frac{3.97 \times 10^{-3}}{0.02} = 0.2\ m$

$\boxed{c = 0.2\ m,\ \frac{t_0}{c} = 0.12,\ \frac{f_0}{c} = 0.02}$

$\boxed{\alpha = 0}$

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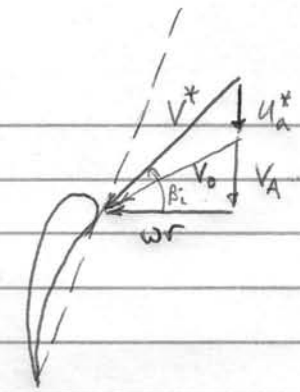

2) a) $\sigma = \dfrac{P_\infty - P_{vap}}{\frac{1}{2}\rho (V^*)^2}$

$P_\infty = P_{atm} + \rho g h = 101{,}000 + 1000 \cdot 10 \cdot 4 = 141{,}000\ Pa$

$(V^*)^2 = (V_A + u_a^*)^2 + (\omega r + u_t^*)^2$ (when $V_T = 0$, given)

$\omega = 120\ \text{rpm} \cdot \left(\frac{2\pi}{60}\ \frac{\text{rad/s}}{\text{rpm}}\right) = 12.56\ \text{rad/s}$

$r = 0.7 \cdot 2\,m = 1.4\,m$ $\Rightarrow$ $\omega r = 17.58\ m/s$

Advance coeff. $J = \dfrac{V_A}{ND} = \dfrac{10}{120 \cdot 4} = 0.02 \ll 1 \Rightarrow u_t^* \ll \omega r$ (page 158)

$\tan \beta_i = \dfrac{V_A + u_a^*}{\omega r}$ (eqn. 214)

$u_a^* = (12.56 \cdot 1.4) \tan(32.1^\circ) - 10\ m/s = 1.03\ m/s$

$V^* = 20.76\ m/s$

$\boxed{\sigma = 0.64}$

$\Delta V_A = \pm 0.5\ m/s$ $\quad \beta_i = 32.1^\circ$

$\tan(\beta_i^+) = \dfrac{10.5 + 1.03}{17.58} \Rightarrow \beta_i^+ = 33.3^\circ$

$\tan(\beta_i^-) = \dfrac{9.5 + 1.03}{17.58} \Rightarrow \beta_i^- = 30.9^\circ$

$\boxed{\Delta\beta_i = \pm 1.2^\circ}$

$\boxed{\Delta\beta_i = 2.4^\circ}$

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2b) For cavitation-free operation, we can assume an airfoil shape, NACA 66, say, and use Figure 39 to find f_o/c and t_o/c:

$$0.64 = \sigma \geq -c_{p\,min} \quad \& \quad \underset{\|}{\overset{2.4^\circ}{}}\Delta\beta_i < \Delta\alpha \;\Rightarrow\; \boxed{\frac{f_o}{c} = 0.04\,,\quad \frac{t_o}{c} = 0.08}$$

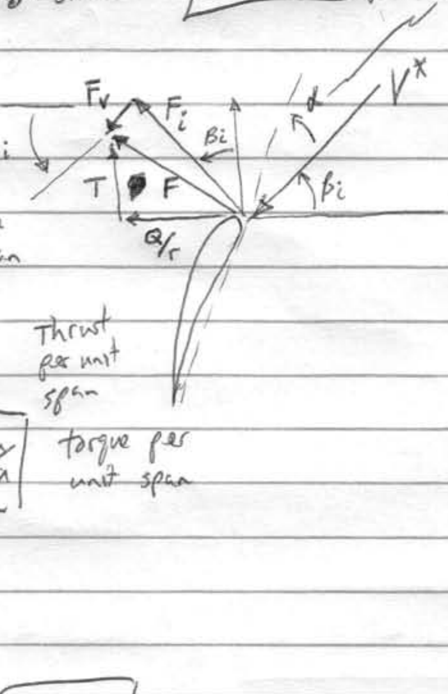
From Figure 38

$$C_L\Big|_{f_o/c=0.02} = 0.1097\left(1-0.83\,\frac{t_o}{c}\right)\left(\underbrace{\alpha}_{\text{assume } \alpha=0} + 2.35^\circ\right) = 0.24$$

$$\boxed{C_L} = \frac{f_o/c}{(f_o/c)_{spec}}\cdot C_L\Big|_{(f_o/c)spec} = \frac{0.04}{0.02}\cdot 0.24 = \boxed{0.48}$$

$$C_L = \frac{2\Gamma}{V^* c} \rightarrow \boxed{c} = \frac{2\Gamma}{C_L V^*} = \frac{2\cdot 2.5}{0.48\cdot 20.76} = \boxed{0.50\ m}$$

NOTE: this is a bad picture, because I assumed $\alpha=0$ above.



2c) $F_i = \rho V^* \Gamma = 51,900\ N/m$ inviscid force per unit span

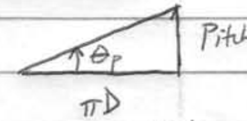
$F_v = \frac{1}{2}\rho (V^*)^2\, c\cdot C_D = 41.5\ N/m$ viscous force per unit span

$$\boxed{T} = F_i\cdot\cos\beta_i - F_v\sin\beta_i = \boxed{43,965\ N/m}$$ Thrust per unit span

$$\boxed{Q} = (F_i\sin\beta_i + F_v\cos\beta_i)\,r = \boxed{38,611\ Nm/m}$$ torque per unit span

d) $\boxed{f_o = 0.02\ m}\,,\quad \boxed{t_o = 0.04\ m}$

$$\boxed{\frac{\text{pitch}}{D}} = \pi\tan\theta_p = \pi\cdot\frac{V_A+u_a^*}{\omega r} = \boxed{1.97}$$

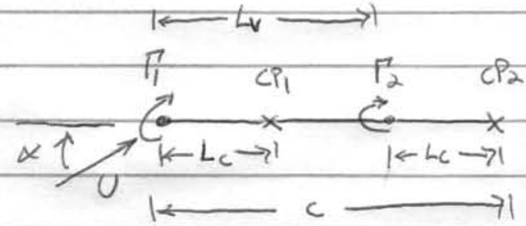


$$\tan\theta_p = \frac{\text{Pitch}}{\pi D} = \frac{V_A+u_a^*}{\omega r}$$

$$\boxed{\theta_p \Longleftrightarrow \beta_i}$$

PS #5 Soln

3)



$\rho = 1\ kg/m^3$, $U = 1\ m/s$, $c = 1\ m$

$L_V = 0.667\ m$, $L_c = 0.333\ m$

$$w_{cp_1} = -\frac{1}{2\pi L_c}\Gamma_1 + \frac{1}{2\pi(L_V - L_c)}\Gamma_2 \quad , \quad \vec{U}\cdot\hat{n}\Big|_{cp_1} = U\sin\alpha$$

$$w_{cp_2} = -\frac{1}{2\pi(L_V + L_c)}\Gamma_1 - \frac{1}{2\pi L_c}\Gamma_2 \quad , \quad \vec{U}\cdot\hat{n}\Big|_{cp_2} = U\sin\alpha$$

$$\cancel{\begin{bmatrix} -0.48 & 0.48 \\ -0.16 & -0.48 \end{bmatrix}\begin{bmatrix}\Gamma_1 \\ \Gamma_2\end{bmatrix}} \qquad \underbrace{\begin{bmatrix} -\frac{1}{L_c} & \frac{1}{L_V - L_c} \\ -\frac{1}{L_V + L_c} & -\frac{1}{L_c} \end{bmatrix}}_{A}\begin{bmatrix}\Gamma_1 \\ \Gamma_2\end{bmatrix} = -\underbrace{\begin{bmatrix}1 \\ 1\end{bmatrix}}_{B}\cdot 2\pi U\sin\alpha$$

$$\begin{bmatrix} -3 & 3 \\ -1 & -3 \end{bmatrix}\begin{bmatrix}\Gamma_1 \\ \Gamma_2\end{bmatrix} = -\begin{bmatrix} 2\pi U\sin\alpha \\ 2\pi U\sin\alpha \end{bmatrix}$$

$$\boxed{\begin{aligned}\Gamma_1 &= +0.5\cdot 2\pi U\sin\alpha \\ \Gamma_2 &= +0.1667\cdot 2\pi U\sin\alpha\end{aligned}}$$

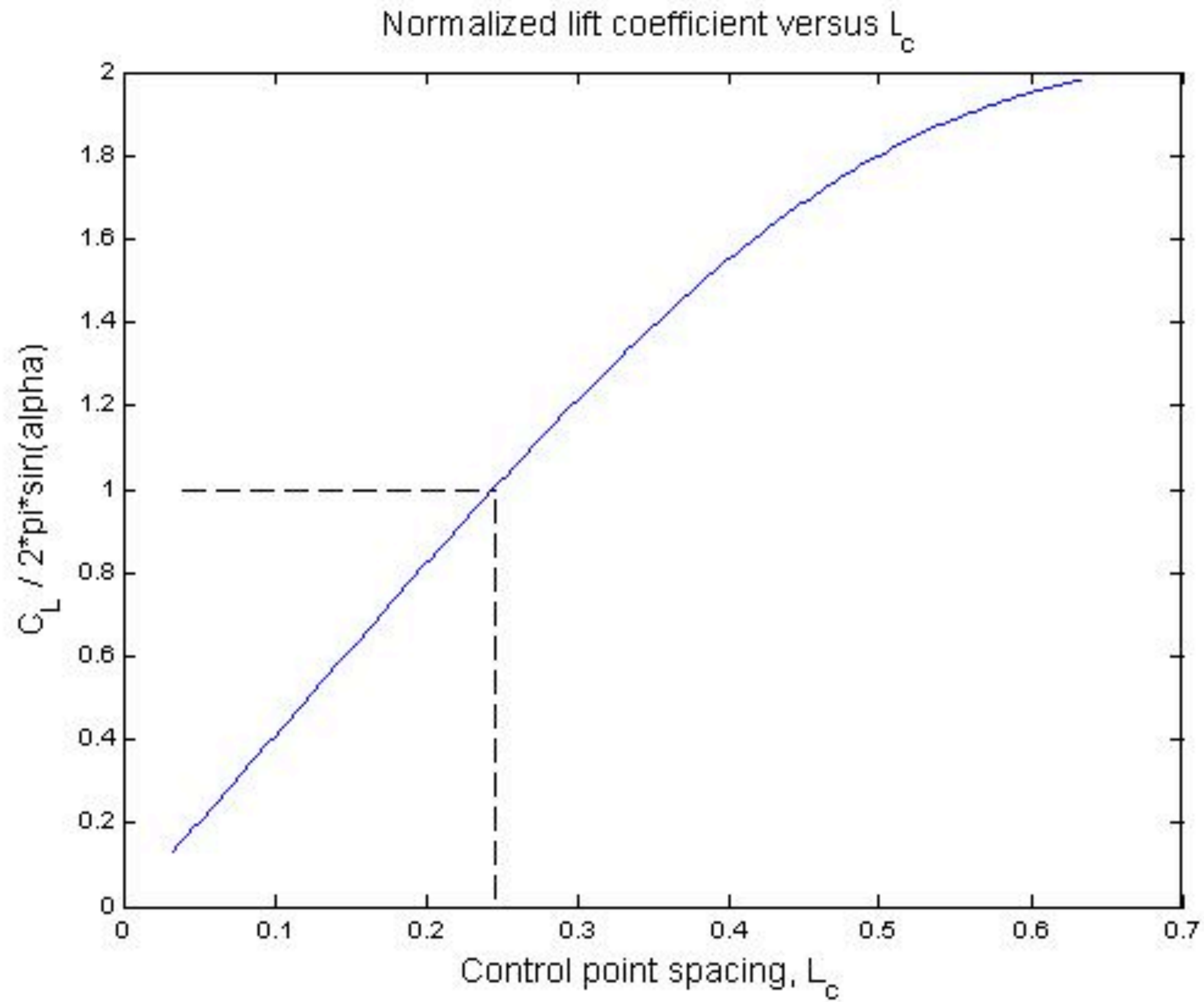
$$\cancel{w_{cv_1} = \frac{\Gamma_2}{2\pi L_V} = \frac{1/6\, 2\pi U\sin\alpha}{2\pi\, 2/3} = \frac{1}{4}U\sin\alpha}$$

$$\cancel{w_{cv_2} = \frac{-\Gamma_1}{2\pi L_V} = \frac{-1/2\, 2\pi U\sin\alpha}{2\pi\, 2/3} = -\frac{3}{4}U\sin\alpha}$$

$$\boxed{C_L} = \frac{\rho U\Gamma}{\frac{1}{2}\rho U^2 c} = \frac{2\Gamma}{Uc} = \frac{2\cdot(0.5+0.1667)\,2\pi U\sin\alpha}{U c^{\,1}} = \boxed{1.33\cdot 2\pi\sin\alpha}$$

c) Accuracy could be improved with more vortices & control points

d) (see matlab)


Normalized lift coefficient versus L_c
C_L / 2*pi*sin(alpha)
Control point spacing, L_c
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2