Problem Set No.5

1. In a Rutherford scattering experiment, one uses a Po^{210} α -source which has $E_\alpha = 5.2$ MeV and produces a collimated beam of intensity 1.1×10^5 counts/min. This beam is then incident on a gold foil of thickness 0.0001". A detector (scintillation) of an area 1.0 cm^2 is placed at a distance of 30 cm away from the foil making a 15° angle from the incident beam direction. Calculate the expected count rate from the detector in counts per minutes.

2. The Rutherford formula is able to account satisfactorily for the scattering of protons from a thin $_{90}Th^{232}$ target up to an incident kinetic energy of 4.3 MeV. Use this observation to estimate a value for the range of nuclear forces which brings about the deviation.

3. On irradiating a thick aluminum target with 7.8 MeV $\alpha -$ particles having a mean range $\overline{R} = 2.5$ mg$/$cm^2, one finds that 8 protons are produced by an (α,p) reaction for every million incident $\alpha -$ particles. What is the mean cross section for this process? Density of aluminum is $\rho = 2.69$ gr$/$cm^3 .

4. Consider elastic scattering of neutrons with moderator atoms of a mass number A. Prove the following sets of relations.

(A) If E_1 and E_2 are the neutron energies before and after the collision in the lab system then

$$\frac{E_2}{E_1} = \frac{A^2 + 2A\cos\theta' + 1}{(A + 1)^2} \tag{1}$$

and

$$\cos\theta = \frac{A\cos\theta' + 1}{\sqrt{A^2 + 2A\cos\theta' + 1}} \tag{2}$$

where θ and θ' are scattering angles in the lab and in the center of mass coordinate systems respectively.

(B) Let $P(E_1 \rightarrow E_2)dE_2$ be the probability that a neutron of energy E_1 before the collision acquires an energy between E_2 and $E_2 + dE_2$ after the collision then

$$P(E_1 \rightarrow E_2) = \frac{1}{1-\alpha} \frac{1}{E_1} \tag{3}$$

where $\alpha = (A-1)^2 / (A+1)^2$. Sketch this probability distribution graphically for A =1 and A = 12. You can prove this important result by simply using the empirical observation that the differential cross-section of neutron scattering in the center of mass system is isotropic.

(C)   Use the fact that the following relation holds

$$P(E_1 \rightarrow E_2)dE_2 = P(\cos\theta')d(\cos\theta') = \frac{1}{2}d(\cos\theta') \qquad (4)$$

to show that the average cosine of the scattering angle in the lab system is given by

$$\overline{\cos\theta} \equiv \int_{-1}^{1} \cos\theta\, P\left(\cos\theta'\right)d\left(\cos\theta'\right) = \frac{2}{3A} \qquad (5)$$

What is the physical implication of this result for the case of hydrogen?

(D)   Show that the average logarithmic energy loss ξ is given by

$$\xi \equiv \overline{\ln\left(\frac{E_1}{E_2}\right)} = 1 + \frac{\alpha}{1-\alpha}\ln\alpha \qquad (6)$$

Show that for large A, the "lethargy" ξ parameter tends to the value 2 (A + 2/3).

(E)   Knowledge of the quaintly ξ makes it possible to estimate the average number of collisions, N necessary to moderate a neutron with an initial energy E_Q to the final energy E, i.e.

$$N\xi = \overline{\ln\left(E_Q / E\right)} \quad \text{or} \quad N = \frac{\overline{\ln\left(E_Q / E\right)}}{\xi}. \qquad (7)$$

From this equation, derive the following table: $\left(E_Q = 2 \text{ MeV} \rightarrow E = 0.0253\text{eV}\right)$

	H	D	Be	C	U
A	1	2	9	12	238
α	0	0.111	0.640	0.716	0.983
ξ	1.000	0.725	0.209	0.158	0.00838
N	18	25	86	114	2172

5.   Assuming that the n-p cross-section in the energy range of 1 ev to 1 MeV is constant and equal to 20 barns. Estimate the mean free path of neutrons in a solid hydrogen moderator. From this knowledge, estimate the distance a neutron would spread out from the place it is born. calculate for the case in which the neutron is born at 1MeV and the terminal energy is 1 ev.