Core Physics Second Part
How We Calculate LWRs

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Method of Attack

- Important nuclides
- Course of calc
  - Point calc(PD + N)➔ϕ➔dN/dt➔N etc
  - Spatial Choice of attack
- Divide problem – Lattice + Core
  - Take advantage of unit cells similarity
  - Calc detailed spectra and flux shape in unit cell
  - Homogenize, average over E ➔few group σ or Σ

Important Classes of Nuclides

- Actinides = fuel + higher actinides
- Fission products
- Structural = Zr Al Fe
- Absorbers = B Er Gd Fe
- Radiologically important

Actinides - 1

- Z = 90 & beyond
- Some fissile (U233, U235, Pu239, Pu241, ...)
  - Some have very large fission cross sections
- Many formed by successive neutron captures
- Some fertile (U238, Pu240, …)
  - Neutron capture → fissile nuclide
- Significant fast fission
- Significant inelastic scattering

Actinides - 2

- Actinides start 4 long chains of (mostly) alpha decay
- 4n thorium series (Th232)
- 4n+1 neptunium - not found naturally
- 4n+2 uranium series (U238)
- 4n+3 actinium series (U235)

Actinides - 3

- Early members have very long half lives
- Tens of thousands to billions of years
- Useful in radioactive dating
- ~ 5 mev per alpha decay
- Some members are gases – radiological hazard in mining, some fuel fabrication, disposal
- Chains responsible for long term problems with waste disposal

Secular Equilibrium

- Short-lived daughters of long-lived parents
- After sufficient time, a pseudo-equilibrium is reached
- Daughter activity is ~ same as that of long-lived parent activity & decays with same half life

Bateman Solution of Linear Decay Chain

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3 \xrightarrow{\lambda_3} N_4 \xrightarrow{\lambda_4}$$

If only N_1 is present at time zero with concentration $N_1(0)$, the activity of the jth nuclide in the chain is:

$$A_j(t) = \lambda_1\lambda_2\lambda_3\ldots\lambda_j N_1(0)^*$$

$$\left\{\frac{e^{(-\lambda_1 t)}}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)\ldots(\lambda_J - \lambda_1)} + \right.$$

$$+ \frac{e^{(-\lambda_2 t)}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\ldots(\lambda_J - \lambda_2)} +$$

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$$+ \frac{e^{(-\lambda_J t)}}{(\lambda_1 - \lambda_J)(\lambda_2 - \lambda_J)\ldots(\lambda_{J-1} - \lambda_J)} \}$$

Reaction Rates

- If we know reaction rates, we know everything
- = N σ φ
- Nomenclature:
  - X = position
  - E = energy
  - B = local burnup
- KEY ASSUMPTION - for a given fuel type:

  Few group σ is only a function of x, B

Cross Sections

- Obtain σ (B) for each nuclide, fuel type from a unit cell or unit assembly calculation
- Unit calculation includes lots of energy detail, lots of local space detail (transport theory)
- Many reduced group cross section sets have been derived for various reactor types
- Ultimately, these come from “evaluated”

All Fewer Group Sets Come From Evaluated Cross Section Sets

- ENDF/B (US)
- JEF (Europe)
- JENDL (Japan)
- For common nuclides, these sets agree well
- Significant differences for uncommon nuclides

NUCLIDE	REACTION	FILE	MAT	THERMAL CROSS SECT. (B)			FAST CROSS SECT. (B)	
				2200 M/S	MAXW.AVG.	RES.INTEG.	14-MEV	FISS AVG.
92-U -235	Total	JEF-2.2	9228	697.5	606.2	556.5	5.862	7.657
		ENDF/B-VI	9228	700.6	612.2	557.5	5.860	7.663
		JENDL-3.2	9228	698.3	608.4	556.3	5.865	7.722
	BROND-2		9241	697.0	611.7	566.1	5.848	7.686
	Elast.	JEF-2.2	9228	15.11	15.02	152.8	2.840	4.409
		ENDF/B-VI	9228	15.12	15.04	152.4	2.840	4.418
		JENDL-3.2	9228	15.06	14.97	152.9	2.869	4.595
		BROND-2	9241	14.05	14.02	153.3	2.860	4.320
	Inel.	JEF-2.2	9228			0.1376	0.4177	1.917
		ENDF/B-VI	9228			0.1375	0.4177	1.917
		JENDL-3.2	9228			0.1549	0.3503	1.785
		BROND-2	9241			0.1838	0.3110	2.017
	n,2n	JEF-2.2	9228				0.5036	0.1369-01
		ENDF/B-VI	9228				0.5036	0.1369-01
		JENDL-3.2	9228				0.5429	0.1348-01
		BROND-2	9241				0.5677	0.1544-01
	n,3n	JEF-2.2	9228				0.3758-01	0.1923-04
		ENDF/B-VI	9228				0.3758-01	0.1922-04
		JENDL-3.2	9228				0.4179-01	0.1649-04
		BROND-2	9241				0.4200-01	0.1954-04
	Fiss.	JEF-2.2	9228	583.2	504.4	271.6	2.060	1.219
		ENDF/B-VI	9228	586.2	509.9	272.2	2.060	1.218
		JENDL-3.2	9228	584.4	506.8	270.9	2.054	1.237
		BROND-2	9241	584.3	510.2	270.0	2.068	1.239
	n,g	JEF-2.2	9228	98.95	86.32	132.0	0.1213-02	0.9519-01
		ENDF/B-VI	9228	99.23	87.28	132.8	0.1204-02	0.9557-01
		JENDL-3.2	9228	98.83	86.66	132.4	0.1607-06	0.9169-01
		BROND-2	9241	98.70	87.47	142.7	0.1000-02	0.9336-01

From JEF Report 14

Depletion

- Balance equation for nuclide N_j
- $dN_j/dt = -N_j\sigma_j\varphi - \lambda_j N_j + N_{j-1}\sigma_{j-1}\varphi + \lambda_{k-1} N_{k-1}$
- But for common fuel nuclides, often all we need is:

  $dN_j/dt = -N_j\sigma_j\varphi + N_{j-1}\sigma_{j-1}\varphi$

  Which is Bateman equations with $\sigma\varphi$ playing the part of λ

  But φ may vary with time

Build the Core Model

- Deplete the unit cell/assembly (with some assumption about how to keep it critical)
- “Homogenize” cross sections over the unit cell or unit assembly
- “Homogenize” cross sections over several ranges of energy to get a “few-group” model as a function of burnup for a each assembly type

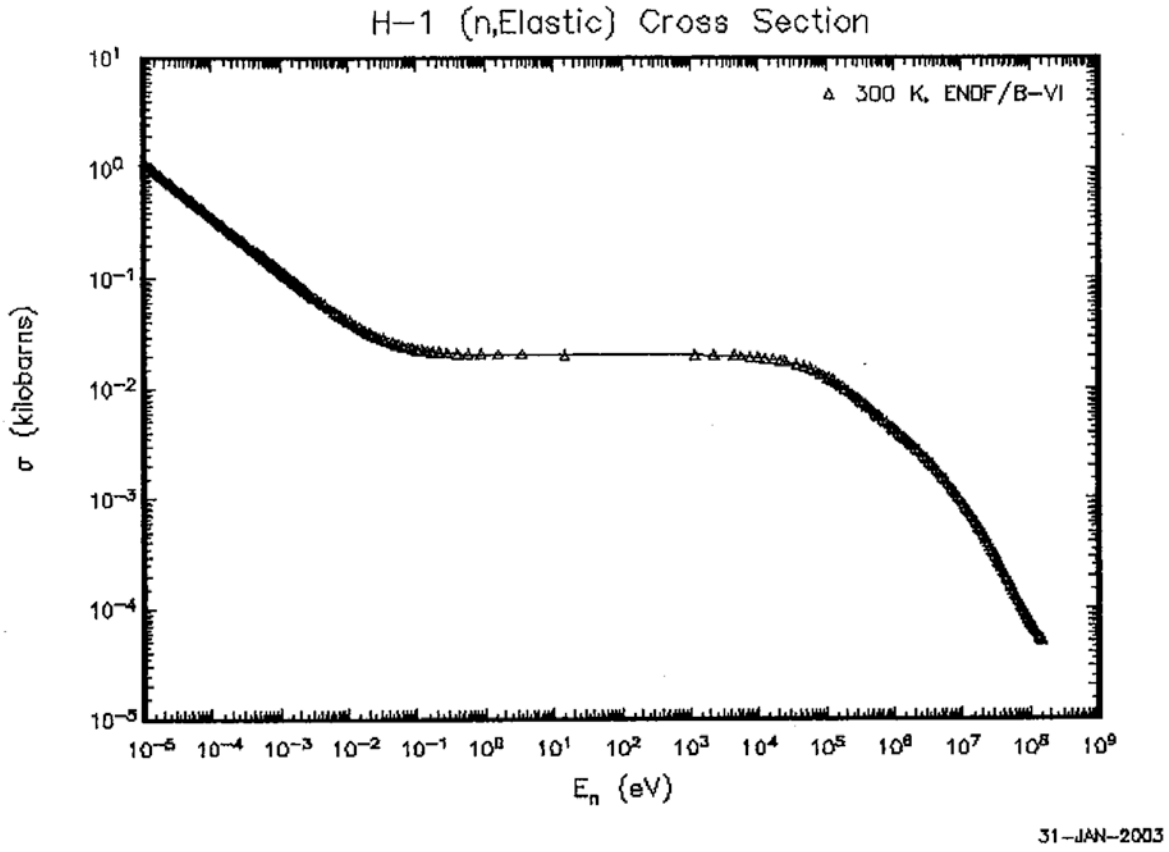
Assembly Burnup Codes

from NEA/NSC/DOC(2002)2

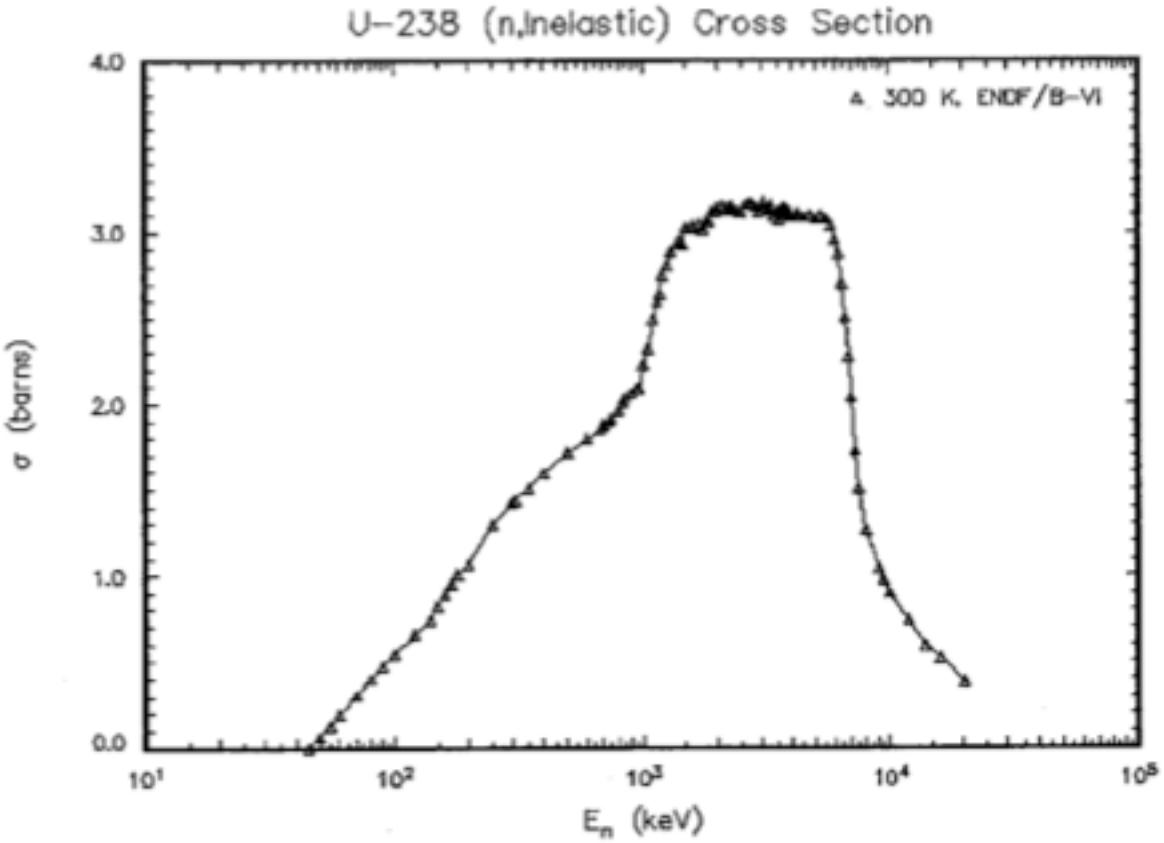
Table 3.4. Burnup codes applied in the benchmark calculations

Burnup Calculation Method	Computer Code	Version (ID*2)	No. of Participants	
Bateman	APOLLO*1	2(**C**)	1	5
	HELIOS*1	1.4(**K**)	1	
	MKENO-BURN	2(**D**)	1	
	WIMS	7(**B, G**)	2	
Matrix Exponential	BOXER*1	(**E**)	1	9
	CASMO	4(**A**)	1	
	FLEXBURN	(**L**)	1	
	KENOREST	1998(**R**)	1	
	SCALE	4.3(**P**), 4.4(**Q**)	2	
	SWAT	(**M, N**)	2	
	TGBLA/ORIGEN2.1	(**H**)	1	
Runge-Kutta-Gill	VMONT	(**J**)	1	1
Total			15	

Effect of Chemical Binding on Hydrogen Scattering Cross Section



High Energy U238 Inelastic Scattering



Core Calculation

- Nowadays – nodal diffusion theory
- Sometimes - finite difference diffusion theory
- Sometimes – monte carlo
  - "Continuous" energy monte carlo now popular
  - Often uses ORIGEN for depletion
  - Doesn't eliminate problem of homogenization

Issues

- Homogenization – how much is needed?
- Double heterogeneity (especially in HTGRs)
- Adequate spatial meshing for strong absorbers where fluxes change rapidly with distance
- Dancoff effect (mutual shielding due to adjacent fuel lumps) for irregular fuel designs

22.251 Systems Analysis of the Nuclear Fuel Cycle

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