12 Magnetic fields

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Please see: Table 1.1 in Jiles, David. *Introduction to Magnetism and Magnetic Materials.*

Magnetism

cgs units	*mks* units
$B = H + 4\pi M$	$B = \mu_0 H + M$ or $B = \mu_0 (H + M$.
B in gauss	B in webers/meter^2 (tesla)
H in oersteds	H in amperes/meter
M in emu/cm^3	M in webers/meter^2 M = amperes/mete
μ (vacuum) = 1	μ_0 (vacuum) = $4\pi \times 10^{-7}$ weber/ampere meter

cgs to *mks*	*mks* to *cgs*
B: 1 gauss = 10^{-4} weber/meter^2	1 weber/meter^2 = 10^4 gauss
H: 1 oersted = 79.6 amperes/meter	1 ampere/meter = 12.57×10^{-3} Oe
M: 1 emu/cm^3 = 12.57×10^{-4} weber/meter^2	1 weber/meter^2 = 796 emu/cm^3
ϕ: 1 maxwell = 10^{-8} weber	1 weber = 10^8 maxwells

Length

1 angstrom (Å) = 10^{-8} cm	1 cm = 0.394 in.
1 micron (μ) = 10^{-6} meter	1 in. = 2.54 cm
= 10^{-4} cm	10^{-3} in. = 1 mil = 25.4 μ
= 10^4 Å	1μ = 39.4×10^{-6} in.

Mass

1 kg = 2.205 lb	1 lb = 0.454 kg

Energy

1 cal = 4.19×10^7 ergs	1 erg = 2.39×10^{-8} cal
1 erg = 10^{-7} joule	1 joule = 10^7 ergs
1 eV = 1.602×10^{-12} erg	1 erg = 6.25×10^{11} eV

SUMMARY

	MKS/SI	cgs	Conversion
$H =$	Ni/l [A/m]	$0.4\pi Ni/l$ [Oe]	80A/m = 1Oe
$B =$	$B = \mu_o(H + M)$ [T]	$B = H + 4\pi M$ [G]	$1T = 10^4G$
$M =$	$M = \chi_m H$ [A/m]	$M = \chi_m H$ [A/m]	80A/m = 1Oe
$\mu =$	$\mu_o(1 + \chi_m)$ [H/m]	$1 + 4\pi\chi_m$ [none]	$\mu_o = 4\pi \times 10^{-7}$H/m
$p_m = \mu_m =$	iA [Am^2]	iA [emu, Gcm^3]	
$U =$	$-\boldsymbol{\mu_m} \cdot \boldsymbol{B}$ [J]	$-\boldsymbol{\mu_m} \cdot \boldsymbol{H}$ [erg]	10^7erg = 1J
$u =$	$-\boldsymbol{M}\cdot\boldsymbol{B}$ [J/m^3]	$-\boldsymbol{M}\cdot\boldsymbol{H}$ [erg/cm^3]	10 erg/cm^3 = $1J/m^3$

Maxwell's equations

$\nabla \cdot \boldsymbol{E} = \rho/\varepsilon$	$\nabla \cdot \boldsymbol{E} = 4\pi\rho/\varepsilon$
$\nabla \cdot \boldsymbol{B} = 0$	$\nabla \cdot \boldsymbol{B} = 0$
$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$	$\nabla \times \boldsymbol{E} = -c^{-1}\partial \boldsymbol{B}/\partial t$
$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{J} + \mu_o\varepsilon\partial \boldsymbol{E}/\partial t$	$\nabla \times \boldsymbol{H} = (4\pi/c)\boldsymbol{J} + (\cancel{\varepsilon}/c)\,\partial \boldsymbol{E}/\partial t$

$\int \boldsymbol{E}\cdot d\boldsymbol{A} = (1/\varepsilon)\int\rho(x-x')d^3x' = q/\varepsilon$	$\int \boldsymbol{E}\cdot d\boldsymbol{A} = (4\pi/\varepsilon)\int\rho(x-x')d^3x' = 4\pi q/\varepsilon$
$\int \mathbf{B}\cdot d\mathbf{A} = 0$	$\int \mathbf{B}\cdot d\mathbf{A} = 0$
$\int \boldsymbol{E}\cdot d\boldsymbol{l} = -\partial/\partial t \int \boldsymbol{B}\cdot d\boldsymbol{A}' = -\partial\phi/\partial t$	$\int \boldsymbol{E}\cdot d\boldsymbol{l} = -(1/c)\partial/\partial t \int \boldsymbol{B}\cdot d\boldsymbol{A}' = -(1/c)\partial\phi/\partial t$
$\int \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_o \int \boldsymbol{J}\cdot d\boldsymbol{A}' = \mu_o i$	$\int \boldsymbol{B}\cdot d\boldsymbol{l} = (4\pi\mu_o/c)\int \boldsymbol{J}\cdot d\boldsymbol{A}' = (4\pi\mu_o/c)\, i$

Field about wire:

$B = \mu_o i/2\pi R$ [T] $\qquad H = 2i/cR$ [Oe]

Solenoid:

$B = \mu_o Ni/l$ [T] $\qquad H = 0.4\pi Ni/l$ [Oe]

Dipole field:

$$\boldsymbol{B} = (\mu_o/4\pi)\,[(2iA\cos\theta/r^3)\, \boldsymbol{e_r} + (iA\sin\theta/r^3)\, \boldsymbol{e_\theta}]$$

Image removed due to copyright restrictions. Please see: Fig. 2.3 in Jiles, David.

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Permeability and susceptibility of various materials

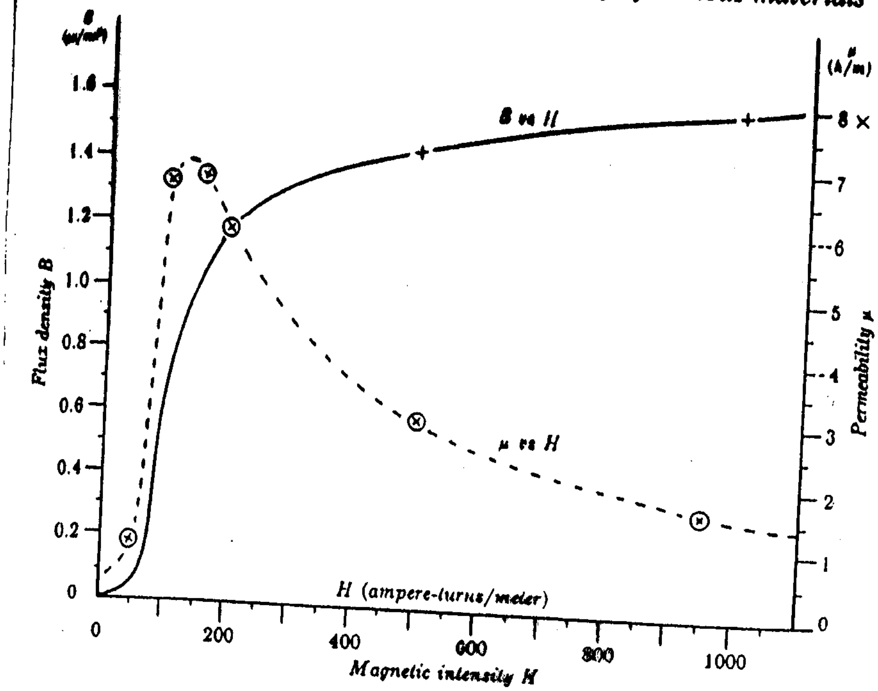


Fig. 2.3 Initial magnetization curve and permeability along the same curve for annealed iron.

UNIFORM B or H FIELD

B field in 0 APPLIED FIELD

B field (vector sum of (a) + (b))

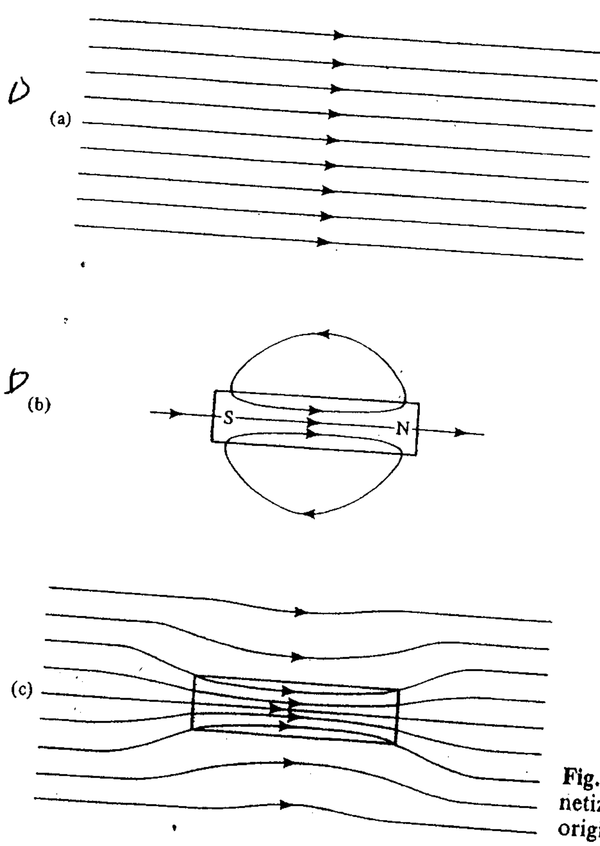


Fig. 2.22 Result of placing a magnetized body, having $\mu > 1$, in an originally uniform field.

FLUX CROWDS INTO MAGNET - ORIGIN OF TERM permeability (μ)

Outside magnet, FIELD (B or H) reduced.

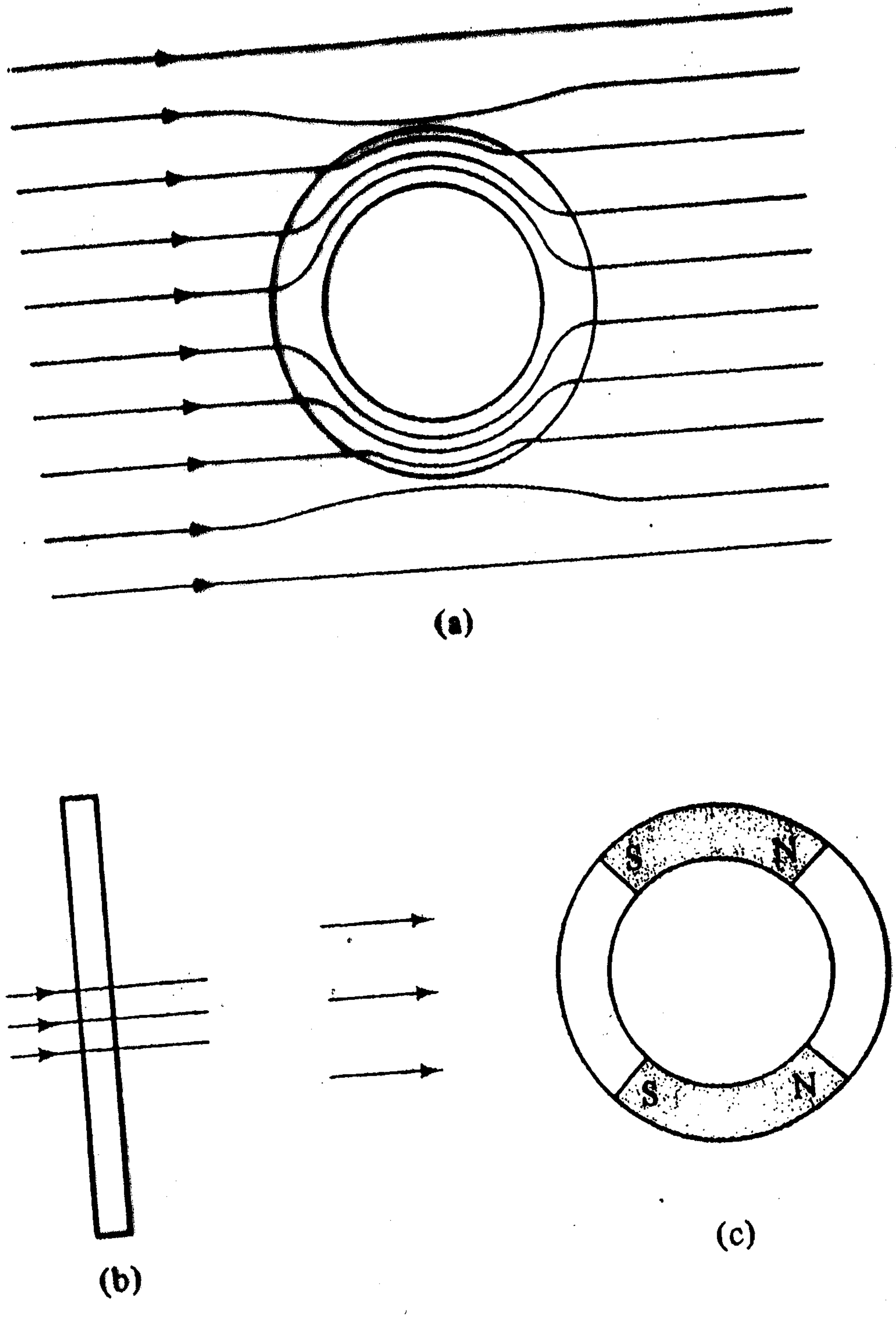
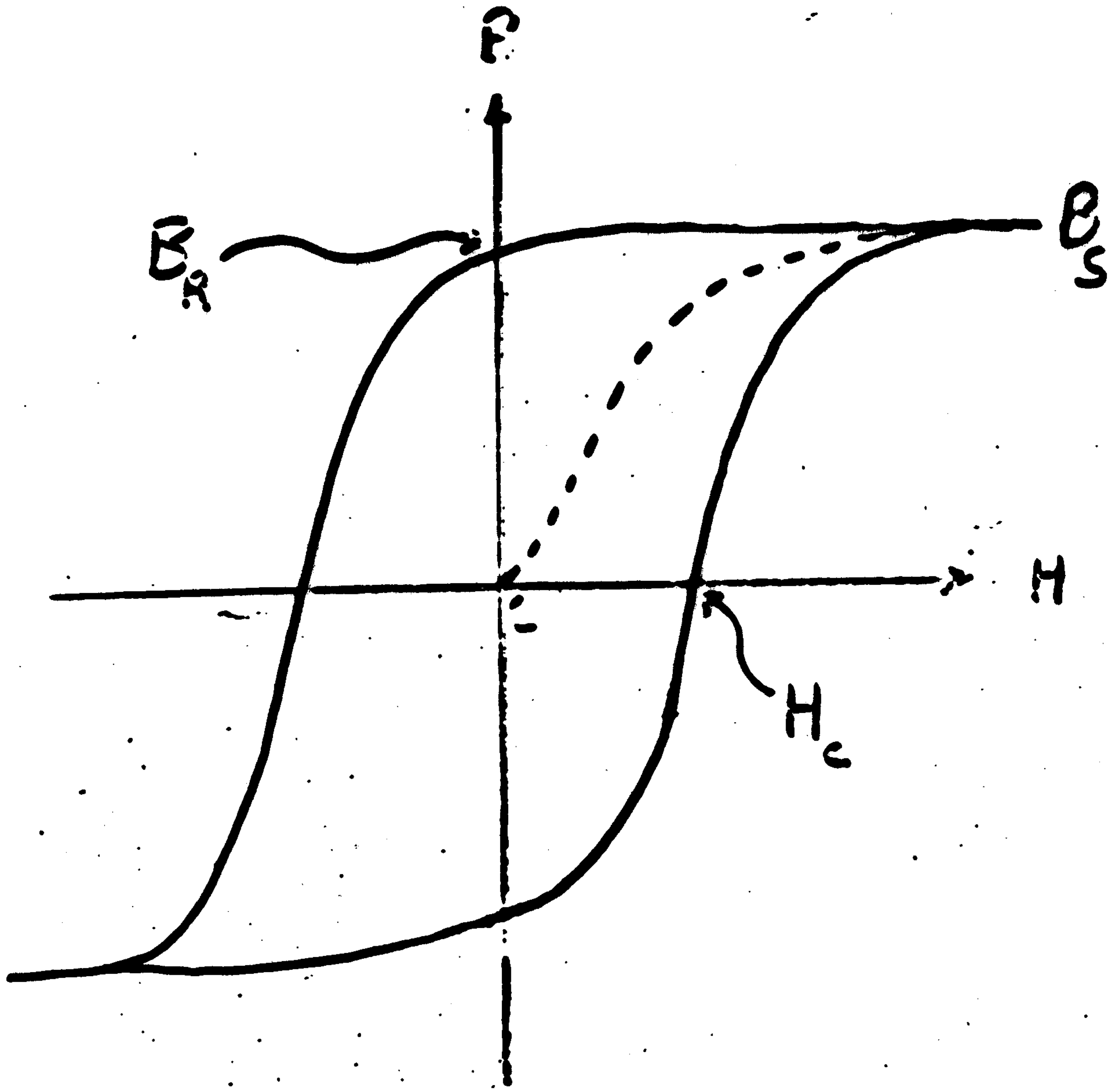


Fig. 2.2



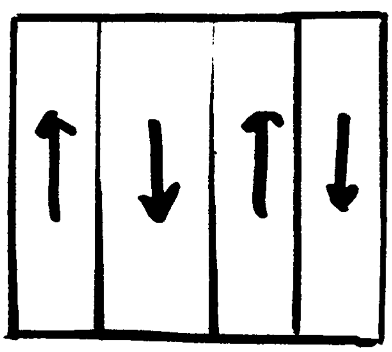
HYSTERESIS LOOP

$$10^{-3} < H_c < 4 \times 10^{4}$$

EXCHANGE ENERGY

$$U_{EX} = -2J \sum S_i \cdot S_{i+\delta}$$

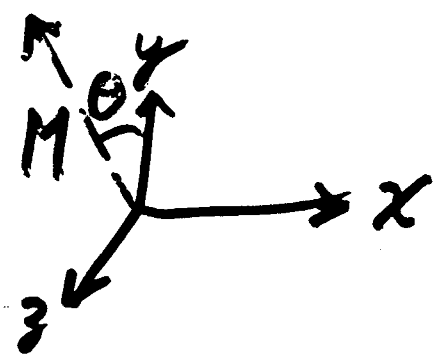
$$\Delta U_{EX} = A' \left[(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2 \right]$$



LIMITS NUMBER OF DOMAINS.

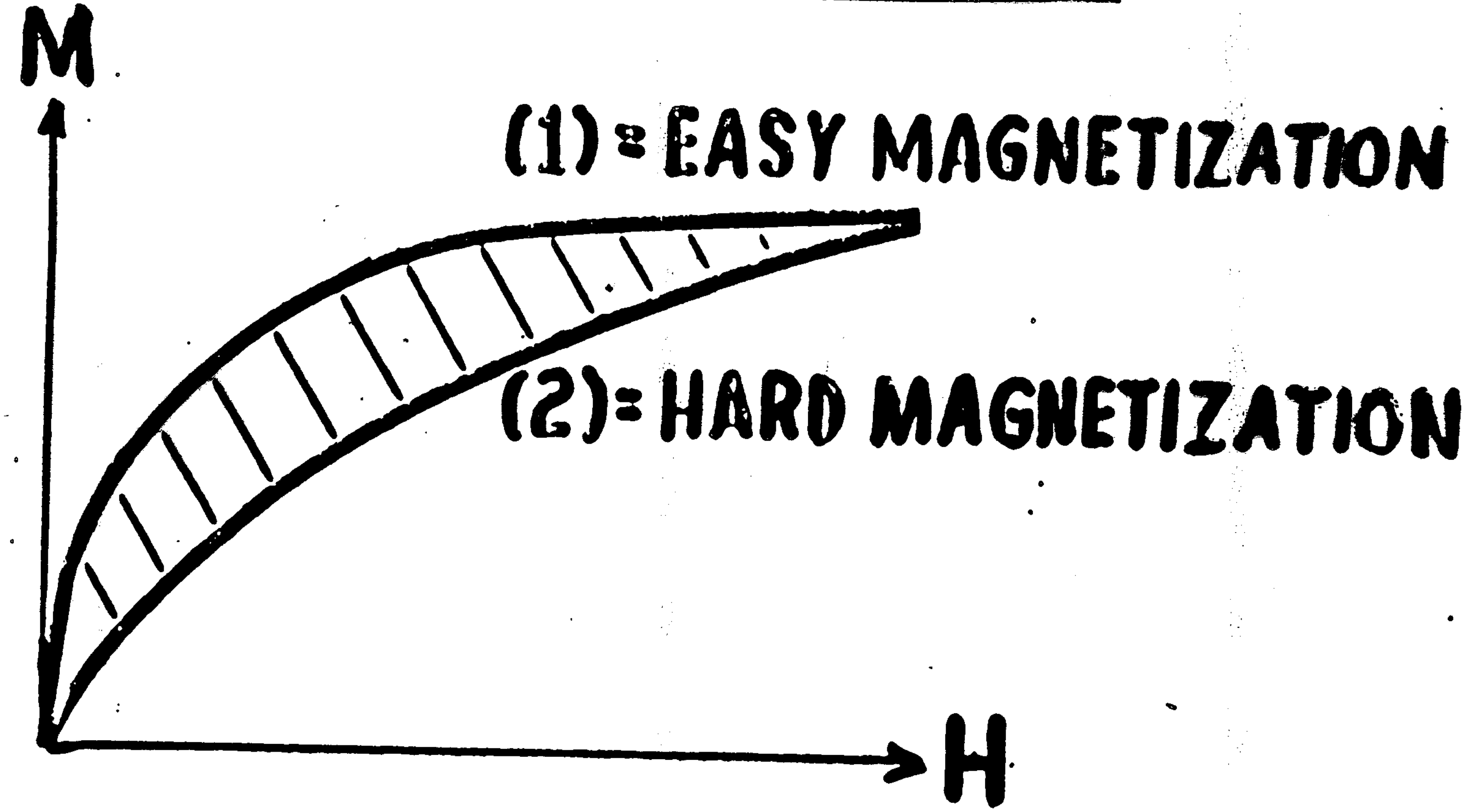
$M = M(x)$,

$M_x = 0$,



$$\Delta U_{EX} = A \left(\frac{\partial \theta}{\partial x} \right)^2$$

ANISOTROPY ENERGY



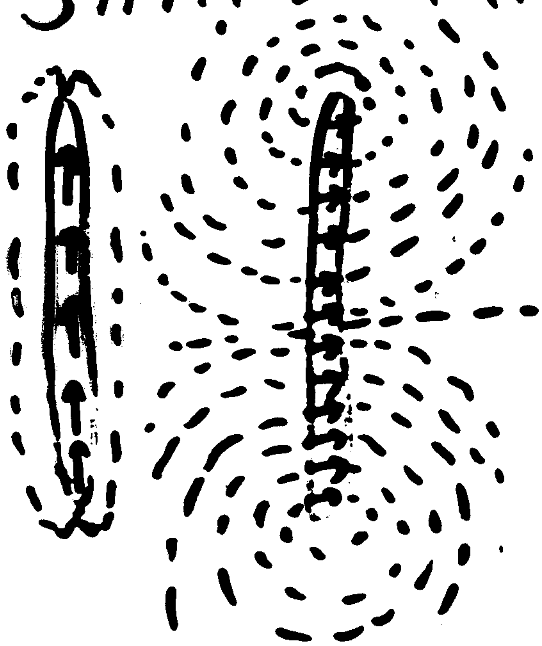
UNIAXIAL SYMMETRV, $U_{anis} = K \sin^2 \theta$
(COBALT)

ANISOTROPY

1) CRYSTALLINE ANISOTROPY

i.e., hcp → c AXIS

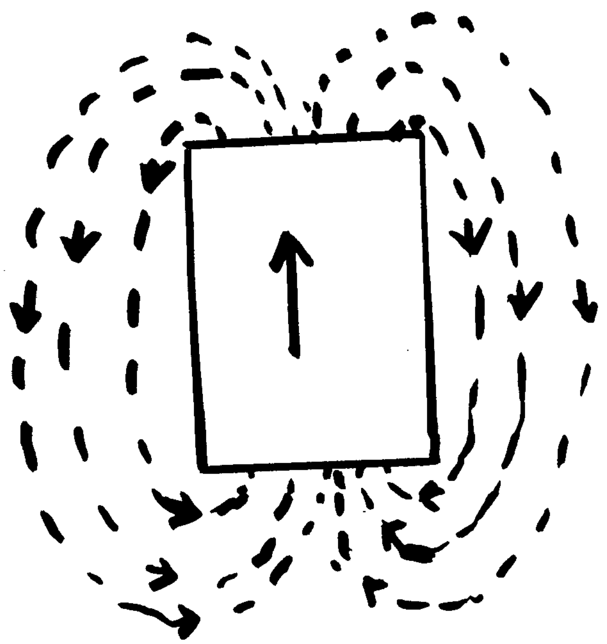
2) SHAPE ANISOTROPY

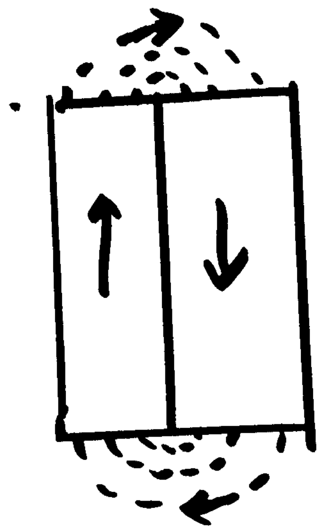


$$H_K \propto 4\pi M$$

$$H_K \equiv H_D$$

MAGNETOSTATIC ENERGY



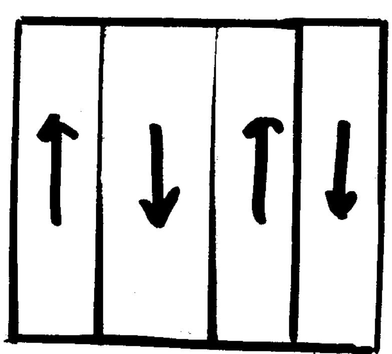


$$U_{M.S.} = -\frac{1}{2}\int H \cdot M \, dv$$

EXCHANGE ENERGY

$$U_{EX} = -2J \sum S_i \cdot S_{i+\delta}$$

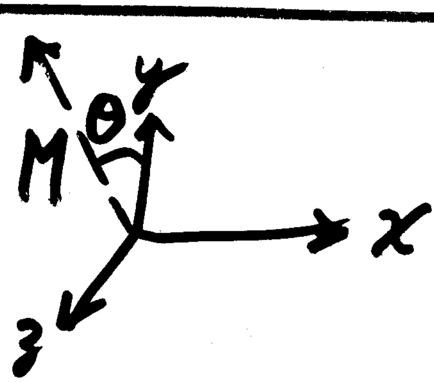
$$\Delta U_{EX} = A' \left[(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2 \right]$$



LIMITS NUMBER OF DOMAINS.

$M = M(x)$,

$M_x = 0$,



$$\Delta U_{EX} = A \left(\frac{d\theta}{dx} \right)^2$$

ANISOTROPY ENERGY ⇒

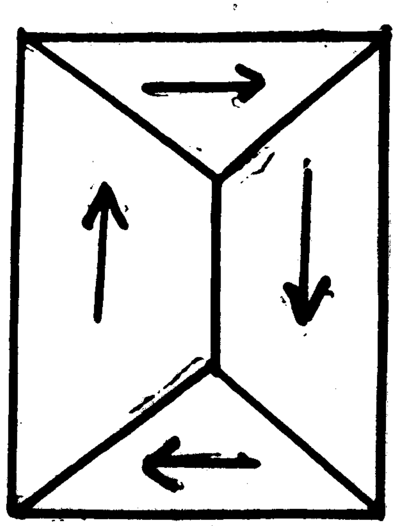
(EASY AXIS OF MAGNETIZATION)

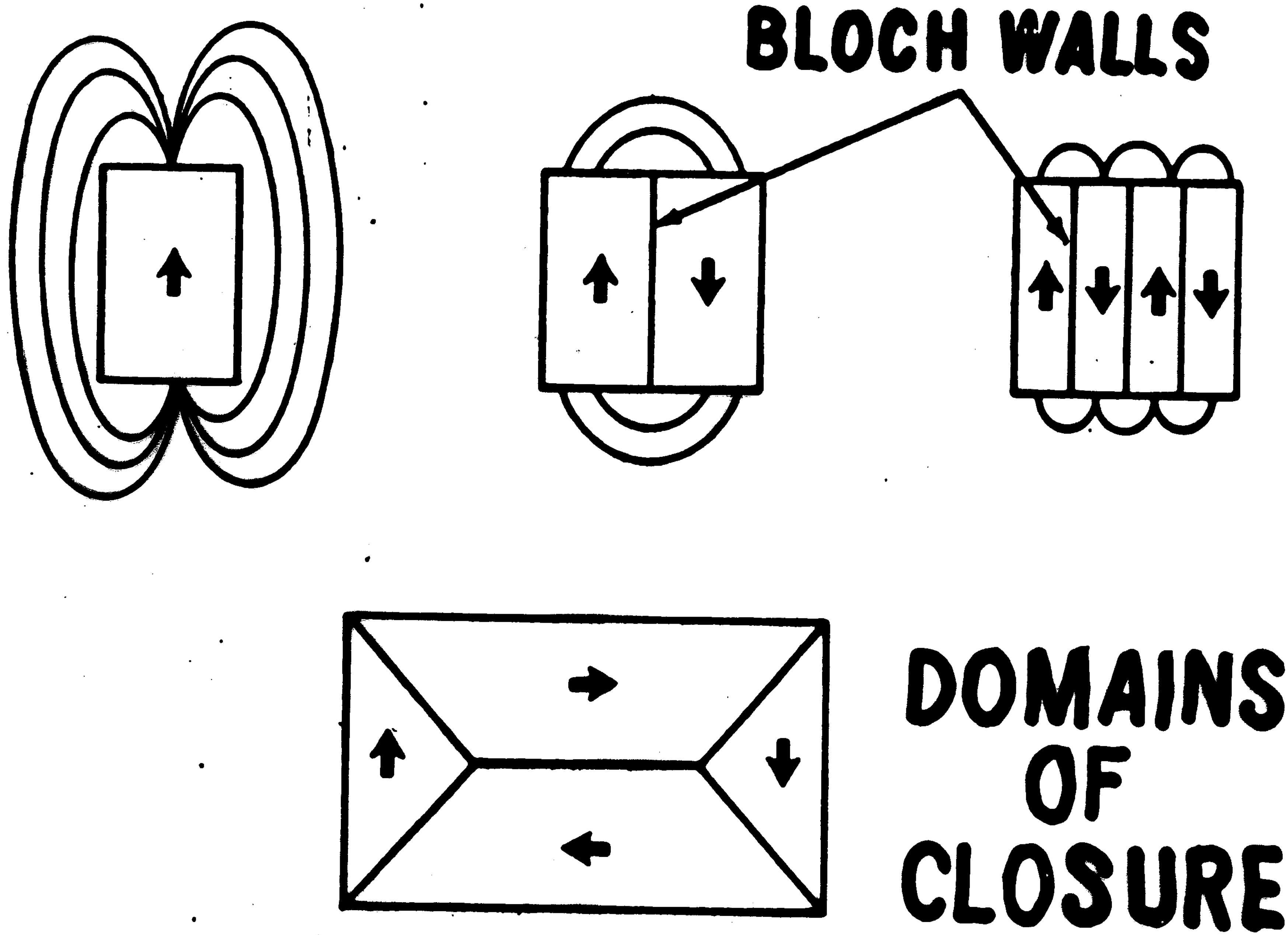
AND

MAGNETOELASTIC ENERGY ⇒

(ELONGATION IN DIRECTION OF MAGNETIZATION)

LIMIT DOMAIN CONFIGURATIONS

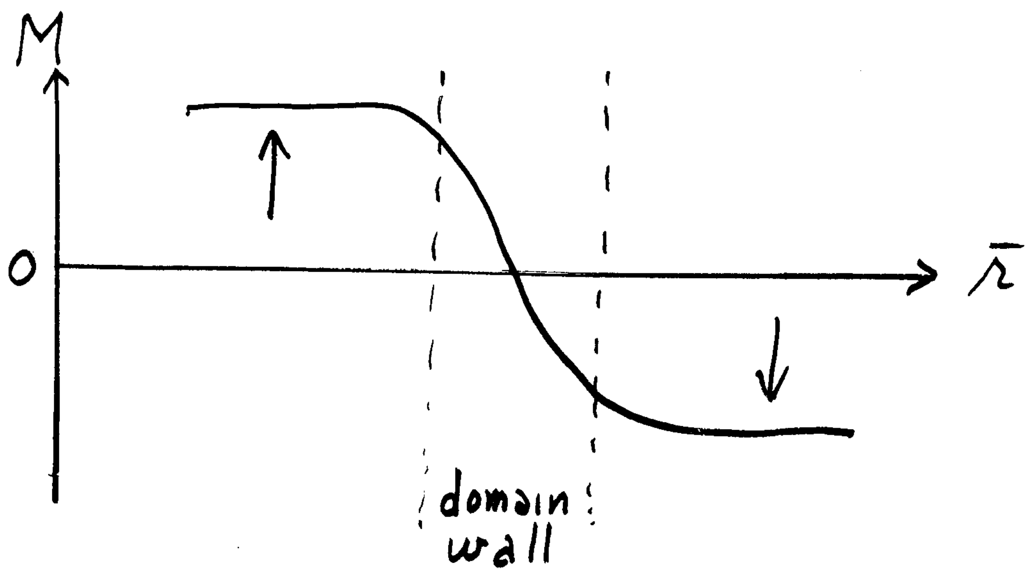



BLOCH WALLS
DOMAINS
OF
CLOSURE

MA NETIC DOMAIN WALL · FORMATION (STATICS)

$$E = \delta \int (\text{ANISOTROPY} + \text{Exchange})\, d\bar{r} = 0$$

$\Rightarrow$

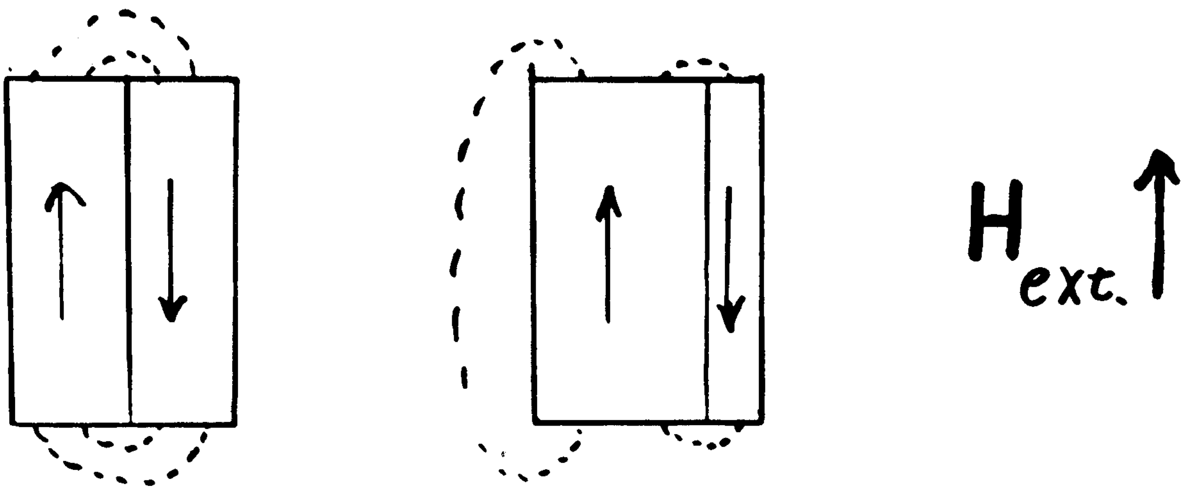


$$\frac{\text{VALL ENERGY}}{\text{UNIT AREA}} = \sigma = 4\sqrt{K \cdot A} \ \ \text{ergs/cm}^2$$

DYNAMICS OF MAGNETIZATION

1) Spin Rotation - Single Domain

2) Domain Growth



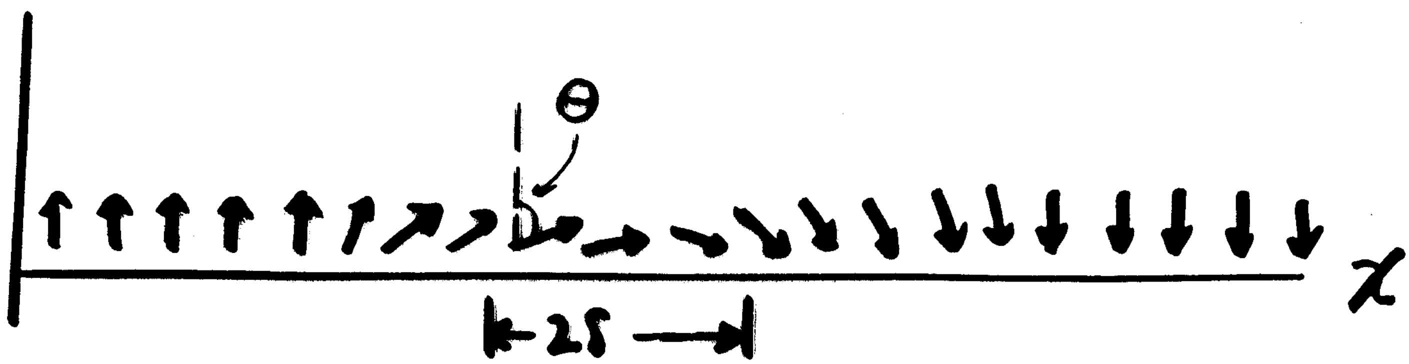
3) Pinning of Domain Walls
   Nucleation of Domain Walls

a) Grain boundaries

b) Edges

c) Impurities

FERROMAGNETIC DOMAIN WALL

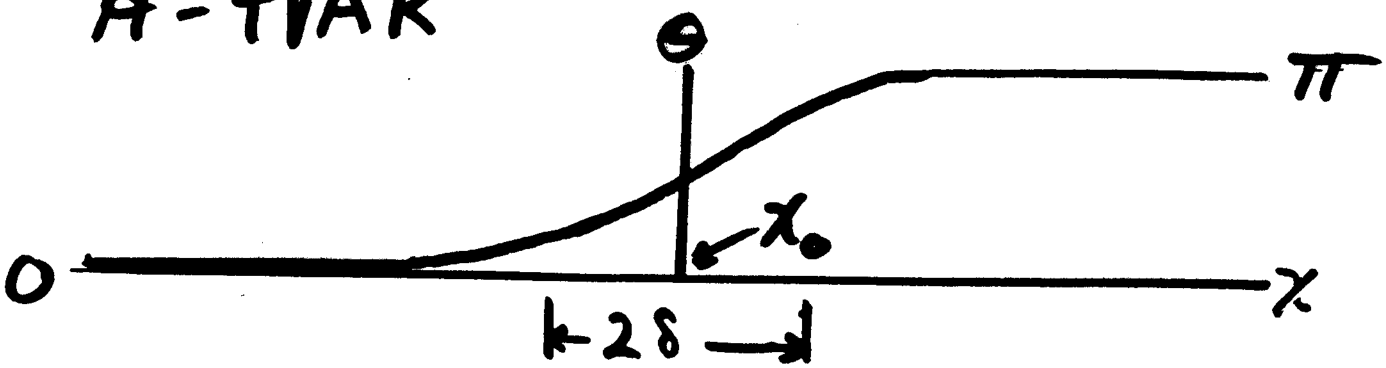


$$\mathcal{H} = \int_{-\infty}^{\infty} \left[A \left(\frac{d\theta}{dx} \right)^2 + K \sin^2 \theta \right] dx$$

$$\delta \mathcal{H} = 0 \Rightarrow \frac{K}{2} \sin 2\theta - A \frac{d^2\theta}{dx^2} = 0$$

$$\theta = \sin^{-1} \operatorname{sech}\left(\frac{x - x_0}{\delta} \right), \quad \delta = \sqrt{A/K} = \text{WALL HALF-WIDTH}$$

$$\mathcal{H} = 4\sqrt{AK}$$



$$\frac{K}{2}\sin 2\theta - A\frac{d^2\theta}{dx^2} = 0$$

$$\int \frac{d\theta}{dx}\,dx \Rightarrow \frac{K}{2}\int \sin 2\theta\, d\theta = A\int \frac{d\theta}{dx}\frac{d\frac{d\theta}{dx}}{dx}\,dx$$

or

$$\frac{A}{2}\left(\frac{d\theta}{dx}\right)^2 + \frac{K}{4}\cos 2\theta = C$$

B.C. $x = \pm\infty$, $\frac{d\theta}{dx} = 0$, $\theta = 0$ or π

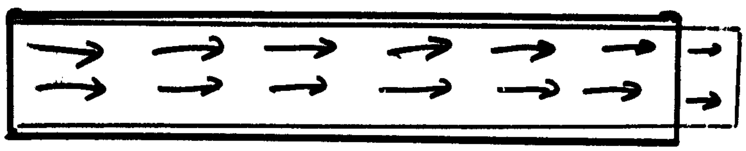
$$\Rightarrow C = \frac{K}{4} \quad \text{or} \quad A\left(\frac{d\theta}{dx}\right)^2 = K\sin^2\theta$$

or $$\int_{\theta_0}^{\theta} \frac{d\theta}{\sin\theta} = \int_{x_0}^{x} \frac{dx}{\delta}, \quad \delta = \sqrt{\frac{A}{K}}$$

B.C. or Symmetry, x_0 = midpoint $\Rightarrow \theta_0 = 90^\circ$

$$\theta = \sin^{-1}\left(\operatorname{sech}\frac{x - x_0}{\delta}\right)$$

MAGNETO-ELASTIC OR MAGNETO-STRICTIVE ENERGY.



Elongates in direction of magnetization

If such elongation not physically possible ⇒ compression; elastic stress

Reverse Process

Elongate mat'l ⇒ induces magnetization in that direction

Compress mat'l ⇒ magnetization in ⊥ direction.

$$E_{ME} = (9/4)\left[(C_{11}-C_{12})\lambda_{100}^{2} - 2C_{44}\lambda_{111}^{2}\right]$$

C = elastic constants, λ = magnetostriction constant

Isotropic $E_{ME} = (3/2)\lambda\sigma\sin^2\theta$