6.301 Solid-State Circuits

Recitation 25: More on Charge Control and Space-Charge Lasers
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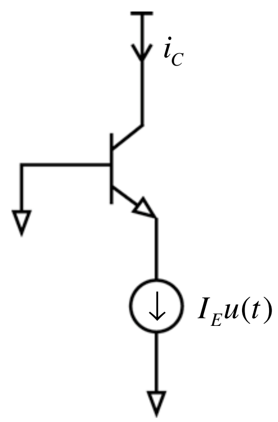
First, let's do some cleanup from the last recitation. We said that we would treat the problem of emitter switching as an example of how to use the charge control equations.



Note that from our experience with Lab 2, we might expect this <u>common</u> <u>base</u> stage to be relatively fast. What does charge control have to say about that?

Use charge-control equations for emitter current. Notice that I follow my own sign convention here…I choose to define current flow out of the emitter as positive.

$$i_E = q_F\left(\frac{1}{\tau_F}+\frac{1}{\tau_{BF}}\right)+\frac{dq_F}{dt} = I_E u(t)$$

If we solve this, we find that

$$q_F = \tau I_E\left(1-e^{-t/\tau}\right)$$

Where τ is defined by

$$\frac{1}{\tau}=\frac{1}{\tau_F}+\frac{1}{\tau_{BF}}$$

Writing this out:

$$\tau = \frac{\tau_F\tau_{BF}}{\tau_F+\tau_{BF}} = \frac{\beta\tau_F^{\ 2}}{\tau_F+\beta\tau_F} = \frac{\beta}{\beta+1}\tau_F = \alpha_F\tau_F$$

So the time constant involved is smaller than τ_F! Recall that the time constant associated with base current drive was τ_{BF}. Now we have all we need to calculate i_C:

$$i_C = \frac{q_F}{\tau_F} = \alpha_F I_E \left(1 - e^{-t/\alpha_F \tau_F}\right)$$

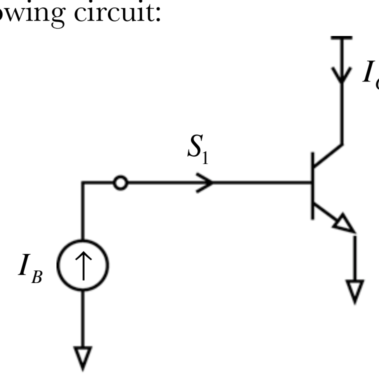
And for i_B, we apply KCL (again remembering the sign convention that I have chosen):

$$\begin{aligned} i_B + i_C = i_E \Rightarrow \ i_B &= i_E - i_C \\ &= I_E - \alpha_F I_E \left(1 - e^{-t/\alpha_F \tau_F}\right) \end{aligned}$$

Initially, then, the base current = i_E!

CLASS EXERCISE

Consider the following circuit:



$$i_C = \frac{q_F}{\tau_F}$$

$$i_B = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt}$$

$$i_E = -q_f\left(\frac{1}{\tau_F} + \frac{1}{\tau_{BF}}\right) - \frac{d_{qF}}{dt}$$

At time t=0, we open switch S_1. Derive and sketch I_C as a function of time.

(Workspace)

Take a moment to remind yourself of what is physically happening. We have assumed that recombination in the base is negligible. Thus, the collector current continues just as long as it takes to clear q_F out of the base via reverse injection.

Space Charge Layers

This, unfortunately, is where things get messy. The upside is that working through the math here will give you a very detailed understanding of what goes on in a bipolar transistor.

Recall that the BC and the BE pn junctions have capacitances associated with their depletion regions. We are able to define a nonlinear, voltage-dependent capacitance associated with these depletion regions, and relate charge and voltage as

$$dQ = C(V)dV$$

To figure out how much charge it takes to get to a voltage V_0, we must integrate

$$\int_0^{Q_0} dQ = \int_0^{V_0} C(V)dV$$

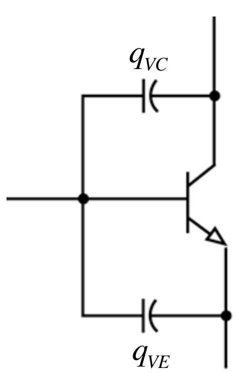
$$Q_0 = \int_0^{V_0} C(V)dV$$

As a check, we note that with a capacitance that is independent of voltage, we have

$$Q_0 = \int_0^{V_0} CdV = C\int_0^{V_0} dV = CV_0$$

Just as we expect.

Now for this BJT, we have



We have to store charge q_{VC} on C_μ, and q_{VE} on C_{je} (part of C_π). This charge has to be supplied to the base, and that is why these capacitances are relevant to us here. For C_μ, we have

$$C_\mu = \frac{C_{\mu 0}}{\left(1 - V_{BC} / \psi_C\right)^{mC}} = C_{\mu 0} \psi_C{}^{m_C} \left(\psi_C - V_{BC}\right)^{-m_C}$$

If we're curious about the charge on C_μ, we must perform the integration

$$\int_0^{Q_{VC}} dQ_{VC} = \int_0^{V_{BC}} C_\mu(V) dV$$

$$\begin{aligned} Q_{VC} &= \left[-\left(\frac{C_{\mu 0} \psi_C{}^{m_C}}{1 - m_C} \right) \left(\psi_C - V_{BC}\right)^{1-mc} \right]_0^{V_{BC}} \\ &= K_C \left[\psi_C{}^{m_C'} - \left(\psi_C - V_{BC}\right)^{m_C'} \right] \end{aligned}$$

Where $K_C = \dfrac{C_{\mu 0} \psi^{m_C}}{1 - m_C}$ and $m_C{}' = 1 - m_C$

That's a lot of work, and for what? Well, for one, we can now write down the charge on these nonlinear capacitors as a function of V_{BE} and V_{BC}. A typical m_C for a base-collector junction is 1/2, and for a good base-emitter junction is 1/3. We have

$$\begin{aligned} Q_{VC} &= K_C \left[\psi_C{}^{1/2} - \left(\psi_C - V_{BC}\right)^{1/2} \right] \\ Q_{VE} &= K_E \left[\psi_E{}^{2/3} - \left(\psi_E - V_{BE}\right)^{2/3} \right] \end{aligned}$$

And the additional charge control terms

$$i_B = \frac{dQ_{VC}}{dt} + \frac{dQ_{VE}}{dt}$$

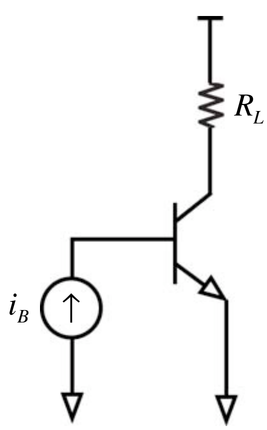
$$i_C = -\frac{dQ_{VC}}{dt}$$

$$i_E = -\frac{dQ_{VE}}{dt}$$

Ultimately, these monstrous expressions for Q_{VC} and Q_{VE} will be useful because, once we compute the total charge necessary to get to a particular V_{BC} or V_{BE}, we will sweep the nonlinearity completely under the rug.

This calls for an example.

Consider the following circuit with a base current drive.



$i_B = 200\mu A$
$R_L = 1k\Omega$
$V_{CL} = 10.2V$
$\beta_F = 100$
$\beta_R = 5$
$\tau_F = 1ns$
$\tau_R = 5ns$
$\tau_S = 29ns$

We ask, how long does it take to get out of cutoff? In order to get out of cutoff, V_{BE} must equal 600mV. In order to get V_{BE} to 600mV, we must charge Q_{VE} and Q_{VC} appropriately. The total amount of charge needed is

$$\Delta Q_{VE} = Q_{VE}(0.6V) - Q_{VB}(0V) = 3.8\,pC$$

$$\Delta Q_{VC} = Q_{VC}(-9.6V) - Q_{VC}(-10.2V) = 0.6\,pC$$

Total charge that must be supplied by i_B to get out of cutoff: 4.4pF.

And here is where we sweep aside the nonlinearity. We say that

$$\Delta t = \frac{\Delta Q}{i_B} = \frac{\Delta Q_{VE} + \Delta Q_{VC}}{i_B}$$

$$= \frac{4.4\,pC}{0.2mA} = 22ns$$

Good for a ballpark figure.