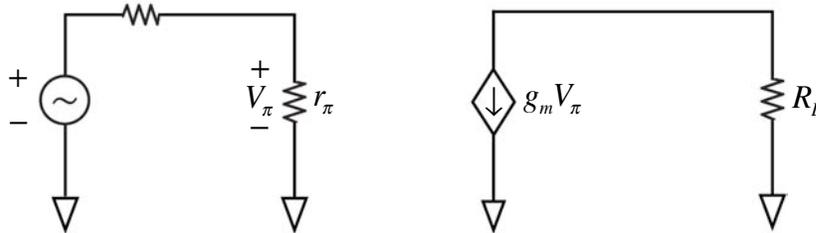# 6.301 Solid State Circuits

Recitation 12: Base-Width Modulation
Prof. Joel L. Dawson

There are times when the most important metric of an amplifier is its low-frequency gain. Based on the model we have given you so far:

We would have you believe that the achievable gain for a single stage is unbounded. That is, for a given gain, we just choose R_L to be as high as necessary. As you might suspect, this is only possible up to a point.

CLASS EXERCISE

Consider the common emitter amplifier:
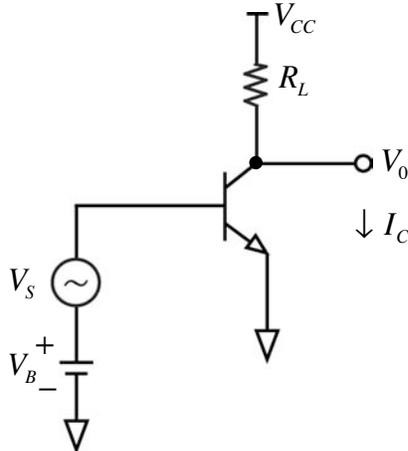
$$g_m = \frac{I_C}{V_T}, \quad a_v = g_m R_L$$

Assume that with the bias voltage V_B, we get perfect control over the collector current.

(1) Suppose that the lowest allowable quiescent voltage at V_0 is zero (ground). Express the maximum I_C we can have in terms of V_{CC} and R_L.

(2) What is the maximum gain we can get out of this stage? What limits us?
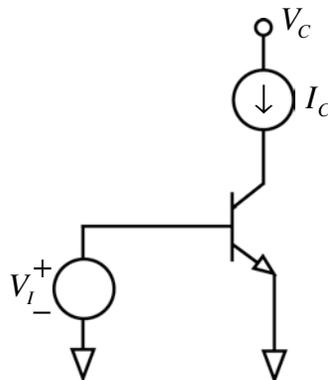
(Workspace)

It is not uncommon, though, for an op-amp to achieve a gain of 10^6 in only two stages. If we allot 10^3 of gain for each stage, that implies a power supply $\geq 1000 \cdot V_T = 25V$. Since we routinely buy op-amps that operate on much smaller power supplies, it is clear that some other tricks are being played. One such trick looks like this:

We need a more complete model of the transistor to understand this circuit, though. Let's jump right in…

We return to device physics. Specifically, the excess minority carrier change in the base.
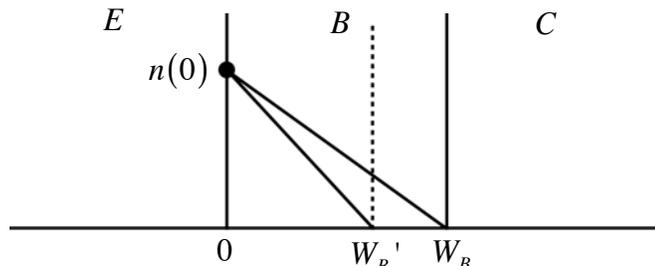
The width of the base is decreased as BC junction is reverse biased.

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When the width of the base decreases, the slope of the carrier distribution in the base <u>increases</u>. This means the diffusion current must also increase. Recall,

$$I_C = \frac{\overbrace{n(0)}^{\text{slope}}}{W_B} \times D_n \times A_E \times q$$

Diffusion constant   Emitter Area   Elementary Charge

$$= \frac{\dfrac{n_i^2}{N_B} e^{\frac{qV_{BE}}{kT}}}{W_B} \times D_n \times A_E \times q$$

Now what we're interested in is $\dfrac{\partial I_C}{\partial V_{CE}}$. We don't see an explicit dependence on V_{CE}, but we know that W_B depends on V_{CE} …

$$\frac{\partial I_C}{\partial V_{CE}} = -\frac{n_i^2}{N_B} e^{\frac{qV_{BE}}{k_T}} \cdot \left(-\frac{1}{W_B^2}\right) D_n A_E q \cdot \frac{\partial W_B}{\partial V_{CE}}$$

Recognizing $\dfrac{n_i^2}{N_B} e^{\frac{qV_{BE}}{k_T}} D_n A_E q = W_B I_C$, we can simplify:

$$\frac{\partial I_C}{\partial V_{CE}} = -\frac{1}{W_B} \frac{\partial W_B}{\partial V_{CE}} I_C$$

Now we've gotten as far as we can without getting into more detailed device considerations. It turns out that $\dfrac{\partial W_B}{\partial V_{CE}}$ is well approximated by a constant. From a unit standpoint, we observe that $\dfrac{1}{W_B} \dfrac{\partial W_B}{\partial V_{CE}}$ must have units of V^{-1}. We thus write

$$\frac{\partial I_C}{\partial V_{CE}} = kI_C = \frac{I_C}{V_A}$$

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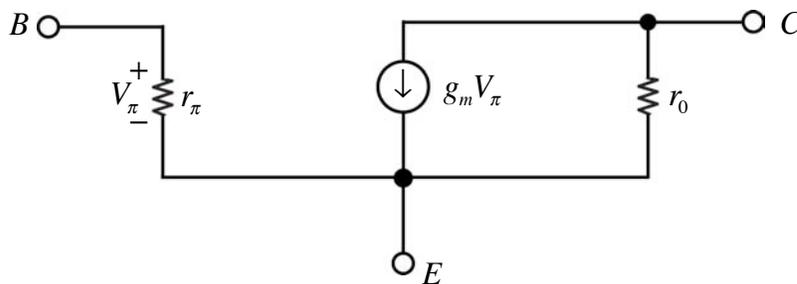Recitation 12: Base-Width Modulation
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Where we call V_A the "early voltage."

What does all of this mean? For one, it means that we will modify our large signal model of the bipolar transistor in the following way:

$$I_C = I_S e^{\frac{qV_{BE}}{kT}} \left(1 + \frac{V_{CE}}{V_A} \right)$$

$$\left[Check: \quad \frac{\partial I_C}{\partial V_{CE}} = \frac{1}{V_A} I_S e^{\frac{qV_{BE}}{kT}} = \frac{I_C}{V_A} \right]$$

Second, it means that we have a new small-signal model:

Where

$$r_0 = \left(\frac{\partial I_C}{\partial V_{CE}} \right)^{-1} = \frac{V_A}{I_C}$$

Typical values of early voltage are 25 to 250 volts. For a collector current of $1mA$, this gives and between $25k\Omega$ and $250k\Omega$.
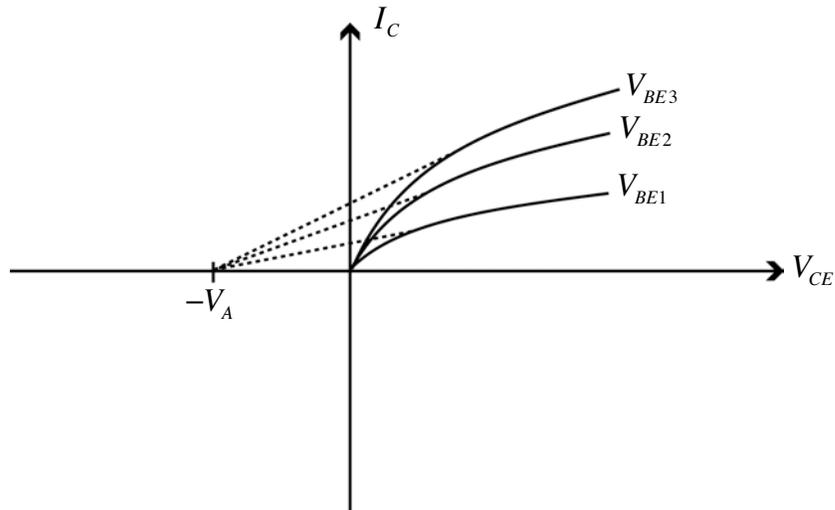
So how does this behavior show up in the laboratory? Most obviously, it shows up on an instrument called a curve tracer.

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A curve tracer automates the graphing of I_C vs V_{CE} parameterized by different values of V_{BE}.

If you extrapolate according to the dotted lines back to where $I_C = 0$, all of the lines meet at $-V_A$.
We can see this mathematically:

$$I_C\big|_{V_{CE}=-V_A} = I_S e^{\frac{qV_{BE}}{kT}}\left(1 - \frac{V_A}{V_A}\right) = 0$$

Looking at the graph, we can see that higher early voltages imply "flatter" I_C vs. V_{CE} curves. And the flatter the curve, the higher r_0.
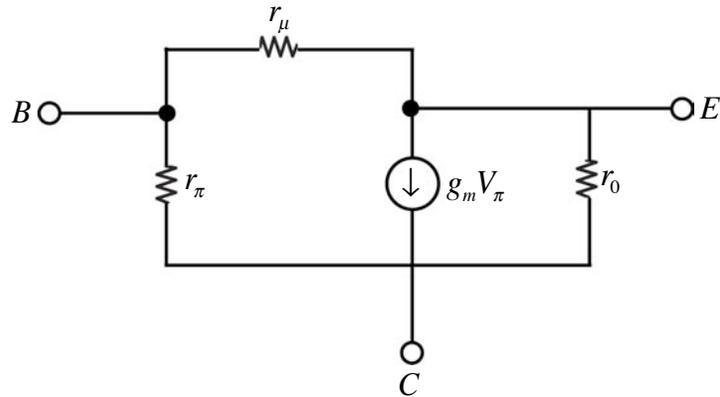
There's one more effect to consider:
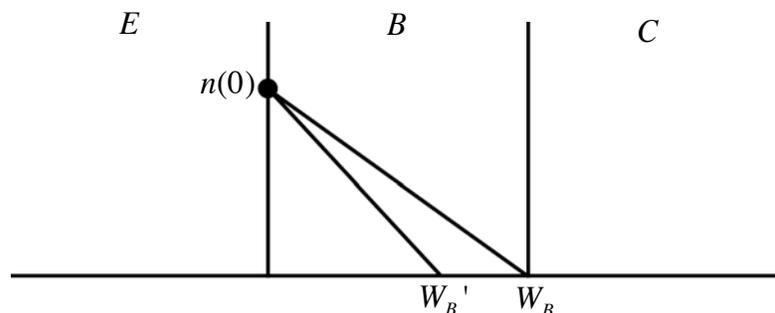
Notice that when we increase V_{CE} and therefore decrease W_B, we <u>decrease</u> the amount of charge in the base that is available for recombination. This means that the base current decreases, and we can capture this behavior with an added resistance between the base and collector:

It turns out that r_μ is typically on the order of βr_0, and usually greater than $10\beta r_0$. We will almost always ignore this resistance.

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Fall 2010