Course 6.336 - Introduction to Numerical Algorithms
(Fall 2003)
Solutions to Problem Set #5

Problem 5.1

a) We know how strut force depends on the displacement, $f(L) = EA_c\frac{L_0-L}{L_0} = L_0 - L$. Projecting strut forces, we form the following system of equations:

$$\begin{cases} \frac{x_1}{L_1}(1-L_1) + \frac{x_1-d}{L_2}(1-L_2) + F_x = 0 \\ \frac{y_1}{L_1}(1-L_1) + \frac{y_1}{L_2}(1-L_2) + F_y = 0 \end{cases} \tag{1}$$

Here L_i are **functions** of x_1 and x_2 and represent lengths of the struts:

$$L_1 = \sqrt{x_1^2 + y_1^2}$$
$$L_2 = \sqrt{(x_1-d)^2 + y_1^2}$$

b) Assuming in (1) $F_x = 0$, and using symmetry of our problem, it's evident that $x_1 = \frac{d}{2}$, which can be eliminated from our system of equations. Therefore our single equation will look like

$$\frac{y_1}{L}(1-L) + \frac{y_1}{L}(1-L) + F_y = 0,$$
$$L = \sqrt{\frac{d^2}{4} + y_1^2}$$

(c) Now we suppose that $d = \sqrt{3}$. As shown above, we need to solve only for y, as $x = d/2$.

We will be solving the following 1-D nonlinear equation for y_1

$$2y_1\left(\frac{1}{\sqrt{(\frac{d}{4}^2 + y_1^2)}} - 1\right) + F_y = 0 \tag{2}$$

We will then solve the equation for different source loads, from $F_y = 0$ to $F_y = -.16$ in increments of -0.02 using a straightforward Newton's solver for each load force increment. We use the previous solution for the initial guess for the next load step.

The following matlab script implements this.

```
function  RecursiveNewton
d = sqrt(3);
dy_eps = 0.001;
f_eps = 0.001;
force_initial = 0.0;
force_step = -0.02;
no_steps = 9;
initial_guess_y = 0.5;
for force_trial = [1:no_steps]
    fy_down(force_trial) = force_initial + (force_trial-1)*force_step;
    [x_down(force_trial),y_down(force_trial)]= ...
OneDNewton(initial_guess_y, d, dy_eps,f_eps, fy_down(force_trial));
    initial_guess_y = y_down(force_trial);
end

function [x,y] = OneDNewton(initial_guess_y, d, dy_eps,f_eps, Fy)
x = d/2;
y = initial_guess_y;
dont_stop = 1;
iters = 0;
while (dont_stop)
    L = sqrt(x^2 + y^2);
    f_y = 2*y*(1-L)/L + Fy;
    df_dy = -2*y^2*L^-3 + 2*(1-L);
    y_now = y  - f_y/df_dy ;
    L = sqrt(x^2 + y^2);
    f_y = 2*y*(1-L)/L + Fy;
    if ( (abs(f_y) < f_eps)  & (abs(y_now -y) < dy_eps))
        dont_stop = 0;
    end
    y = y_now;
    iters = iters + 1;
end
```

This produces a plot of position versus force as shown below.

(d) Now, we wish to source step from $F_y = -.16$ and slowly decrease in magnitude the y-directed force in the same increment size, .02, until there is no force. We will start with the final answer found in part c) for our first guess for the system where $F_y = -.14$.

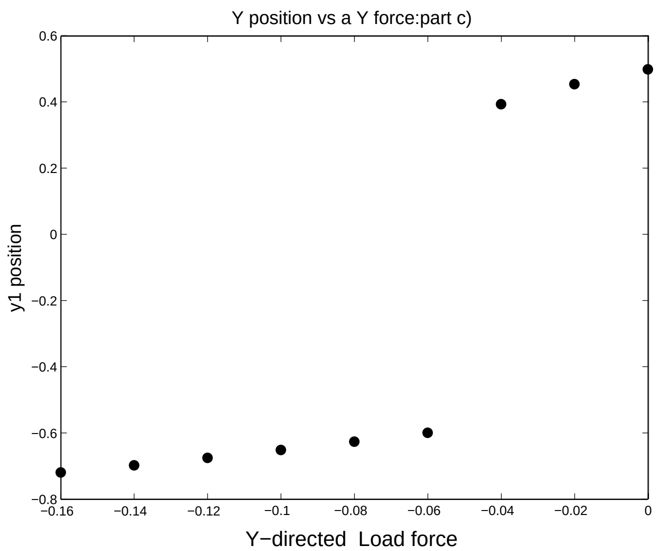
The following code, appended to the end of the function Recursive Newton, in part (c), implements this.

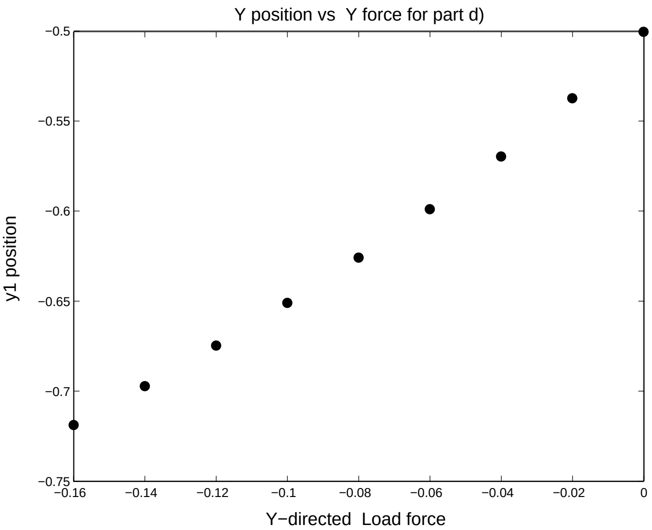
```
force_initial = fy_down(force_trial);
force_step = 0.02;
for force_trial = [1:no_steps]
    fy_up(force_trial) = force_initial + (force_trial-1)*force_step;
    [x_up(force_trial),y_up(force_trial)]= OneDNewton(initial_guess_y, d, dy_eps,f
    initial_guess_y = y_up(force_trial);
end
```

This produces a plot of position versus force as shown below.

(e) We note that in part (d), the final result is that $y1 = -0.7185$ for $F_y = 0$, while in part (c), $y1 = 0.5000$ for $F_y = 0$. This is because for all of the negative forces applied during our continuation schemes, there is not a unique solution for any of the applied forces, but rather, at two solutions: a solution where $y1$ is positive, and another corresponding to a solution where $y1$ is positive (and for $F_y = 0$, there are 3 solutions, the one at $y1 = 0$ being highly unstable).

As you can see from this result, the solution which the Newton solver converges to is highly dependent upon the initial guess. In part c), we started with a guess that converged to the positive solution. As we backtracked, to that initial point, however, our initial guesses were such that we always found the "stretching" solutions with negative $y1$.




Y position vs Y force for part d)
y1 position
Y−directed Load force
−0.5
−0.55
−0.6
−0.65
−0.7
−0.75
−0.16
−0.14
−0.12
−0.1
−0.08
−0.06
−0.04
−0.02
0