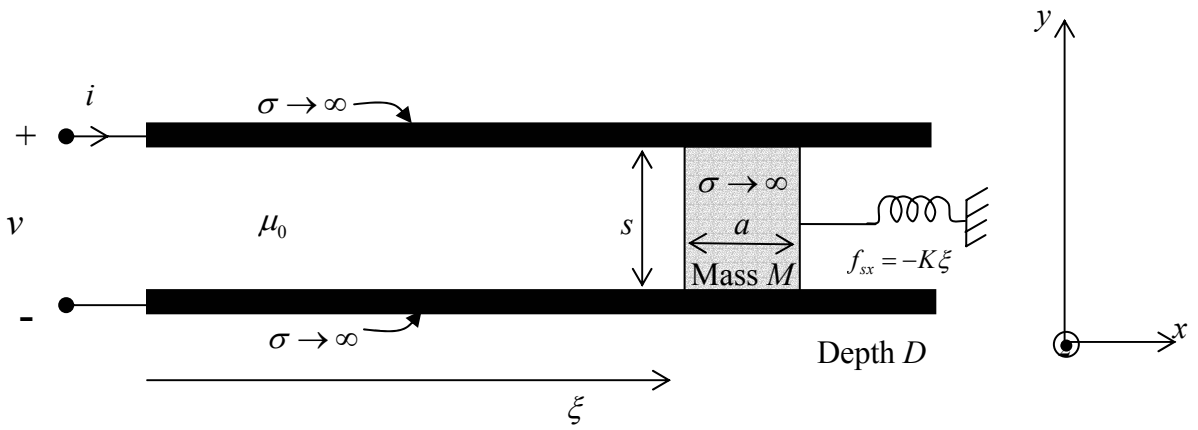
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Quiz 2 April 21, 2004
Spring Term 2004, 4-6PM

6.641 Formula Sheet Attached in the study materials section.You are also allowed to use the formula sheet that you prepared for Quiz 1 plus an additional 8 ½"x 11" formula sheet (both sides) that you have prepared for Quiz 2.

Problem 1

Two parallel perfect conductors of depth *D* and spacing *s* enclose a perfectly conducting ($\sigma \to \infty$) block of width *a* and mass *M* that is free to move with negligible friction force in the *x* direction. The block distance from the left hand edge of the parallel conductors is ξ. The block is also constrained by a linear spring with spring constant *K*. The spring exerts no force when $\xi = 0$ so that the spring force in the *x* direction is $f_{sx} = -K\xi$.



a) What is the self-inductance $L(\xi)$?

b) At time $t = 0$, when $\xi(t = 0) = \xi_0$, a current $i = I_0$ flows. At this time, the terminals are short-circuited so that $v(t) = 0$ for $t \geq 0$. What is the magnetic flux, Φ_0, in the free space region for $0 < x < \xi_0$ at time $t = 0_+$? If the block is moved to position $\xi = \xi_0 / 2$, what is the current *i* and magnetic flux in the free space region, $0 < x < \xi_0 / 2$.

c) Under the short-circuited conditions of part (b), what is the magnetic force, $f_x(\xi)$ on the perfectly conducting block as a function of ξ, Φ_0, μ_0, and geometric parameters?

d) What is the total magnetic and spring forces acting on the block?

e) What is the equilibrium position, ξ_{eq}, of the movable block?

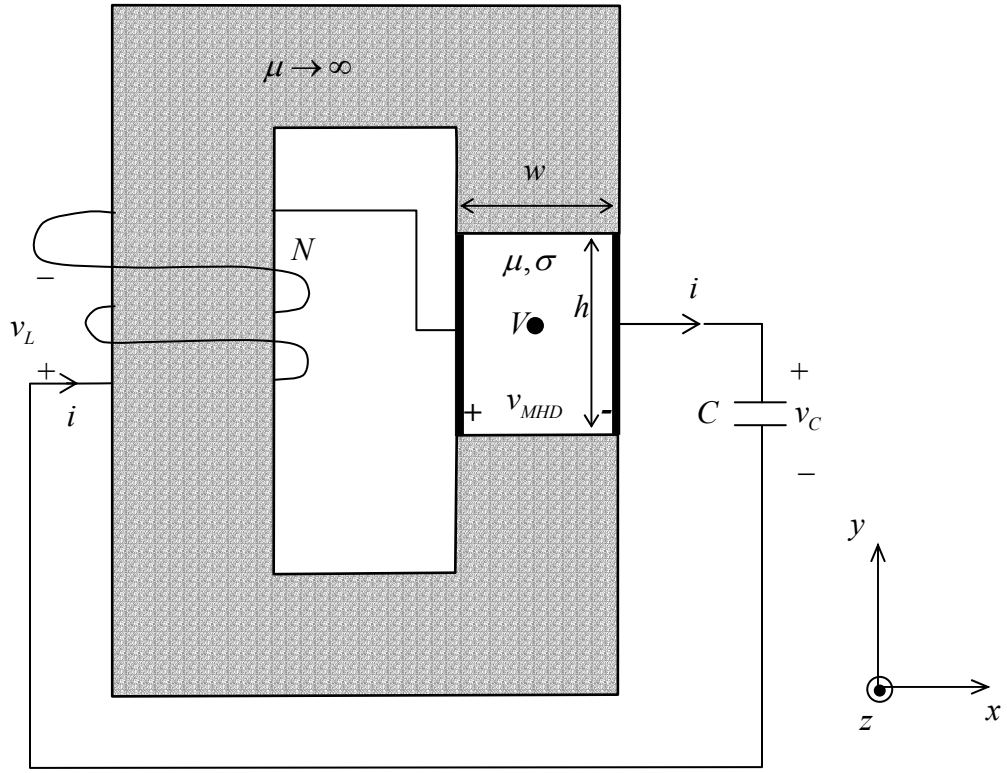
f) Is this position stable or unstable?

g) At time $t = T$, a step perturbation force, $f_p(t) = f_0 u(t - T)$, is exerted on the stationary movable block at its equilibrium position ξ_{eq}, so that $\xi(t) = \xi_{eq} + \xi'(t)$, where $\xi'(t) \ll \xi_{eq}$ $u(t - T)$ is a unit step-function with step at $t = T$. To first order, find $\xi'(t)$.

Hint: To first order, $f(\xi) = f(\xi_{eq}) + \left.\frac{df}{d\xi}\right|_{\xi_{eq}} \xi'$.

Problem 2

A magnetic circuit of depth *D* contains a magnetohydrodynamic (MHD) generator with material of ohmic conductivity σ, magnetic permeability μ, width *w* and height *h*. The MHD conducting material is moving at velocity $\bar{v} = V\bar{i}_z$. The electrical terminals of the MHD machine is connected in series with a capacitor of capacitance *C* and with an *N* turn coil. The voltage across the MHD machine terminals is v_{MHD}, across the capacitor is v_C, and across the *N* turn coil is v_L with polarities defined as shown in the figure below.



Depth *D*

a) If a current *i* flows through the coil, what is the vector magnetic field $\bar{B}$ in the MHD machine?
b) What is the self-inductance of the *N* turn coil?
c) What is v_{MHD} in terms of current *i*, σ, *V*, μ, *N*, and geometric parameters?
d) What is the governing differential equation for *i*?
e) Under what conditions is the system self-excited so that any initial current at $t = 0$ grows to infinity with increasing time? Express your answer in terms of σ, *V*, μ, *N*, and geometric parameters.
f) For what range of capacitance *C* will the current also oscillate as it grows? What is the oscillation frequency? Express your answers in terms of σ, *V*, μ, *N*, and geometric parameters.