0.1 PREFACE

The text is aimed at an audience that has seen Maxwell's equations in integral or differential form (second-term Freshman Physics) and had some exposure to integral theorems and differential operators (second term Freshman Calculus). The first two chapters and supporting problems and appendices are a review of this material.

In Chap. 3, a simple and physically appealing argument is presented to show that Maxwell's equations predict the time evolution of a field, produced by free charges, given the initial charge densities and velocities, and electric and magnetic fields. This is a form of the uniqueness theorem that is established more rigorously later. As part of this development, it is shown that a field is completely specified by its divergence and its curl throughout all of space, a proof that explains the general form of Maxwell's equations.

With this background, Maxwell's equations are simplified into their electroquasistatic (EQS) and magnetoquasistatic (MQS) forms. The stage is set for taking a structured approach that gives a physical overview while developing the mathematical skills needed for the solution of engineering problems.

The text builds on and reinforces an understanding of analog circuits. The fields are never static. Their dynamics are often illustrated with step and sinusoidal steady state responses in systems where the spatial dependence has been encapsulated in time-dependent coefficients (of solutions to partial differential equations) satisfying ordinary differential equations. However, the connection with analog circuits goes well beyond the same approach to solving differential equations as used in circuit theory. The approximations inherent in the development of circuit theory from Maxwell's equations are brought out very explicitly, so that the student appreciates under what conditions the assumptions implicit in circuit theory cease to be applicable.

To appreciate the organization of material in this text, it may be helpful to make a more subtle connection with electrical analog circuits. We think of circuit theory as being analogous to field theory. In this analogy, our development begins with capacitors– charges and their associated fields, equipotentials used to represent perfect conductors. It continues with resistors– steady conduction to represent losses. Then these elements are combined to represent charge relaxation, i.e. "RC" systems dynamics (Chaps. 4-7). Because EQS fields are not necessarily static, the student can appreciate R-C type dynamics, where the distribution of free charge is determined by the continuum analog of R-C systems.

Using the same approach, we then take up the continuum generalization of L-R systems (Chaps. 8–10). As before, we first are given the source (the current density) and find the magnetic field. Then we consider perfectly conducting systems and once again take the boundary value point of view. With the addition of finite conductivity to this continuum analog of systems of inductors, we arrive at the dynamics of systems that are L-R-like in the circuit analogy.

Based on an appreciation of the connection between sources and fields afforded by these quasistatic developments, it is natural to use the study of electric and magnetic energy storage and dissipation as an entree into electrodynamics (Chap. 11).

Central to electrodynamics are electromagnetic waves in loss-free media (Chaps. 12–14). In this limit, the circuit analog is a system of distributed differential inductors and capacitors, an L-C system. Following the same pattern used for EQS and MQS systems, fields are first found for given sources– antennae and arrays. The boundary value point of view then brings in microwave and optical waveguides and transmission lines.

We conclude with the electrodynamics of lossy material, the generalization of L-R-C systems (Chaps. 14–15). Drawing on what has been learned for EQS, MQS, and electrodynamic systems, for example, on the physical significance of the dominant characteristic times, we form a perspective as to how electromagnetic fields are exploited in practical systems. In the circuit analogy, these characteristic times are RC, L/R, and $1/\sqrt{LC}$. One benefit of the field theory point of view is that it shows the influence of physical scale and configuration on the dynamics represented by these times. The circuit analogy gives a hint as to why it is so often possible to view the world as either EQS or MQS. The time $1/\sqrt{LC}$ is the geometric mean of RC and L/R. Either RC or L/R is smaller than $1/\sqrt{LC}$, but not both. For large R, RC dynamics comes first as the frequency is raised (EQS), followed by electrodynamics. For small R, L/R dynamics comes first (MQS), again followed by electrodynamics. Implicit is the enormous difference between what is meant by a "perfect conductor" in systems appropriately modeled as EQS and MQS.

This organization of the material is intended to bring the student to the realization that electric, magnetic, and electromagnetic devices and systems can be broken into parts, often described by one or another limiting form of Maxwell's equations. Recognition of these limits is part of the art and science of modeling, of making the simplifications necessary to make the device or system amenable to analytic treatment or computer analysis and of effectively using appropriate simplifications of the laws to guide in the process of invention.

With the EQS approximation comes the opportunity to treat such devices as transistors, electrostatic precipitators, and electrostatic sensors and actuators, while relays, motors, and magnetic recording media are examples of MQS systems. Transmission lines, antenna arrays, and dielectric waveguides (i.e., optical fibers) are examples where the full, dynamic Maxwell's equations must be used.

In connection with examples, about 40 demonstrations are described in this text. These are designed to make the mathematical results take on physical meaning. Based upon relatively simple configurations and arrangements of equipment, they incorporate no more complexity then required to make a direct connection between what has been derived and what is observed. Their purpose is to help the student observe physically what has been described symbolically. Often coming with a plot of the theoretical predictions that can be compared to data taken in the classroom, they give the opportunity to test the range of validity of the theory and to promulgate a quantitative approach to dealing with the physical world. More detailed consideration of the demonstrations can be the basis for special projects, often bringing in computer modeling. For the student having only the text as a resource, the descriptions of the experiments stand on their own as a connection between the abstractions and the physical reality. For those fortunate enough to have some of the demonstrations used in the classroom, they serve as documentation of what was done. All too often, students fail to profit from demonstrations because conventional note taking fails to do justice to the presentation.

The demonstrations included in the text are of physical phenomena more than of practical applications. To fill out the classroom experience, to provide the engineering motivation, applications should also be exemplified. In the subject as we teach it, and as a practical matter, these are more of the nature of "show and tell" than of working demonstrations, often reflecting the current experience and interests of the instructor and usually involving more complexity than appropriate for more than a qualitative treatment.

The text provides a natural frame of reference for developing numerical approaches to the details of geometry and nonlinearity, beginning with the method of moments as the superposition integral approach to boundary value problems and culminating in energy methods as a basis for the finite element approach. Professor J. L. Kirtley and Dr. S. D. Umans are currently spearheading our efforts to expose the student to the "muscle" provided by the computer for making practical use of field theory while helping the student gain physical insight. Work stations, finite element packages, and the like make it possible to take detailed account of geometric effects in routine engineering design. However, no matter how advanced the computer packages available to the student may become in the future, it will remain essential that a student comprehend the physical phenomena at work with the aid of special cases. This is the reason for the emphasis of the text on simple geometries to provide physical insight into the processes at work when fields interact with media.

The mathematics of Maxwell's equations leads the student to a good understanding of the gradient, divergence, and curl operators. This mathematical conversance will help the student enter other areas– such as fluid and solid mechanics, heat and mass transfer, and quantum mechanics– that also use the language of classical fields. So that the material serves this larger purpose, there is an emphasis on source-field relations, on scalar and vector potentials to represent the irrotational and solenoidal parts of fields, and on that understanding of boundary conditions that accounts for finite system size and finite time rates of change.

Maxwell's equations form an intellectual edifice that is unsurpassed by any other discipline of physics. Very few equations encompass such a gamut of physical phenomena. Conceived before the introduction of relativity Maxwell's equations not only survived the formulation of relativity, but were instrumental in shaping it. Because they are linear in the fields, the replacement of the field vectors by operators is all that is required to make them quantum theoretically correct; thus, they also survived the introduction of quantum theory.

The introduction of magnetizable materials deviates from the usual treatment in that we use paired magnetic charges, magnetic dipoles, as the source of magnetization. The often-used alternative is circulating Ampèrian currents. The magnetic charge approach is based on the Chu formulation of electrodynamics. Chu exploited the symmetry of the equations obtained in this way to facilitate the study of magnetism by analogy with polarization. As the years went by, it was unavoidable that this approach would be criticized, because the dipole moment of the electron, the main source of ferromagnetism, is associated with the spin of the electron, i.e., seems to be more appropriately pictured by circulating currents.

Tellegen in particular, of Tellegen-theorem fame, took issue with this approach. Whereas he conceded that a choice between two approaches that give identical answers is a matter of taste, he gave a derivation of the force on a current loop (the Ampèrian model of a magnetic dipole) and showed that it gave a different answer from that on a magnetic dipole. The difference was small, the correction term was relativistic in nature; thus, it would have been difficult to detect the effect in macroscopic measurements. It occurred only in the presence of a time-varying electric field. Yet this criticism, if valid, would have made the treatment of magnetization in terms of magnetic dipoles highly suspect.

The resolution of this issue followed a careful investigation of the force exerted on a current loop on one hand, and a magnetic dipole on the other. It turned out that Tellegen's analysis, in postulating a constant circulating current around the loop, was in error. A time-varying electric field causes changes in the circulating current that, when taken into account, causes an additional force that cancels the critical term. Both models of a magnetic dipole yield the same force expression. The difficulty in the analysis arose because the current loop contains "moving parts," i.e., a circulating current, and therefore requires the use of relativistic corrections in the rest-frame of the loop. Hence, the current loop model is inherently much harder to analyze than the magnetic charge–dipole model.

The resolution of the force paradox also helped clear up the question of the symmetry of the energy momentum tensor. At about the same time as this work was in progress, Shockley and James at Stanford independently raised related questions that led to a lively exchange between them and Coleman and Van Vleck at Harvard. Shockley used the term "hidden momentum" for contributions to the momentum of the electromagnetic field in the presence of magnetizable materials. Coleman and Van Vleck showed that a proper formulation based on the Dirac equation (i.e., a relativistic description) automatically includes such terms. With all this theoretical work behind us, we are comfortable with the use of the magnetic charge–dipole model for the source of magnetization. The student is not introduced to the intricacies of the issue, although brief mention is made of them in the text.

As part of curriculum development over a period about equal in time to the age of a typical student studying this material (the authors began their collaboration in 1968) this text fits into an evolution of field theory with its origins in the "Radiation Lab" days during and following World War II. Quasistatics, promulgated in texts by Professors Richard B. Adler, L.J. Chu, and Robert M. Fano, is a major theme in this text as well. However, the notion has been broadened and made more rigorous and useful by recognizing that electromagnetic phenomena that are "quasistatic," in the sense that electromagnetic wave phenomena can be ignored, can nevertheless be rate dependent. As used in this text, a quasistatic regime includes dynamical phenomena with characteristic times longer than those associated with electromagnetic waves. (A model in which no time-rate processes are included is termed "quasistationary" for distinction.)

In recognition of the lineage of our text, it is dedicated to Professors R. B. Adler, L. J. Chu and R. M. Fano. Professor Adler, as well as Professors J. Moses, G. L. Wilson, and L. D. Smullin, who headed the department during the period of development, have been a source of intellectual, moral, and financial support. Our inspiration has also come from colleagues in teaching– faculty and teaching assistants, and those students who provided insight concerning the many evolutions of the "notes." The teaching of Professor Alan J. Grodzinsky, whose latterday lectures have been a mainstay for the course, is reflected in the text itself. A partial list of others who contributed to the curriculum development includes Professors J. A. Kong, J. H. Lang, T. P. Orlando, R. E. Parker, D. H. Staelin, and M. Zahn (who helped with a final reading of the text). With "macros" written by Ms. Amy Hendrickson, the text was "Tex't" by Ms. Cindy Kopf, who managed to make the final publication process a pleasure for the authors.