

1. a)  $Q = 8a\lambda$ ,  $\lim_{r \rightarrow \infty} \Phi = \frac{Q}{4\pi\epsilon_0 r}$ ,  $E_r = \frac{Q}{4\pi\epsilon_0 r^2}$

b)  $d\Phi = \frac{\lambda dx'}{4\pi\epsilon_0 [x'^2 + a^2 + z^2]^{3/2}}$   $-a \leq x' \leq a$

$$\begin{aligned} \Phi &= \frac{\lambda}{\pi\epsilon_0} \int_{-a}^a \frac{dx'}{[x'^2 + a^2 + z^2]^{3/2}} \\ &= \frac{\lambda}{\pi\epsilon_0} \ln [x' + \sqrt{a^2 + z^2 + x'^2}] \Big|_{-a}^{+a} \\ &= \frac{\lambda}{\pi\epsilon_0} \left[ \ln [a + \sqrt{2a^2 + z^2}] - \ln [-a + \sqrt{2a^2 + z^2}] \right] \end{aligned}$$

2. a)  $\nabla \cdot (\epsilon(x) \vec{E}) = \frac{d}{dx} (\epsilon(x) E_x(x)) = \rho_f = 0$

$\epsilon(x) E_x(x) = C(t)$

$E_x(x) = \frac{C(t)}{\epsilon(x)} = \frac{C(t)}{\epsilon_0} e^{-\alpha x}$

$$\begin{aligned} \int_0^d E_x(x) dx &= V(t) = \frac{C(t)}{-\alpha \epsilon_0} \Big|_0^d = -\frac{C(t)}{\alpha \epsilon_0} (e^{-\alpha d} - 1) \\ &= \frac{C(t)}{\alpha \epsilon_0} (1 - e^{-\alpha d}) \end{aligned}$$

$C(t) = \frac{\alpha \epsilon_0 V(t)}{1 - e^{-\alpha d}}$

$E_x(x) = \frac{\alpha \epsilon_0 V(t)}{(1 - e^{-\alpha d}) \epsilon_0 e^{\alpha x}} = \frac{\alpha V(t) e^{-\alpha x}}{(1 - e^{-\alpha d})}$

b)  $\mathcal{D}_3(x=0, t) = \epsilon_0 E_x(x=0, t) = \frac{\alpha \epsilon_0 V(t)}{(1 - e^{-\alpha d})}$

$\mathcal{D}_3(x=d, t) = -\epsilon_0 e^{\alpha d} E_x(x=d, t) = -\epsilon_0 e^{\alpha d} \frac{\alpha V(t) e^{-\alpha d}}{(1 - e^{-\alpha d})} = -\frac{\alpha \epsilon_0 V(t)}{(1 - e^{-\alpha d})}$

c)  $C = \frac{A \mathcal{D}_3(x=0, t)}{V(t)} = \frac{A \epsilon_0 \alpha}{(1 - e^{-\alpha d})}$