PROFESSOR: So today we're going to talk about the dynamics of running. And ask questions as freely as possible, and also if my handwriting's bad, let me know. It generally is. So. So this is actually a really fun lecture. We have a lot of cool videos. I don't know if any of you have seen Raibert hoppers and stuff before, but we have some cool models and some cool analysis too. So hopefully, you'll be as excited as I am to participate in this learning experience. So here we go.

So the basic model we'll be looked at today in several different versions of this is the SLIP model. This is the spring-loaded inverted pendulum. All right. It's like the running version of the rimless wheel or compass gait. It's pretty simple. You just have a mass at the end of a pendulum. You have a spring here, spring constant k. The spring is a rest length r naught and a-- yeah, we already said the spring constant. And so then this-- the coordinates, then, are this length r and the angle theta from the upright.

So what we're going to want to look at here is not just this model, but also, in the same way we had for the rimless wheel, as you remember, is a 1-D iterated map. So you probably remember that for the compass gait that lets you look at the stability of it, and there's all these interesting dynamics that are captured by that simple system. So we're going to want to look at the 1-D iterated map, and the other thing I'm going to try to convince you is that this is a biologically plausible system.

So if you look at the rimless wheel, it feels like something that's walking, compass gait even more so, [INAUDIBLE] knees. These things all-- just intuitively, you can believe that they're-- sorry-- a-- whoa, that's probably going to be bad-- that you could believe that they're walking systems. But this one maybe doesn't seem to you immediately like a running system. Maybe it seems like a jumping system or bouncing system or something, but actually, it actually is a pretty good model for running systems.

So the two things that we're going to have to go through, how do you get a 1-D iterated map? Now that maybe is a bit surprising for you because if you look at this, the full state space of this inverted pendulum right now, it has states of r-- sorry-- r theta, r dot, theta dot. Now that's four parameters. You remember when you slice through and you pick a surface of section, you can cut it down by one. So for the rimless wheel, maybe you have theta, theta dot. You cut out theta dot, you're in pretty good shape.

Here we have four, but we're actually going to show that we can actually get this down to a 1-D system and actually look at the iterated map for this. So that's kind of cool. And the other thing is that also, this system bounces and takes off, so actually, we need two coordinate systems. One is this r theta, r dot, theta dot. And then when it's in flight, we're going to have x, y, x dot, y dot, all right. And both of these are 4-D, so going to have to deal with that either way.

But first the convincing part. So the justification here comes from a field called comparative biomechanics. Now this is a pretty cool field. They look at all sorts of different systems and try to figure out these commonalities in the kinematics or the dynamics or certain properties and figure out, what are the fundamental common features of these different biological systems?

So this running model, actually, you can look at it in the context of things from cockroaches, which are on the order of a gram in mass, horses, which are on the order of-- well, some of them, at least, are 135 kilograms. It's not that heavy, really, so you could imagine maybe heavier horses. Crabs.

Now you can think about the kinematics of these system and just how wildly different they are. Cockroaches, six-legged, bouncing all around, [INAUDIBLE] tripod gait. Horses, quadrupeds. Crabs, actually they run sideways with eight legs. This is different information, so I won't put it there. But crabs actually will run sideways using their eight legs. It's actually pretty cool. I don't know if you've seen it.

And then humans, which maybe is the one that we most-- well, I don't know, that's probably unfairly biocentric. So we're humans. And all of these, despite the very different kinematics, despite, what is this, five orders of magnitude in mass, they all have incredible dynamic similarity.

Let's hope I'm not getting graded on spelling, because I'm pretty sure that's way off. [INAUDIBLE] And so if you look at it in the right way, you can actually see that all of these systems, cockroaches, horses, crabs, camels, cats, bunny rabbits, all these things actually look very similar.

And so Bob Full, some of you may know Bob Full, I think at Berkeley. He looks at a lot of things, especially the cockroach work. He has a lot of cool work on cockroaches. And he also has this paper which-- let's see if I can find what page this on.

Sorry, here. There we go. Is that clear up there? Ooh. Sorry.

So here, you can look at the-- oh, let me dim the lights a bit. If you look at the plot on the left, this is speed and the stride length. So if you plot speed and stride lengths-- here they've got gerbils, dogs, and camels, but you can do this for a very wide variety of systems. You can see that they're all bounced over the place. You don't have-- I mean, they're all behaving very differently, so that's not necessarily the right way to look at them if you want to see what is fundamentally similar between them.

If you look at the right, this is the [INAUDIBLE] number. In this case, you can see it's just the ratio-- well, it's 2 times the ratio of kinetic energy to potential energy. So it's just this non-dimensional quantity. If you plot this and you look at the relative stride length, which is-- you can see they all collapse very tightly on this line. And so there's actually-- that's impressive agreement. You see that these are-- camels, dogs, and gerbils are all-- look very similar when you look at them in this non-dimensional way.

So all these systems, orders of magnitude, again, of difference in masses, stride lengths too, probably, all look very similar when you compare them in the right framework. And that's what comparative biomechanics tries to do, in addition to other things, obviously.

So here's something that's really cool and that connects us back to our SLIP model. If we look at the next page, you'll see right here, here in-- this is the relative spring constant for individual individual leg [INAUDIBLE] systems, cockroach to kangaroo, five orders of magnitude. You can see that they're all pretty similar. This is huge spectrum of masses, and yet in the framework of the SLIP model's spring constant, they all behave pretty similarly, humans, all these things.

And so if you look at these systems through this simple model, they all not only look similar to one another, but their dynamics and their center of mass [INAUDIBLE] are pretty well described by this kind of behavior. Sorry. By that kind of behavior. So hopefully that will be somewhat compelling argument that this model is representative of actual running behavior, not just something that comes out of nowhere. And a lot of work was done on this in the '80s and '90s on trying to study this model and connect it to animals and that sort of stuff.

I don't know if you've seen Raibert's work, but he has robots that are very similar to these sort of SLIP model that run and jump and actually are from the '80s and such like that actually are some of the really cool robots. And I'll show you some of those. So he actually didn't even call them SLIP. He actually made them before SLIP, and that motivated some of the work on SLIP models.

But I can show. Here, I'll show you a picture of a robot so you can see the kind of cool stuff that we're getting to. I don't know if-- are many of you familiar with the Leg Lab? They should be around. Yeah, they had a lot of cool robots.

And here, see if I can give you just a little picture [INAUDIBLE] Ah. Oh, so that's the biped robot. This thing's bouncing, but that's not really the picture I wanted to show you. Ah, here we go. Yeah.

So this little robot, you can see that's tracing its foot. This up here is tracing its center mass. And you see how center mass actually compresses right above the foot. And that's what you expect here. It lands, and it squishes down.

And so instead of when you're walking and you vault over this leg-- like you think about that rimless wheel. Its center mass rolls up over its foot. This one, both intuitively in the model and you can see right here this robot, which is similar to the model in many ways, is squishing down into its foot, all right. And that actually is one of the ways of looking at the difference between what is running, what is walking.

So one of the definitions for running is that you're supposed to have an aerial phase. But not every running animal has an aerial phase. So this is-- actually, I think comparative biomechanicians look at this center of mass to center of pressure trajectory, because-- I don't know if you know Groucho Marx. Groucho Marx had this funny run. Maybe I'll do it.

So [INAUDIBLE] but sort of like this-- you know? That kind of goofy thing where your feet aren't coming up the ground? That doesn't seem natural. But actually, some animals do that. There's actually elephants that do that Groucho run. So maybe if you're big enough, it makes sense.

But yeah, so [INAUDIBLE] this is very different from walking. And it's actually quite similar to running, and actually, if you look at these animals, it captures a lot of their dynamics. So and another important thing is that these kind of robots which came around in the '80s were the similar kind of work in robotics, because again, the same time [INAUDIBLE] these slow, careful walking robots.

Again, here are these robots that are flying through the air. I'll show you some videos later, but they did flips and all kinds of crazy stuff. I mean, they're throwing themselves around wildly, which is not what people thought of when they thought of these legged robots.

Oh, and the other thing-- this is cool too. If you look at animals, like we're talking about this springs and stuff like that, that's not just some sort of obtuse model that comes out of nowhere. If you look at horses and stuff, well, animals, actually, are full of springs. Their tendons can store up energy and stuff. And actually, horses have a big tendon that runs through their leg all the way up, I think, behind their hip.

And apparently, one of the limiting factors of trying to run fast is that you can't pull your leg forward fast enough to get to the next stride. And so what they actually can do is they can preload when they're pushing, and then that tendon actually acts like a spring. It actually stores energy and lets them swing faster than they would be able to if they just used the motor. And if you look at the oxygen intake of these animals, you can see that they're behaving much more efficiently than you could hope to accomplish if you just had-- if you were simulating a spring with a bunch of motor-- with your muscles acting like a spring.

So it's actually they're physical springs. Actually, this tendon in there actually have this actual behavior. So hopefully-- maybe I'm dwelling on that point too much, but I think that's really cool, that these animals actually have this kind of spring behavior. You can see it in their dynamics, and actually, this model captures a lot of that.

Oh, and here's one other thing that Russ [INAUDIBLE] and it's pretty cool, too. If you think about-- this is sort of tangent, but if you think about how birds sit on purchase forever, right, I mean, it seems like that'd be tiring to be able to hang there forever just squeezing, right? But they actually have tendons, too, such that when they sit on the perch and their weight squishes their legs down, it actually will clamp their talons in and let them hold on.

So that's cool, too, that these animals have all these interesting passive structures and springs, stuff like that, that help them do all these kind of things. And so it's not like the, just use your muscles and just dominate, all these kind of things. There's a lot of passive structures that do a lot of this for free. So hopefully, you all think that's awesome.

All right. All right, so getting back to the system. Looking at this-- sorry-- at this model again-- did you see where I put my chalk? All right. So going back to the SLIP system-- let's see. Again, the state space here-- is it-- yeah, it's still dark in here.

So going back to this system, again, we have r theta, r dot, theta dot, and in the aerial phase, x, y, x dot, y dot. All right, I'll bring this one back down.

And let me put in the assumptions really clearly of what this model is going to have, so the assumptions for the SLIP model. So one is that you have a massless-- whoa, that's terrible-- massless leg and toe. All right? So all your mass is concentrated in that [INAUDIBLE] at the body.

You have an ideal lossless spring in your leg. And what that means, that effectively, your collisions with the ground, unlike all the walking models, are perfectly elastic. All right? And now you can see that that's not an extra assumption that is derived from these. When it hits, even though this toe sticks in right away and [INAUDIBLE] inelastic collision with that toe, because there's no mass there, there's no energy, there's not momentum in it-- sorry. This is [INAUDIBLE] you don't lose any energy.

And so it's actually conservative as it runs through. And then if you-- it's flying through the air, there's no drag or anything, that's conservative when it flies through the air as well. And so this system actually is completely conservative in its full operation. So yeah, so it's very different than a rimless wheel.

And also, then the other assumption we have to make is that the leg instantly goes to the desired theta and rest length. So again, when it's massless, the rest length, it will do that automatically. But you have to assume that [INAUDIBLE] teleported to that theta. So you don't worry about collisions with the ground or everything like that. Your leg just goes to the theta touchdown. So the moment you take off, it's in this new configuration.

So now what we have to do is, now that we have the system and the model, we want to pick our surface of section, because our goal here is to turn this into a 1-D iterated map, right, because that's what this is all about. We're going to try to figure out, how do we pick a spot where you can just look at one section and describe all the dynamics, just being like, OK, we're here. Simulate to the next one. And then we can just iterate this map and figure out the fixed points, figure out a lot of things, as we did with the rimless wheel.

So does anyone have an idea of what a good surface of section would be? No? What special configurations are there that we can look at?

AUDIENCE: Take-off.

PROFESSOR: Take-off. What else? That's one.

AUDIENCE: Full compression.

PROFESSOR: Full com--

AUDIENCE: Or rest length [INAUDIBLE]

PROFESSOR: At the rest length? That's true. Full compression you could do, but that would be like there'd be a 0 r dot or something like that. But yeah. No, I mean, you could. There's a lot of these special configurations.

But the one that you really want to look at, the one that collapses things down the most is if you look at the apex. So you look at the maximum height of this flight. So we can just look at that y. And the reason we can do this I can describe right here.

So we look in the flight phase. Now, first of all, x, we don't really care about what x is for the stability. Doesn't matter where it is along that position. That doesn't affect the dynamics directly. So we don't care about x.

So y, we definitely do care about. This matters. x dot, well, x dot, because we know it's conservative and we know that since at the apex y dot equals 0, then x dot is purely a function of y, right? We can look at the total energy of the system, which is 1/2 m V squared plus mgy.

We know y dot is 0, so that means that V is just x dot. And so you can write x dot is a function of y. And so here, we know this is 0. We don't care about this. This one we can find directly from y.

And so everything that happens between one apex, another apex, we know if we just know y. You see that? Yes.

AUDIENCE: We're always adjusting the angles [INAUDIBLE] landing so that it is vertical at the [INAUDIBLE]

PROFESSOR: It's set at whatever desired angle we're at. So there's some nominal angle we want on this touchdown. So let's say it's 30 degrees or whatever. And so that means that then you take off and your angle-- your leg goes to 30 degrees, and then you hit there.

And so the touchdown angle is always the same. And so there's no control in this at the moment. It's just the passive stability of this balancing system. Does that makes sense to everyone how we can collapse all these things?

AUDIENCE: Professor?

PROFESSOR: Oh, yeah.

AUDIENCE: [INAUDIBLE] why don't we care about x again? Because--

PROFESSOR: Because that doesn't affect the stability of the system. So if we're trying to get somewhere and hit a target, yeah, then we have to look at x in our controller. But because the x doesn't figure in the dynamics anywhere, right, doesn't matter, because again-- sorry, this is something I probably should have mentioned in the beginning.

It's not going downhill or anything like that. So how far it goes in x doesn't matter. The dynamics are going to be the same invariant to that, if that makes sense. And so because of that, I mean, x will be changing, but it won't affect the next apex height, because it doesn't figure into the dynamics like that. Does that make sense? All right.

So we can represent the whole thing just using y. I guess that's the important thing to realize. So what do I want to erase? I'll erase this stuff. I'll throw this back up. Hmm. This isn't ideal.

All right, this is not going to get better than that. All right. So we just go through this transition and we go from apex to apex, we can look at is we have y apex at n. Then we need to figure out how that translates into the y, x dot, and y dot at that apex. All right, so we need to map this one piece of information into all of these.

And then we can map that into the y, x dot, y dot at the touchdown, so when it comes in and hits. Now we have to change our coordinate system to r theta, r dot, theta dot. This is still a touchdown, and we swing that through in the stance phase. Then we get that to r theta, r dot, theta dot at take-off.

So my D and my O are as different as I can make them look. Then at take-off again, we switch to our aerial phase. We go back to y, x dot, y dot at takeoff. And that brings us, then, to a y, x dot, y dot at apex. And then here, of course, we grab y n plus 1 at apex.

[PHONE RINGING]

So these transitions, I'll number of them and make this a little bit clearer. That's one, two, three, four. Sorry. So all right. And the key now is to figure out how to push our system throughout all these transitions so we can figure out what this mapping is going to be.

So some of these are pretty easy to do, and some of them can be quite tricky to do. But all of them are quite manageable. So what you look at first is the energy, which is our kinetic plus potential, again, is 1/2 mv squared plus mgy. So at the first step, if you want to convert-- if we want to take y apex and turn it into these guys again, we can have the energy of the apex we know is 1/2 m x dot, again, because we know y dot 0, plus mgy is equal to a constant. And so then this means that we can say, all right, x dot equals this function of y.

So we can figure out what x dot is, [INAUDIBLE] and then we know also-- sorry. We know y dot equals 0. And so here, then, that gets our first transition.

Two then, if we want-- we know that x dot at touchdown-- so this is the ballistic phase coming down from apex-- is equal to x dot apex, because there's no forces on it, so it's just going to keep carrying forward in x. And then we know y touchdown is going to be r naught cosine theta naught. So this is what I was talking about with there's the desired theta. So your touchdown, you know you're going to touch down at this desired theta. And this, then, again, since we know our energy, we can get y dot touchdown, all right.

So here that gets our transition from apex to touchdown. Now this is just a coordinate transformation. It's pretty easy to do. Just make sure you get your velocities right. You have to decompose them properly. But you can just transform those coordinates, and then you get that without too much difficulty.

But then you get to the tricky one, and the tricky one-- [INAUDIBLE] The tricky one is the stance phase. So how do you push yourself from when you come in and hit to squishing down and launching back out? So that's the involved part of this. So four is stance dynamics.

AUDIENCE: Is that y a touchdown [INAUDIBLE]

PROFESSOR: Pardon?

AUDIENCE: y [INAUDIBLE]

PROFESSOR: Oh, yeah, this is touchdown. Sorry. So to get to two, to get this falling phase, you know x doesn't change. There's no forces in the x direction. y touchdown, you fix this angle. That's instantaneous warp of the leg.

So you know when it touches down, it's going to be this high, because that's when touchdown's defined, is when the toe hits the ground. And then here, you know your height. You know this. And again, conservation of energy will give you this at the y dot touchdown since that just allows you to figure out your state when you come in.

So stance dynamics. So here, you've got a idea of what you expect the system to do. You have a spring. And if you're running in steady state, I believe you take off the same angle you come in. But so what you can imagine happens is this thing swings around like this, but the center of mass goes in and comes back out. So again, it's doing that compression and then launching itself, all right.

So I think it's a assumption Raibert did for the steady state operation. It comes in, out at the same angle, and serves as a compression in the middle. And so again, this kind of behavior is, again, one of those ways of defining running, like for the Groucho kind of running, is that you see the center of mass come in to the center of pressure, as opposed to vaulting over it. If this was a stiff leg, it would go like this. So that distinction is one way of defining running.

So we can define take-off, so we know our final condition. So take-off happens when r equals r naught. So when you get back to your rest length, you assume that your body then, it goes back to the ballistic phase. All right, then we can write down our energies here.

Now again, we're in the polar coordinates now, so our energies look a little bit different. Our kinetic energy looks like that, and our potential energy is this term, and what else do we need? What else?

AUDIENCE: Spring potential.

PROFESSOR: Spring, yeah. So it's k over 2 x-- sorry-- r minus r naught squared. And so using that, you probably remember Lagrange. It's not that bad of a system.

You can get the equations of motion. The equation of motion here, you get-- oh, sorry. And so you have all your expected terms, centrifugal terms, Coriolis terms, all that if you try to make sense of what this is.

And so if you want to get from touchdown to take-off, you can't get a closed form solution from this. You can simulate pretty easily, obviously. But you don't get the closed form, so you can't get this analytical kind of mapping. But if you use a small angle and small displacement approximation, you can linearize the system, and you can get a closed form solution for this touchdown to take-off phase, all right.

And so that's this assumption that theta is much, much less than 1. Your delta r over r0 is much less than 1. And so then you can get to a closed form solution. It's kind of ugly. It's not really ugly. I mean, it's not so ugly as to be prohibitive, but I'm not going to write it down. But that lets you actually get a closed form return map, all right.

And something cool about that is that you get these two fixed points. You get one stable fixed point and one unstable fixed point. And-- hmm. And actually, a number of people thought that a system like this, this conservative system, shouldn't be able to get that sort of stability, because if you remember in the rimless wheel, the energy loss was critical towards achieving the stability. When you went faster, you hit harder, you lost more energy. And when you went slower, you didn't lose much energy, so you were able to speed up as you went down this ramp.

So the fact that you can achieve stability in this system is kind of cool, too, because it's completely conservative, yet somehow, dynamics are able to actually push it towards a repeatable state. [INAUDIBLE] an then looking at-- we're going to see this return map. This is from [INAUDIBLE] Geyer's thesis work, I believe. And it's pretty cool, if I can get to it for you.

There we go. There we go. So this is actually the return map for this small angle approximation you can get analytically. And you see there's this unstable fixed point, and then down there, there's that stable fixed point. So you zoom in, and you can see the stability region, that that unstable one is defining the boundary of the stability for that.

The apex height, why does it get to that minimum there? Why doesn't it go below that 0.87? Anyone know? Why don't we look at it below that?

AUDIENCE: Because that's the maximum range?

PROFESSOR: Yeah, that's your height at the cosine theta length of the leg. And so you can't get below that. And then the weird thing is, and this is something I was talking about with Russ just before he went home, was why does it keep climbing like that?

AUDIENCE: Why does it keep climbing [INAUDIBLE]

PROFESSOR: Well, because this is conservative. So how can you have an instability that pushes you up to larger and larger apex heights?

AUDIENCE: [INAUDIBLE] to a more vertical--

PROFESSOR: At some point, though, it should cap. There should be another stable fixed point bouncing straight up. There isn't. The thing is that, what we think is that that unstable one right there is your vertical bouncing. And the thing is that, actually, when you linearize these, they don't quite conserve energy anymore.

This actually can be a non-conservative term. So we think that's what this is from. So it really shouldn't be climbing up like that, because when you simulate these things, you don't get above that second fixed point. It rolls up to this unstable one.

And so then-- but looking at this again, now here's another strange thing. It looks like it should be globally stable, then, right? Because if in practice we can't get above that unstable fixed point-- and if you look at this apex axis, right, the higher it is, the slower it moves. So at your minimum apex height, right, that's when you're moving fastest. It looks like it's stable throughout the entire operating regime, right?

The fastest it can move is that little guy on the far left where it's apparently stable. And the slowest it can move is this vertical bouncing, [INAUDIBLE] fixed point. So how do you explain that?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Well, that's the thing is that it-- that's pretty much exactly the issue is that your failure mode can come from coming in, squishing really low-- I'm going to fall in my face here, so I should let everyone see it. You can compress and get really low, and then you bounce forward. And if you bounce forward at such a directory that you never get high enough for your leg to be out, touch it down, then you're just going to bounce forward, land on your face. You can't actually warp your leg. Does that make sense?

So that can be your failure mode. So you bounce in, you shoot across, and then if you never have an apex high enough, you'll never get your leg ahead of you. So that's the failure mode. So that's actually then what we see when we simulate it.

And the simulation actually looks a little bit different. There's actually-- this guy looks pretty good, but if you simulate it and you have slightly different parameters, that curve there comes down a little bit lower, and so you're actually able to be from the top and bounce in and overshoot and go unstable. So it's not like you're guaranteed stability by starting to bounce vertically, either. So this will change with different energies, different parameter settings, and everything like that.

But the takehome thing I think that's cool is that, even this simplified perspective of it, you get this stability in this conservative system. You get this stability that you can see in the simulation, and that even though it's not dampening on any energy or anything like that, is able to bounce along and right itself. And apparently, what that's related to-- apparently, people didn't used to think that was possible.

But the coordinate transform is actually enough to get that kind of stability, is that-- it's called piecewise holonomic. And by switching these coordinates, you're actually able to get the-- that allows the dynamics to stabilize themselves, apparently, as opposed to a normal holonomic system apparently couldn't actually converge in back towards this fixed point. If you think about the pendulum [INAUDIBLE] like that, that's not going to converge [INAUDIBLE] It has to conserve the energy.

But apparently, with piecewise holonomic systems, it's available. But you can read some papers and get into that. But I don't know much more than I just told you.

All right, so here's another thing that's interesting, is that if you look at-- if you want to model those Raibert hoppers, right, they obviously aren't completely conservative. They do have some mass in the toe and in the leg, and so when they hit, they do lose some energy, because when that toe hits and sticks to the ground, it's going to be a dissipative reaction.

So you can look at this model by, I think, [INAUDIBLE] and Buehler in '91. I'm going to use my big chalk again. Ah, and here's my old big chalk. So looking at this. So here we have a toe mass. So we have the mass of the body and mass of the toe.

And this spring can possibly be non-linear if you want. And this was largely-- they did some analytics, but a lot of it was computationally driven, the analysis they did of this system. And so the thing is, because of this hit here, you have a loss due to the toe mass. So must add energy.

And I do this with a control on the spring. You can vary your spring constant like that. You can add energy to the system. And so the control in the spring is their only control then.

And you actually can get similar stability properties to the SLIP model. You can get these when you simulate it. You can find the same stable fixed point, unstable fixed point kind of behavior.

And so this is all what we've been talking about. The image we have is this sagittal plane dynamics, right? The sagittal plane is this plane, and so this idea of running through like that. Getting a little bit aerial phase so I look less ridiculous.

And so the other thing, though, is that it's good for the lateral plane. If you look at the lateral plane dynamics, certain animals, the same kind of SLIP model and that kind of behavior can actually be witnessed. So this is, again, something Bob Full spend a lot of time on.

If you look at a cockroach, which has a tripod gait-- here's a cockroach. I wish they actually looked like that. Probably wouldn't be as disturbing. There. And so what you have is, they use three legs at a time, and they're springy legs.

As it runs, if you look at the dynamics, you actually can see the-- you actually can see that these legs are acting like springs. And actually, the same SLIP behavior you'd expect, you witness in the cockroaches. And so there's some cool things here. Let me show you.

So these-- have any of you heard of preflex, as opposed to reflex? It's like pithy kind of name, I guess. So the preflex is where actually, you can actually look at the time constant required for the monosynaptic reflex, which is the electrical signal to go through and actually to the spinal cord and back. So not to the brain and to the full path, but just the quick reflex kind of response. And actually, some of these creatures actually respond faster than that.

So the theory is, is that it's actually a musculoskeletal response. So it's not even-- it's not controlled at all. It's not going to the muscles or anything like that. It's tendons. It's the springiness in the leg. And that is actually what's providing some of the control here and some of the stability. And so there's a really incredibly awesome video that I am definitely going to show you, regardless of how inconvenient this thing comes out.

AUDIENCE: It's not an anticipatory neural.

PROFESSOR: It's not. And that's because you can actually-- can perturb them almost instantaneous, slam with perturbation. And so they can't anticipate this. And you actually can see, within one step, a cockroach step, they'll spring back, and their center of mass trajectory will get back on track. And so it's not like they see something coming or there's-- they're going to step something that's going to hit them. It's that there's just-- they're running along, a perturbation, and then they start bouncing right away.

I'm sorry about this. I don't know what's happening here. Yeah. I'll have to-- [INAUDIBLE]

AUDIENCE: So in that model, is it now the spring is actually [INAUDIBLE] Because now that you have x, y, z, and it's at an angle, [INAUDIBLE]

PROFESSOR: Yeah, I mean you--

AUDIENCE: Is that how you do that, or--

PROFESSOR: If you're doing the lateral plane, I think you have to have some sort of springiness like that that can push you through. But you can actually look at the cockroach running along the sagittal plane, and you don't need the [INAUDIBLE] You can--

AUDIENCE: Oh, [INAUDIBLE]

PROFESSOR: Yeah, you can treat several legs if they're just like one spring in the tripod. If there's something called-- I don't know if you've seen-- you know Bob Full's work at all and these templates and anchors, where the anchors are the more complicated system, and the templates are these really simplified systems? So your SLIP little thing here can be like a template, which is a really minimalist model but captures some of the essential dynamics and does so in a concise way.

And you can look at just a cockroach as following this kind of running. I mean, and it actually captures a lot of it. I mean, there's more complicated ones. If you look at the lateral plane dynamics, I mean, you probably need a more complicated model. But so--

AUDIENCE: What did you say this model was called again?

PROFESSOR: This is a SLIP. Oh, but [INAUDIBLE] and Buehler came with this. So this is-- yeah, it has a mass of the toe. I don't know if it has a different-- I don't think it has a different name. Yeah, so let me get to this. There you go. All right.

So first, the less exciting thing, I think, right here, but still pretty cool. So measuring the force on these cockroaches' steps, you could imagine, is difficult. Something they did, actually, to facilitate these experiments is they actually have these guys running on Jello. I think he set up diffraction [INAUDIBLE] on this jello.

And so you see that, how it changes color when they're pushing? They're actually able to figure out the magnitude and the direction of the force to some level of accuracy by looking at that. And apparently, orange jello works really well. I don't know what it is about orange jello, but if you ever want to analyze the forces on a cockroach's legs, start with orange jello, if that's the only thing you get out of this lecture.

But yeah, so that's pretty cool, but that's just analyzing when it's turning. They can figure out-- that's to look at some of the springiness and how the force response and the center of mass response connect. But here. This is one of the greatest experiments of all time right here.

All right, so this is looking at the perturbation experienced by these cockroaches. So what they did, they bolt a cannon to the back, because pulling strings and stuff like that, it's not fast enough. It's nowhere near fast enough. This cockroach is running. Aw, come on. What's going on here?

All right. Really sorry. It's running. Bam. Perturbation. The little cannonball actually hits it on the way back, you see.

[LAUGHTER]

That's not really fair. That's double perturbation. But they tracked the center of mass of this guy, and actually, you could see it gets back on in less than a step. And if you look at the time scale of this and the time scale of their monosynaptic reflex, it happens too quickly. And so they think it's actually compliance in the legs and everything like that that's just passively tuned such that it gets banged to the side and rights itself.

AUDIENCE: Will you put this on the course website?

PROFESSOR: Actually, I think-- I mean, it's Bob Full's video, but he may have it available. But a great thing is that the guy who did this thing-- Devin did it. And he said-- and this is all he said, so he didn't go through the full thing. But he's like, you'd be surprised by how little gunpowder is necessary to perturb a cockroach.

[LAUGHTER]

And so you could only imagine the first cockroach they got out there. And they're like ah, this is about the right amount and just blows the cockroach up or launches it across the room or-- so that would have been a fun trial in an experiment to watch from a different room, I think. Yeah, we'll watch this one more time.

AUDIENCE: [INAUDIBLE]

PROFESSOR: [INAUDIBLE] But you see, that's just a fantastic little experiment. And the perturbation, as you see-- I mean, it doesn't know that's coming. That hits it like that. It just responds almost instantaneously. So I think that's really cool.

And that shows this-- I mean that's SLIP in the lateral plane. And it shows that these springs and this compliance in the actual-- in the animal can actually do things that just a control couldn't do, that the time scale of the response can be faster than control could achieve. I don't know if I have-- yeah, I don't think I have these [INAUDIBLE]

So does anybody have a question? Yeah. OK, so that's nature having this spring bouncy compliance. But you can see Raibert's hoppers, back in just the '80s and '90s, did amazing things, too. If you look at-- let's see, where's this 1-D monoped? There. Some of these videos are pretty crappy, but there.

So that's just a little monoped running around. Do you see? And I mean, it's constrained to be in the plane. It's on a boom, but it could fall down forward and everything like that. It's not like it's free to roll. So it's achieved the stability bouncing around that circle.

But thought they could do more than that if you look at-- they built bipeds. This is-- I think, yeah, Russ grabbed this from a VHS tapes. So this is actually what's funny is that I think this is the fastest biped around, but it wasn't quite that fast. That's skipping frames, but I think it's still-- unless something has changed in the last year. It runs like 2.2 meters a second, which I think is actually still the fastest biped. But Russ would know if anything has changed.

AUDIENCE: Is it spinning around?

PROFESSOR: Pardon?

AUDIENCE: [INAUDIBLE] I'm sorry. It looks like it's spinning around but it's not [INAUDIBLE]

PROFESSOR: Yeah. Oh. But check out that. So here-- oh, come on. There. So they figured they could do more.

Here's a biped. So this one isn't on a boom. Bam. Check that out. Look at this one more time. Running there, [INAUDIBLE] speed, like gymnastics. That's like a robot doing a front flip.

Then here, this is pretty impressive, too. Yeah. Right over those stairs like nobody's business.

AUDIENCE: Is there a [INAUDIBLE]

PROFESSOR: If what were out of phase? If the--

AUDIENCE: [INAUDIBLE]

PROFESSOR: If the stairs were out of phase? I don't know. You could probably imagine setting up stairs that would make it easier and stairs that would make it harder. So yeah, you probably could come up with something that would be problematic if it had to hop. Well, maybe could hop on one foot for a while. I don't know. It's on a boom.

But yeah, so I mean, these are-- I mean, even now, you can look at these things, and you're-- I mean, they would be amazing if someone just achieved this, but these were back in the '80s and '90s that people did this. And so I mean, you've probably seen Big Dog. Raibert worked on Big Dog, too, and it's like 20 years later than this stuff. And it's still state of the art, that kind of control and similar control in a lot of ways.

We can do some more of these. Oh. Oh, there's one that's-- let's see. and the thing is that not only do these guys do these crazy things, but their robustness is really pretty incredible. I mean, a lot of these, like the Honda robots walk on, carefully, just ground and everything like that.

Here. [INAUDIBLE] towing him along. It's probably not the most comfortable. It looks like it's a little bit less smooth. But I mean, it's running along sidewalks outside of MIT.

They have-- this other one's [INAUDIBLE] just running by itself on grass and everything, too. Oh, the quadrupeds are pretty cool, too. That's a quadruped running down the hallway. And they actually did a cool paper called "Four Legs Running as One," "Four Legs that Run as One," something-- "Running With Four Legs as if it Were One"-- there we go-- that used the same kind of control ideas but actually can get these quadrupeds and actually run based on the same ideas. And you see that thing's moving pretty fast and pretty robustly right down hallway like that. So it's really pretty amazing capabilities, even today.

There's one I really wish I could find for you. Yeah. And so interesting thing that some more recent work, they've-- oh, let's look at [INAUDIBLE] Yeah. Check that out. Isn't that just amazing, right?

And actuators that these guys use is hydraulics at the hips and then pneumatics along the leg, so [? [FFFT] ?] with a pneumatic. Need that so you can rocket yourself that far. But it's just incredible. Yeah.

So something that people are working on more recently is this thing called SLIP walkers. So you can imagine if you have a compass gait with springy legs, right, as it's spring constant gets very high, it starts behaving just like a compass gait. If the spring constant goes to infinity, it's rigid, it's going to be just like a compass gait. If you let those strings get looser and looser, it's going to start being more like one of these bouncy bipeds, right, more like this SLIP behavior.

And so actually, you can get stability properties-- I mean, it can be stable through a relatively broad range of bounciness so that the same robot can vault over its legs and walking and then start bouncing and start running. It actually could do that transition. And that's a cool anything also that you see. I should have brought this up at the beginning when we were talking about biology.

But those transitions are interesting, because if you look at the efficiency of human locomotion-- so you can measure O2 consumption, and you can see how efficient it is to walk at different speeds and to run at different speeds. You can look at the efficiency curve for-- I believe this is speed, and this is, let's say, efficiency. And so you have something with walking and some curve like this. And then running, you have some curve like this.

I mean, quantitatively, these could be wrong, but this is a qualitative feature. So if you look at the efficiency of a person walking and you just put them on a treadmill and just tell them, start walking, speed it up, speed it up, and tell them just to transition to running whenever they want, they'll transition right here. That switches.

And so you know the feeling, I'm sure, where you're walking faster and faster and suddenly it just feels more comfortable to just start jogging. Apparently, that happens when the efficiency of that motion outweighs that of walking, because obviously you can walk where it's uncomfortable to walk and run where it's uncomfortable to run. But the natural transition happens at that efficiency, bifurcation.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Pardon? Yeah, that should be cost. Now it's even more skewed. There we go. Thank you.

Yeah, so [INAUDIBLE] SLIP walkers, I think. Hopefully, you can get some of this in transition, but I'm not sure exactly where they are right now. I don't know if they have any videos of those. There's some papers about them.

And I think that's all the stuff that Russ wanted me to convey. So we just have some idea of this culture of these cool running models and these springy things and these preflexes and stuff, and in nature, the fact that springs are critical to locomotion of horses and the stability of the cockroach running and everything like that. So if anyone has any questions, I'd love to answer them about anything.

AUDIENCE: When you were saying how they were-- the transitioning using the spring model between walking to running--

PROFESSOR: Oh, the SLIP walkers?

AUDIENCE: Hmm?

PROFESSOR: The SLIP walkers?

AUDIENCE: Yeah, the SLIP walkers. Is it still [INAUDIBLE] elastic collision, or do they [INAUDIBLE]

PROFESSOR: I think in practice. I think they have idealized legs. I think they've done simulations idealized legs, too. But you can do either way, I'm pretty sure. I haven't looked at that work as closely as I probably should have. But yeah, for the walking, vaulting behavior, you need either a toe thing that's going to-- or you can let your spring constant go to infinity. And then you'll have a rigid impact, right.

So you can treat it that way, too. But then yeah, if you loosen it up, then you're not going to have the intermediate still lossy behavior. But yeah, I think they have done stuff with idealized legs and probably non-idealized ones, too. So anything else?

AUDIENCE: If you get it just right, can you adjust the spring constant so that you'll get the push off right before landing?

PROFESSOR: Oh, you mean toe off, kind of, or the toe-off thing that pushes you forward that [INAUDIBLE] efficiency?

AUDIENCE: Right.

PROFESSOR: I think you should-- I mean, I'm not sure about how the tuning would relate to the walking things. But when you have these idealized legs with the spring, I mean, you can't be more efficient than if you're vaulting over them. So I don't know if it connects to the-- I mean, it seems like it could connect to that toe off launch, but I'm not sure if that is explicit, just by having your leg not be a strike with the dissipative inelastic collision is going to help you.

But I don't know if you get the same kind of-- yeah, it seems like you should, but I don't know if that exact point-- I haven't seen that exact point. But it could easily be addressed in other papers. Yeah, I should-- I don't know if I have the references right now, but I can make sure that Russ gives those to your or get those to you myself next lecture if you want to look up some of the SLIP walkers, because that's pretty new stuff, I think. Yeah.