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8.012 Physics I: Classical Mechanics
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012 Fall 2007

# Final Exam
# Monday, December 17, 2007

NAME: ________________________________________

Instructions (PLEASE READ THESE CAREFULLY):

1. Do all **EIGHT (8)** problems. You have **3 hours**.
2. **Show all work**. Be sure to **circle your final answer**.
3. Read the questions carefully
4. **All work must be done in the white books provided**.
5. No books, notes, calculators or computers are permitted
6. A sheet of useful equations is provided on the last page.

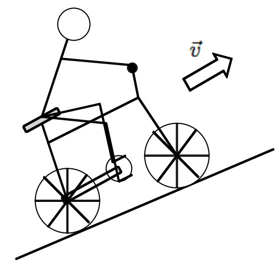
## Your Scores

| Problem | Maximum | Score | Grader |
|---|---|---|---|
| 1 | 10 | | |
| 2 | 10 | | |
| 3 | 15 | | |
| 4 | 10 | | |
| 5 | 15 | | |
| 6 | 15 | | |
| 7 | 10 | | |
| 8 | 15 | | |
| Total | 100 | | |

Problem 1: Quick Short Answer Problems [10 pts]

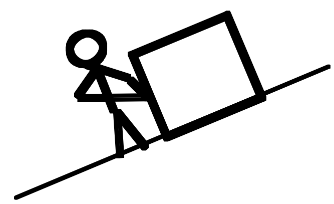
Answer all five problems. You do not need to show any work. **Be sure to record your answers in your solution book, NOT on this exam.**

(a) [2 pts] A bicycle rider pedals up a hill with constant velocity v. In which direction does friction act on the wheels?



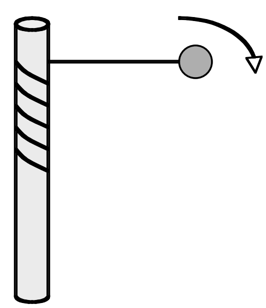
Uphill | Downhill | Friction does not act while the bike moves at constant velocity

(b) [2 pts] An 8.012 student pushes a heavy object up a hill, and is prevented from slipping by friction between her shoes and the surface of the hill. While she is walking up, she picks up one of her feet. Will she be:



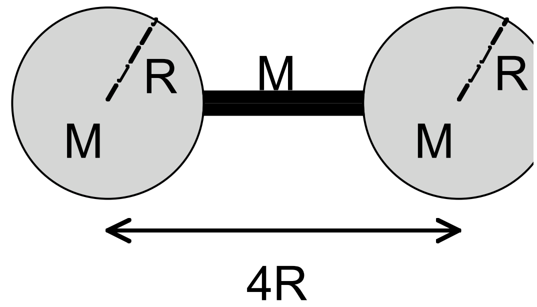
More likely to slip | Less likely to slip | Neither, as there is no change in the friction force acting

(c) [2 pts] A ball attached to rope is twirled around a stick as shown in the diagram at right. Ignore gravity and friction. Which of the following quantities is conserved in the motion of the ball? Be sure to write down all of the choices below that apply.

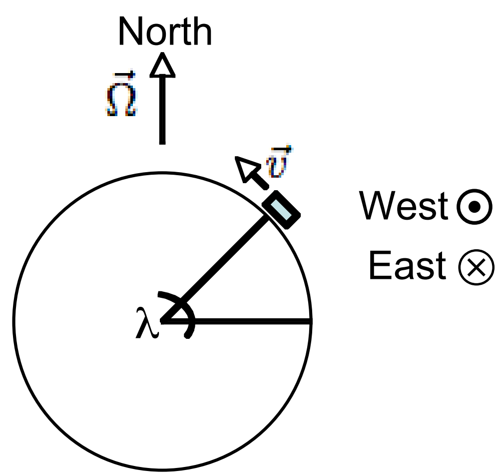


Energy | Momentum | Angular Momentum | None of these are conserved

(d) [2 pts] *(Challenging)* Consider two uniform disks of mass M, radius R and negligible thickness, connected by a thin, uniform rod of mass M. The centers of the disks are separated by a distance 4R. Reproduce the diagram at right in your solution book and draw the principle axes of this object centered at its center of mass [1 pt], indicating the axis about which torque-free rotations are unstable [1 pt]. Note: you do not need to calculate the moment of inertia tensor to solve this problem.

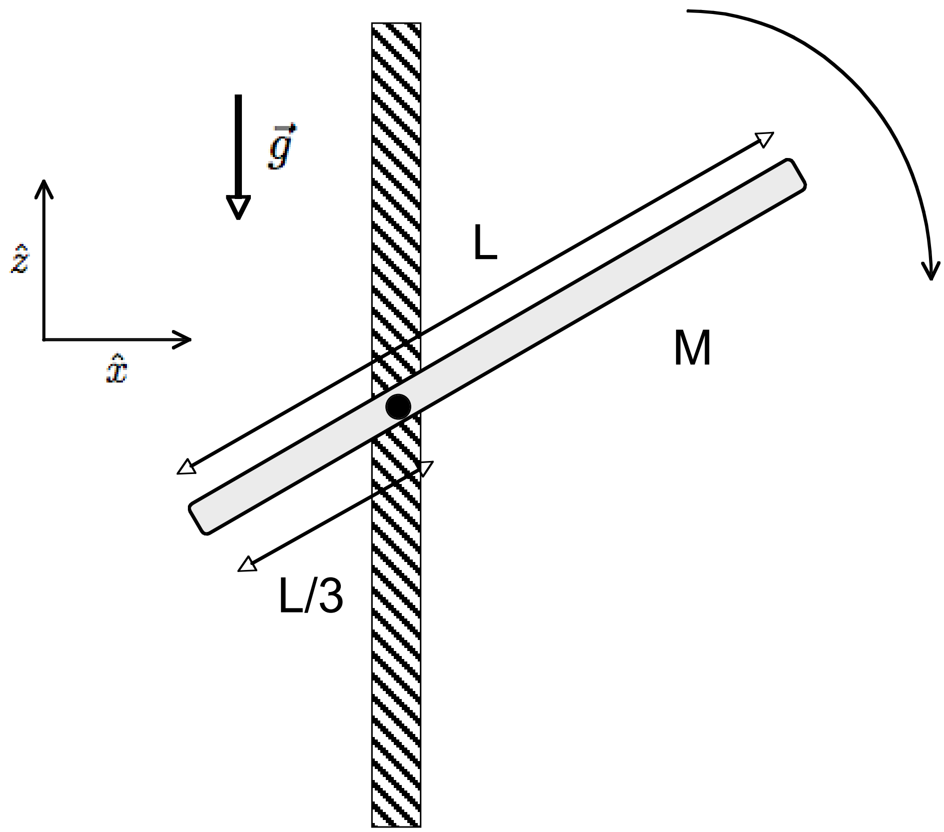



(e) [2 pts] A car at latitude λ on a rotating Earth drives straight North with constant velocity v as indicated in the diagram. In which direction does friction between the car's tires and the road act on the car to counteract the Coriolis force on the car?




East West Coriolis force does not act on the car

Problem 2: Swing Bar Pendulum [10 pts]

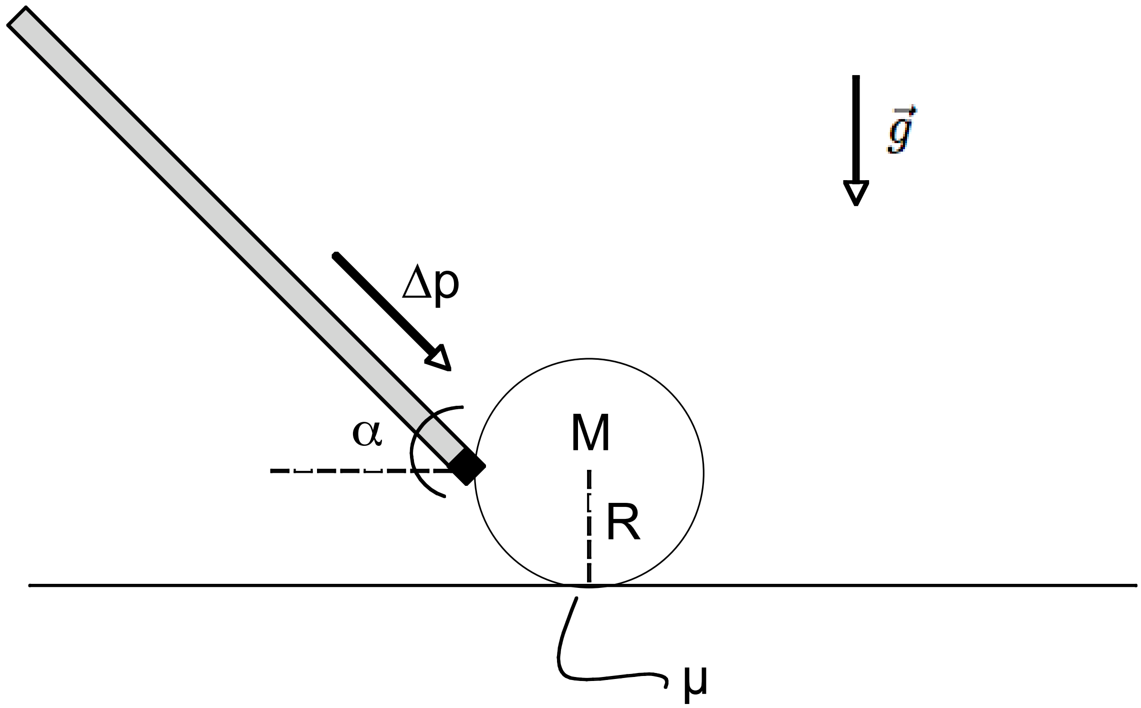



A uniform bar of mass M, length L, and negligible width and thickness is pivoted about a fixed post at a point 1/3 along the length of the bar (see figure). The bar is initially released from rest when it is tipped just slightly off of vertical, causing it to swing downwards under the influence of constant gravitational acceleration g as shown above. Ignore friction.

(a) [5 pts] What is the total force the swinging bar exerts on the fixed post when it passes through horizontal? Express your answer as a vector with components in the coordinate system indicated above.

(b) [5 pts] What is the angular rotation rate of the bar as it swings past it lowest point (i.e., oriented vertically)?

Problem 3: Cue Ball Spin [15 pts]



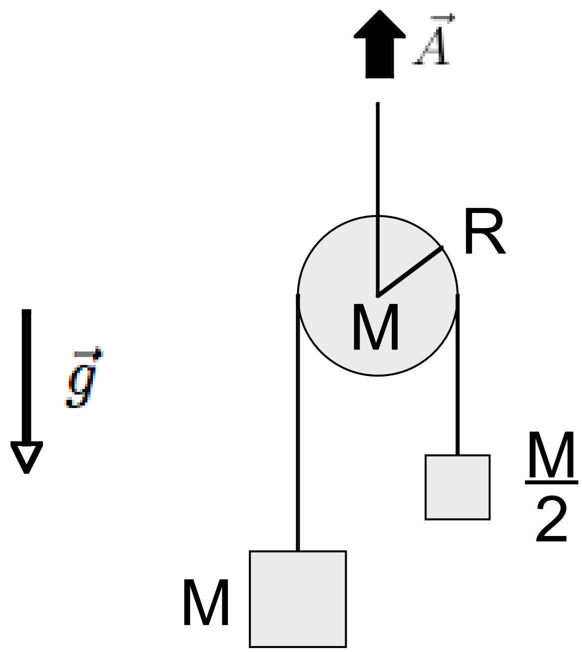

A billiards player strikes a cue ball (a uniform sphere with mass M and radius R) with a cue stick at the middle of the ball (i.e., at a height R above the table) and at an angle α with respect to horizontal. The strike imparts an impulse Δp on the ball in the direction of the strike, causing it to move toward the right as well as "backspin" – spin in the direction opposite of rolling motion. The coefficient of kinetic friction between the ball and table surface is $\mu$. Assume that the ball does not rebound off of the table after the strike, and that constant gravitational acceleration g acts downwards.

(a) [5 pts] What is the initial speed and angular rotation rate of the cue ball after it is struck?

(b) [5 pts] For what angle α will the ball eventually come to rest?

(c) [5 pts] For the case of part (b), how far does the ball travel before it comes to rest?

Problem 4: The Accelerated Atwood Machine [10 pts]

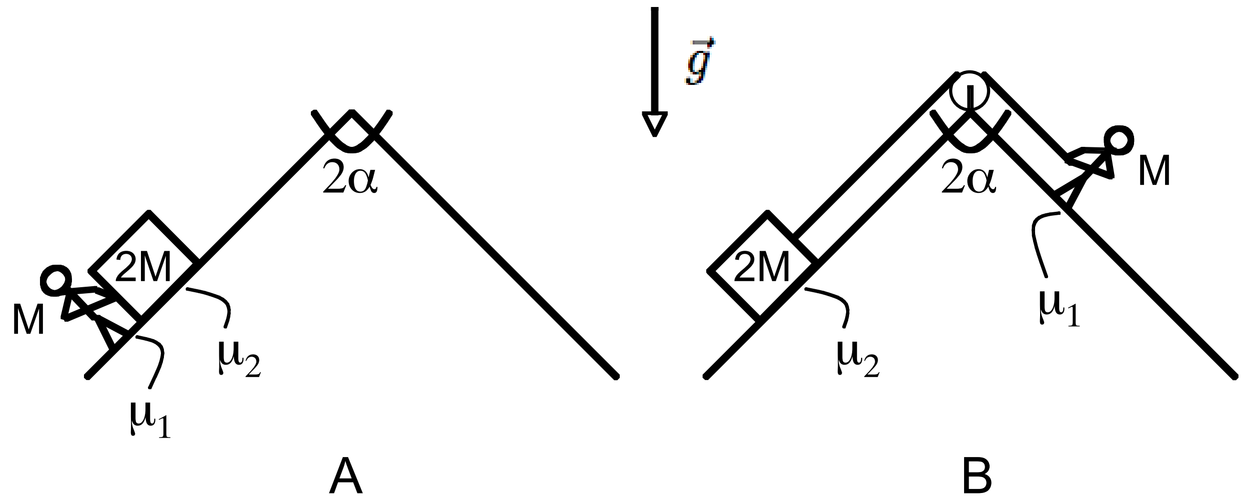



An Atwood machine consists of a massive pulley (a uniform circular disk of mass M and radius R) connecting two blocks of masses M and M/2. Assume that the string connecting the two blocks has negligible mass and does not slip as it rolls with the pulley wheel. The Atwood machine is accelerated upward at an acceleration rate A. Constant gravitational acceleration g acts downward.

(a) [8 pts] Compute the net acceleration of the two blocks in an inertial frame of reference in terms of g and A. Do not assume that tension along the entire string is constant.

(b) [2 pts] For what value of A does the block of mass M remain stationary in an inertial frame?

Problem 5: What is the Best Way to Move a Heavy Load up a Hill? [15 pts]



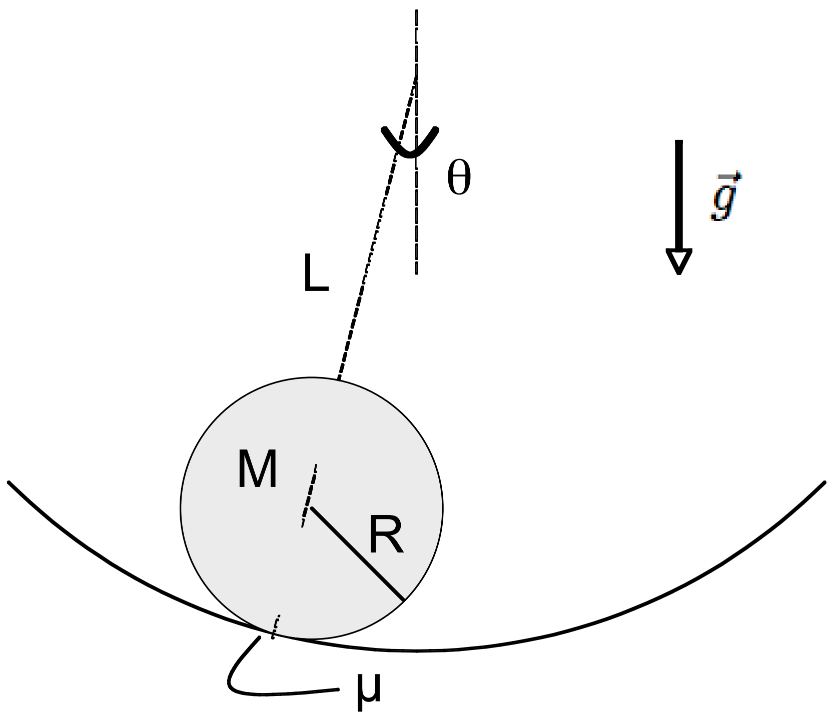

Two students, each of mass M, are attempting to push a block of mass 2M up a symmetric triangular hill with opening angle 2α. Student A pushes the load straight up; student B pulls the load up by running a massless rope through a massless, frictionless pulley at the top of the hill, and pulling on the rope from the other side. The maximum coefficient of friction (assumed here to be equal to the coefficient of kinetic friction) is $\mu_1$ between the students' shoes and the hill, and $\mu_2$ between the block and the hill. Assume $\mu_1 > 2\mu_2$. Constant gravitational acceleration g acts downwards.

(a) [5 pts] For what minimum angle $\alpha_{min}$ (i.e., maximum steepness) does neither student need to apply any force to hold the load in place?

(b) [5 pts] Calculate and compare the forces each student must exert on the block to move it up the hill at constant velocity. Does either student have an advantage here?

(c) [5 pts] Calculate and compare the minimum angles $\alpha < \alpha_{min}$ that each student is able to move the block up the hill at constant velocity without their shoes slipping on the hill surface. Does either student have an advantage here?

Problem 6: Ball Rolling in a Bowl [15 pts]



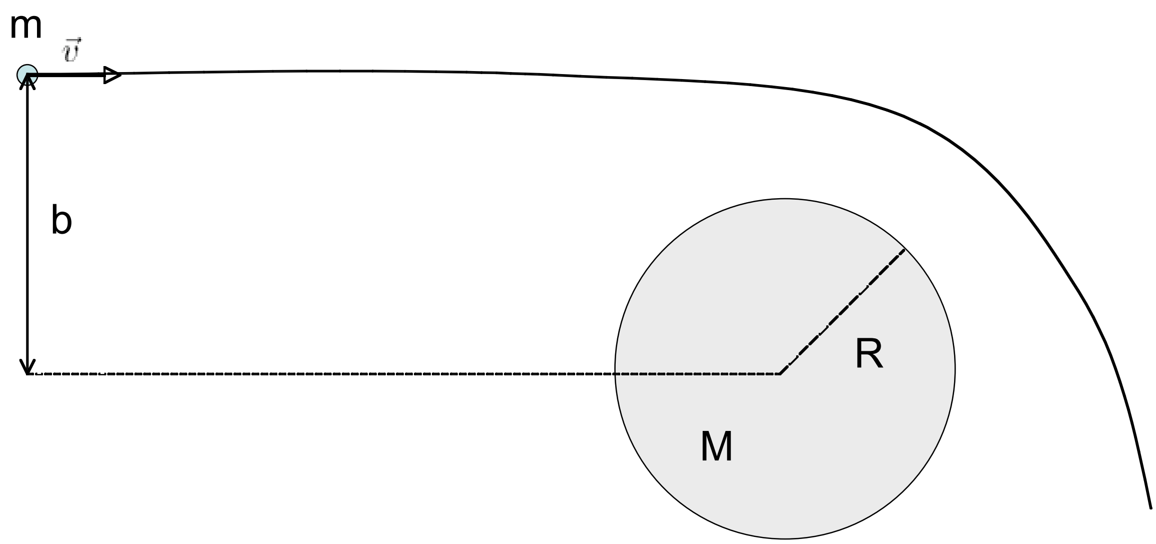

A solid uniform ball (a sphere) of mass M and radius R rolls in a bowl that has a radius of curvature L, where L > R. Assume that the ball rolls without slipping, and that constant gravitational acceleration g points downward.

(a) [5 pts] Derive a single equation of motion in terms of the coordinate θ (the position angle of the ball with respect to vertical) that takes into account both translational and rotational motion, for any point along the ball's trajectory. Be careful with your constraint equation!

(b) [5 pts] Find the position angle of the ball along the bowl's surface as a function of time in the case that θ is small. Assume that the ball is started from rest at a position angle $\theta_0$.

(c) [5 pts] At what maximum initial position angle $\theta_0$ can the ball be placed and released at rest and still satisfy the rolling without slipping condition throughout its motion? Note that $\theta_0$ does not have to be small in this case.

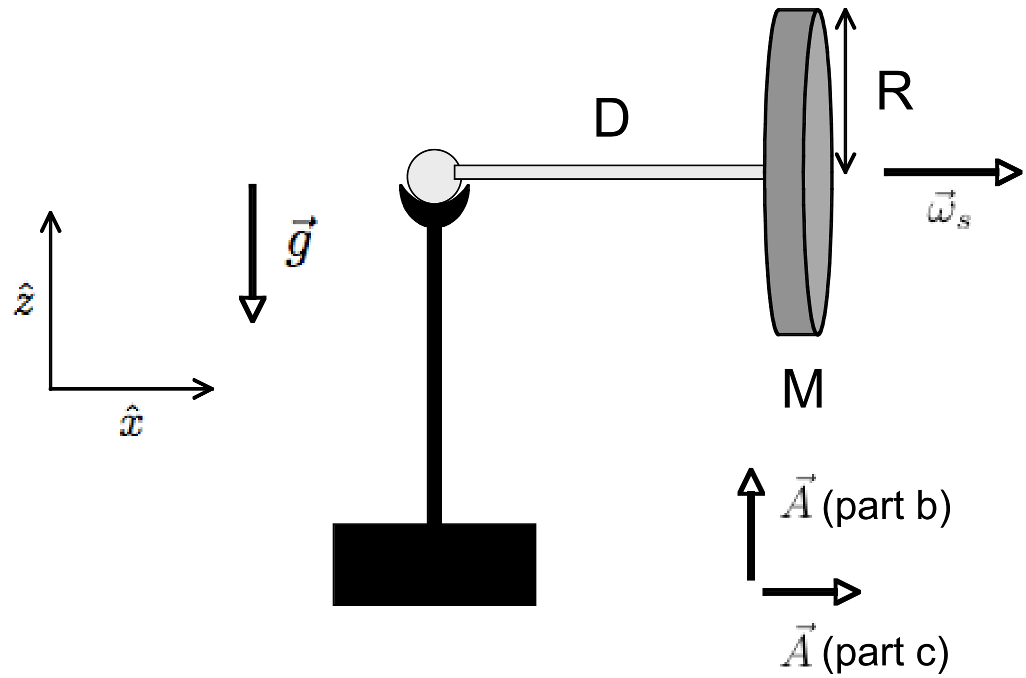
Problem 7: Gravitational Focusing in an Asteroid Field [10 pts]




A small asteroid of mass m moves with speed v when it is far away from a large planet of mass M. The "impact parameter", b, is the distance between the centers of mass of the planet and asteroid perpendicular to the initial velocity vector of the asteroid (see figure). As the asteroid passes by the planet, its path is deflected by the gravitational force acting between the two bodies. Assume that M >> m, so that the planet does not move appreciably as a result of a single interaction with an asteroid.

(a) [5 pts] What is the minimum impact parameter $b_{min}$ that allows the asteroid to miss the planet? Hint: in this case the asteroid just grazes the planet surface.

(b) [5 pts] *(Challenging)* Now assume that the planet moves with velocity v through a space that is filled with many small asteroids of mass m that are distributed uniformly with average number density N (this parameter has dimension $[L]^{-3}$) and are initially at rest. Any asteroid that collides with the planet sticks to it and adds to the planet's total mass. Determine the rate at which the planet gains mass (dM/dt) as it moves through the asteroid field as a function of v. You may express your answer in terms of $b_{min}$. Note: this is an important problem in understanding the formation of planets like the Earth, Problem 8: The Accelerated Gyroscope [15 pts]



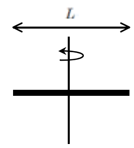

A gyroscope wheel consists of a uniform disk of mass M and radius R that is spinning at a large angular rotation rate $\omega_s$. The gyroscope wheel is mounted onto a ball-and-socket pivot by a rod of length D that has negligible mass, allowing the gyroscope to precess over a wide range of directions. Constant gravitational acceleration g acts downward. For this problem, ignore both friction and nutational motion; i.e, assume the gyroscope only precesses uniformly. For all parts, express your solution as a vector (magnitude and direction) with components in the coordinate system shown above.

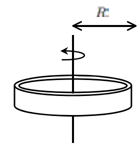
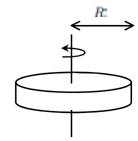
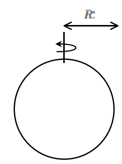
(a) [5 pts] Calculate the total angular momentum vector of the uniformly precessing gyroscope in the orientation show above; i.e., the total of the spin and precession angular momentum vectors.

(b) [5 pts] The pivot mount is accelerated upward with magnitude A. Calculate the precession angular velocity vector in this case.

(c) [5 pts] *(Challenging)* The pivot mount is accelerated toward the right with magnitude A. Calculate the precession angular velocity vector in this case.

# USEFUL EQUATIONS

| | |
|---|---|
| Velocity in polar coordinates | $\vec{\dot{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$ |
| Acceleration in polar coordinates | $\vec{\ddot{r}} = \ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta} + 2\dot{r}\dot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$ |
| Center of mass of a rigid body | $\vec{R} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{1}{M}\int \rho \vec{r} dV$ |
| Volume element in cylindrical coordinates | $dV = r dr d\theta dz$ |
| Kinetic energy | $K = \frac{1}{2}Mv^2 + \frac{1}{2}\vec{\omega} \cdot \bar{I} \cdot \vec{\omega}$ |
| Work | $W = -\int \vec{F} \cdot d\vec{r} = -\int \vec{\tau} \cdot d\vec{\theta}$ |
| Potential Energy (for conservative forces) | $U = -\int \vec{F}_c \cdot d\vec{r}$ where $\vec{\nabla} \times \vec{F}_c = 0$ |
| Angular momentum | $\vec{L} = \vec{r} \times \vec{p} = \bar{I} \cdot \vec{\omega}$ |
| Torque | $\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$ Fixed axis rotation: $\tau_z = I_{zz}\dot{\omega}$ |
| Moment of inertia for a uniform bar (about COM) |   $I_{zz} = \frac{1}{12}ML^2$ |

| | |
|---|---|
| Moment of inertia for a uniform hoop (about COM) |  $I_{zz} = MR^2$ |
| Moment of inertia for a uniform disk (about COM) |  $I_{zz} = \frac{1}{2}MR^2$ |
| Moment of inertia for a uniform sphere (about COM) |  $I_{zz} = \frac{2}{5}MR^2$ |
| Scalar parallel axis theorem | $I = I_{COM} + MR^2$ |
| Velocity from rotation | $\vec{v} = \vec{\omega} \times \vec{r}$ |
| Moments of inertia tensor (permute x→y→z) | $I_{xx} = \sum_i m_i(y_i^2 + z_i^2) = \int dV \rho(y^2 + z^2)$ |
| Products of inertia tensor (permute x→y→z) | $I_{xy} = -\sum_i m_i x_i y_i = -\int dV \rho xy$ |
| Fictitious force in an accelerating frame | $\vec{F}_f = -m\vec{A}$ |
| Fictitious force in a rotating frame | $\vec{F}_f = -2m\vec{\Omega} \times \vec{v}_{rot} - m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ |
| Time derivative between inertial and rotating frames | $\frac{d\vec{B}}{dt}_{(in)} = \frac{d\vec{B}}{dt}_{(rot)} + \vec{\Omega} \times \vec{B}$ |
| Taylor Expansion of f(x) | $f(x) = f(a) + \frac{1}{1!}\frac{df}{dx}\vert_a \ (x-a) + \frac{1}{2!}\frac{d^2 f}{dx^2}\vert_a \ (x-a)^2 + \cdots$ |