MIT OpenCourseWare
http://ocw.mit.edu

### 8.012 Physics I: Classical Mechanics

 Fall 2008For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

# MASSACHUSETTS INSTITUTE OF TECHNOLOGY <br> Department of Physics 

## Exam 3

## NAME:

$\qquad$

Instructions:

1. Do all FIVE (5) problems. You have $\mathbf{9 0}$ minutes.
2. SHOW ALL WORK. Be sure to circle your final answer.
3. Read the questions carefully.
4. All work must be done in this booklet in workspace provided. Extra blank pages are provided at the end if needed.
5. NO books, notes, calculators or computers are permitted. A sheet of useful equations is provided on the last page.

> Your Scores

| Problem | Maximum | Score | Grader |
| :---: | :---: | :---: | :---: |
| 1 | 10 |  |  |
| 2 | 20 |  |  |
| 3 | 25 |  |  |
| 4 | 25 |  |  |
| 5 | 20 |  |  |
| Total | 100 |  |  |

## Problem 1: Quick Multiple Choice Questions [10 pts]

For each of the following questions circle the correct answer. You do not need to show any work.
(a) [2 pts] A bicycle rider pedals up a hill with constant velocity v . In which direction does friction act on the wheels?


Uphill Downhill
Friction does not act while the bike moves at constant velocity
(b) [2 pts] A gyroscope is set to spin so that its spin vector points back to the mount. The gryoscope is initially set at an angle $\alpha$ below horizontal and released from rest. Gravity acts downward. In which direction does the precession angular velocity vector point once
 the gyroscope starts to precess?

Up Down Into the page Out of the page
(c) [2 pts] A ball attached to rope is twirled around a stick as shown in the diagram at right. Ignore gravity and friction. Which of the following quantities is conserved in the motion of the ball? Be sure to write down all of the choices below that apply.


Momentum

None of these are conserved
(d) [2 pts] A force F is applied to two identical dumbbells, each comprised of two masses separated by a thin rod. The force is applied at the center of mass for the left dumbbell and directly on one of the masses for the right dumbbell. If the forces are applied for the same duration of
 time, which dumbbell acquires the greatest center of mass velocity?

They both have the same velocity
(e) [2 pts] For the same dumbbells in part (d), which one acquires the greatest kinetic energy from the applied force?

Left dumbbell Right dumbbell

They both have the same kinetic energy

## Problem 2: Catch Pendulum [20 pts]



A pendulum consists of a ball of mass M attached to the end of a rigid bar of length 2 d which is pivoted at the center. At the other end of the bar is a container ("catch"). A second ball of mass $\mathrm{M} / 2$ is thrown into the catch at a velocity v where it sticks. For this problem, ignore the mass of the pendulum bar and catch, and treat the balls as if they were point masses (i.e., neglect their individual moments of inertia).
(a) [5 pts] What is the initial angular rotation rate of the pendulum after the incoming ball is caught?
(b) [ 5 pts$]$ How much total mechanical energy is lost when the incoming ball gets stuck in the catch?
(c) [10 pts] What minimum velocity does the incoming ball need in order to invert the pendulum (i.e., rotate it by $180^{\circ}$ )?

## Problem 3: Swinging bar [25 pts]



A uniform bar of mass M and length d is pivoted at one end. The bar is released from rest in a horizontal position and allowed to fall under constant gravitational acceleration.
(a) [5 pts] How much work does the contact force apply to the system as a function of angle?
(b) [ 5 pts$]$ What is the angular speed of the bar as a function of angle?
(c) [5 pts] What is the angular acceleration of the bar as a function of angle?
(d) [10 pts] What are the vertical and horizontal forces the bar exerts on the pivot as a function of time angle?

Problem 4: Atwood machine [25 pts]


An Atwood machine consists of a fixed pulley wheel of radius R, a rim of mass M and 6 spokes each with length $R$ and mass $M$. An effectively massless string passes around the pulley wheel and connects two plates of mass M and 2 M . The lighter plate is initially on the ground when the system is released from rest. Constant gravitational force acts downwards, and assume that the string never slips.
(a) [5 pts] Calculate the moment of inertia of the pulley wheel.
(b) [10 pts] Calculate the speed of the more massive plate when it hits the ground (there are multiple ways to solve this).
(c) [10 pts] Assume that when it strikes the ground, the heavier plate sticks. Furthermore, the string remains "stuck" to the wheel, so it continues to pull the lighter plate up. How high does the lighter plate go?

## Problem 5: Rolling a coin [20 pts]



A coin (uniform solid circular disk) with mass M and radius b is set to roll in a circular path of radius R on a table surface $(\mathrm{R}>\mathrm{b})$. The coin is given a spin angular velocity $\omega_{\mathrm{s}}$, and as it rolls it is found to tilt at a small angle $\alpha$ with respect to vertical. Assume that the coin does not slip; constant gravity acts downward.
(a) [5 pts] Determine the "precession" angular velocity of the rolling motion, $\Omega$, both magnitude and direction.
(b) [5 pts] Determine the total angular velocity vector, $\vec{\omega}$, including both spin and precession terms, in a polar coordinate system centered on the center of the coin's trajectory.
(c) $[10 \mathrm{pts}]$ Solve for the angle $\alpha$ in terms of the quantities given in the diagram above. Be sure to examine all equations of motion!
(d) [10 pts BONUS] Determine the total angular momentum vector, $\vec{L}$, in terms of the quantities given in the diagram above, in a polar coordinate system centered on the center of the coin's trajectory. For simplicity, assume that $\alpha$ is very small (i.e., only consider first order terms in $\alpha$ ).

