Now that we have all the pieces in place, we'd like to apply the momentum principle to the rocket problem.

Recall that the momentum principle is that the external force on the rocket causes-- on the system-- causes the momentum of the system to change.

And the fundamental definition of a derivative is to look at the change in momentum between sometimes t plus delta t-- our two states that we've identified-- divided by delta t.

Now recall that we had the momentum of the system at time t was the mass of the rocket times d-bar of the rocket a t.

I'll just quickly show mass of the rocket V-rt.

And we had the system at time t plus delta t where we had delta M-fuel equals minus delta M-rocket Here we had M-r plus delta M. And we had the velocity in the ground frame, which we saw was equal to-- so let's write out the momentum of the system at time t plus delta t.

That was a little bit longer.

p system at t plus delta t had two pieces.

It had M-r plus delta M-r times V of r of t plus delta t.

And we were subtracting-- now, because we made this change, that's minus then delta M-r u plus V-bar of t plus delta t.

And this, recall, was the velocity of the fuel.

Now we're in position to apply our momentum principle, because we have expressions for the momentum of the system at time t and at time t and at time t plus delta t.

So now this will be a big expression.

So we'll write external force is the limit as delta t goes to 0.

Now here, our first term, is of M of r plus delta M-r times the velocity of the rocket at time t plus delta t.

And now we have the fuel term, minus delta M-r times u plus V of r of t plus delta t.

And we have to subtract from that and I'll indicate that with a slightly different color.

M-r V of r of t.

And the whole thing, we're dividing by delta t.

Now let's look at this expression first because there is some very nice simplifications.

The first thing we can see, let's look at this term delta M-r times V of r.

Notice we have minus delta M-r V of r here.

So those two terms cancel.

And we're just left with three other terms and let's write them out.

So now we have the limit as delta t goes to 0.

And I'm going to combine terms in the following way.

M-r times V of r of t plus delta t.

And over here, I have M-r minus V of r.

So I have a minus V of r of t divided by delta t.

And now I have one more term here.

And I'm going to write this as minus the limit as delta t goes to 0 of delta M-r over delta t.

Remember, this term cancelled.

We only have u, the speed of the fuel relative to the rocket.

And in both cases-- this is our first term and here's our second term.

Let's look at these limits.

Notice in here, we're just taking V of r of t plus delta t minus V of r-t divided by delta t.

And that's precisely the definition of the derivative of the velocity of the rocket.

So this first term, our expression becomes the external force is equal to the mass of our system times V of r, the derivative of the velocity of the rocket.

And the second term, this is the rate that the fuel is changing in the rocket.

This is rather the rate that the mass of the rocket is changing.

So that's the derivative dM-r dt times the relative velocity of the fuel with respect to the rocket.

So this equation here-- I'll box it off-- is called the rocket equation, which we've derived from the momentum principle using our momentum diagrams.

And you can see that this is what people refer to as rocket science.