Class 3 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1.
Toss a coin 4 times. Let $A =$ 'at least three heads' and $B =$ 'first toss is tails'.

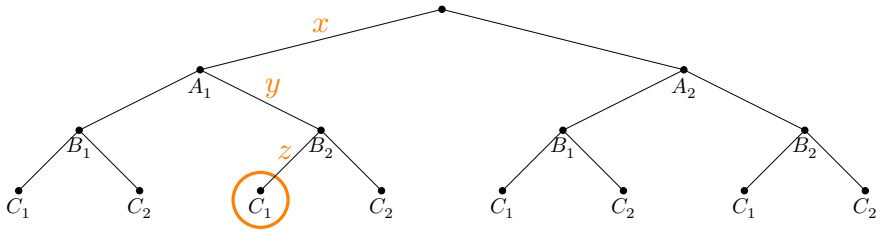
1. What is $P(A|B)$?

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/5

2. What is $P(B|A)$?

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/5

Concept question 2. Trees 1.



1. The probability x represents

(a) $P(A_1)$ (b) $P(A_1|B_2)$ (c) $P(B_2|A_1)$ (d) $P(C_1|B_2 \cap A_1)$.

Concept question 3. Trees 2.
2. The probability y represents

(a) $P(B_2)$ (b) $P(A_1|B_2)$ (c) $P(B_2|A_1)$ (d) $P(C_1|B_2 \cap A_1)$.

Concept question 4. Trees 3.
3. The probability z represents

(a) $P(C_1)$ (b) $P(B_2|C_1)$ (c) $P(C_1|B_2)$ (d) $P(C_1|B_2 \cap A_1)$.

Concept question 5. Trees 4.
4. The circled node represents the event

(a) C_1 (b) $B_2 \cap C_1$ (c) $A_1 \cap B_2 \cap C_1$ (d) $C_1|B_2 \cap A_1$.

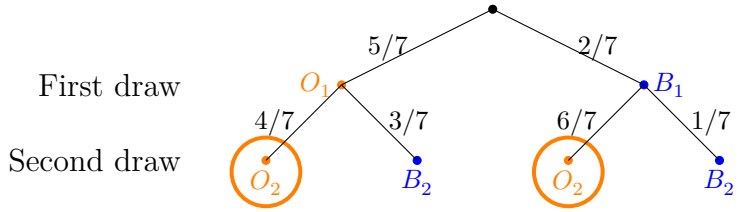
In class examples

Class example 1.

- Organize computations
- Compute total probability
- Compute Bayes' formula

Example. Game: 5 orange and 2 blue balls in an urn. A random ball is selected and replaced by a ball of the other color; then a second ball is drawn.

1. What is the probability the second ball is orange?
2. What is the probability the first ball was orange given the second ball was orange?



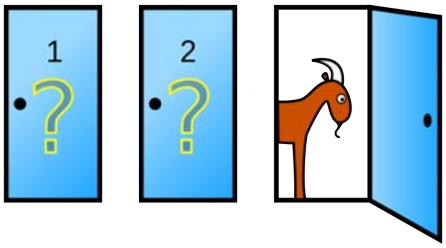
Board questions

Problem 1. Monty Hall

- One door hides a car, two hide goats.
- The contestant chooses any door.
- Monty always opens a different door with a goat. (He can do this because he knows where the car is.)
- The contestant is then allowed to switch doors if they want.

What is the best strategy for winning a car?

(a) Switch (b) Don't switch (c) It doesn't matter



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Organize the Monty Hall problem into a tree and compute the probability of winning if you always switch.

Hint first break the game into a sequence of actions.

Problem 2. Independence

Roll two dice and consider the following events

- A = 'first die is 3'
- B = 'sum is 6'
- C = 'sum is 7'

A is independent of

(a) B and C (b) B alone (c) C alone (d) Neither B or C.

Problem 3. Evil Squirrels

Of the one million squirrels on MIT's campus most are good-natured. But one hundred of them are pure evil! An enterprising student in Course 6 develops an "Evil Squirrel Alarm" which they offer to sell to MIT for a passing grade. MIT decides to test the reliability of the alarm by conducting trials.

- When presented with an evil squirrel, the alarm goes off 99% of the time.
- When presented with a good-natured squirrel, the alarm goes off 1% of the time.

(a) If a squirrel sets off the alarm, what is the probability that it is evil?

(b) Should MIT co-opt the patent rights and employ the system?

Problem 4. Dice Game

1. The Randomizer holds the 6-sided die in one fist and the 8-sided die in the other.
2. The Roller selects one of the Randomizer's fists and covertly takes the die.
3. The Roller rolls the die in secret and reports the result to the table.

Given the reported number, what is the probability that the 6-sided die was chosen? (Find the probability for each possible reported number.)

18.05 Introduction to Probability and Statistics
Spring 2022