Class 4 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1.

Suppose X a random variable with the CDF shown.

values of X:	1	3	5	7
cdf $F(a)$:	0.5	0.75	0.9	1

What is $P(X \leq 3)$?

(a) 0.15 (b) 0.25 (c) 0.5 (d) 0.75

What is $P(X = 3)$? Same distribution as above

(a) 0.15 (b) 0.25 (c) 0.5 (d) 0.75

Board questions

Problem 1. Computing expectation

Suppose X is a random variable with the following pmf.

X:	1	2	3
pmf:	1/4	1/2	1/4

Find $E[X]$ and $E[1/X]$.

Problem 2. Interpreting expectation

(a) Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

(b) Would you pay \$5 to participate in a lottery that offers a 10% percent chance to win \$100 and a 90% chance to win nothing?

Hint: find the expected value of your winnings in each case.

Problem 3. Musical chairs or linearity of expectation

Suppose that there are n people at your table and everyone got up, ran around the room, and sat back down randomly (i.e., all seating arrangements are equally likely).

What is the expected value of the number of people sitting in their original seat?

Problem 4. Bernoulli

(a) Suppose $X \sim \text{Bernoulli}(p)$. Find $E[X]$.

(This is important! Remember it!)

(b) Suppose $Y = X_1 + X_2 + ... + X_{12}$, where each $X_i \sim \text{Bernoulli}(0.25)$. Find $E[Y]$.

Problem 5. Don't let one failure stop you

Let $X = \#$ of successes before the *second* failure of a sequence of independent Bernoulli(p) trials. Find the pmf of X.

Hint: this requires a small amount of counting.

In class examples and discussion

Gambler's fallacy. [roulette]
Fallacy: If black comes up several times in a row then the next spin is more likely to be red.

Truth: P(red) remains the same. The roulette wheel spins are independent.

Hot hand
Theory: NBA players get 'hot'.

Data: The data show that player who has made 5 shots in a row is no more likely than usual to make the next shot.

Extra problems

Extra 1. Suppose $X \sim \text{Binomial}(n, p)$, i.e. the number of successes in n, independent Bernoulli(p) trials. Explain why X is the sum of n Bernoulli(p) random variables.