

Name _____

18.05 Exam 2

5 part I (concept) questions, 5 part II problems. Do all your work on the paper provided

No books or calculators.

You may have one side of a 8.5×11 piece of paper with any information you like on it.

Probability tables: Use the probability tables at the end of this exam to do probability computations for normal, t and chi-square distributions.

Cheat Sheet

Problem 0. (5 pts) **Be sure to attach your cheat sheet to your test.**

Part I: Concept Questions

Answer the Part I questions on **directly on the exam pages**.

Problem I.1. (10 pts: 2,2,2,2,2) Determine which of the following concepts/statements are Frequentist and which are Bayesian. Note: a concept can be either or both. Give a short explanation for each answer.

(a) P-value

(b) Prior distribution

(c) Average of data

(d) There is a 3% probability that the average weight is between 92.3 mg and 100 mg.

(e) The odds in favor of H_0 against H_A are 1 to 3.

Problem I.2. (6 pts: 3,3) Suppose you run a significance test at significance level 0.025, and that the test has a power of 95%.

For each part of this problem, give your answer and a short explanation.

(a) Assuming the null hypothesis, what is the probability of a type I error?

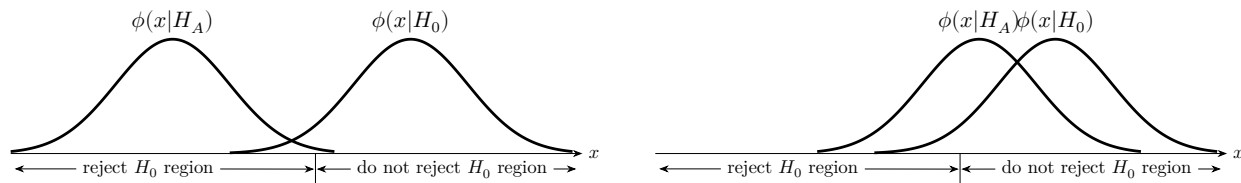
(i) 0.025 (ii) 0.05 (iii) 0.95 (iv) 0.975 (v) cannot be determined from the information given.

(b) Assuming the alternative hypothesis, what is the probability of a type II error?

(i) 0.025 (ii) 0.05 (iii) 0.95 (iv) 0.975 (v) cannot be determined from the information given.

Problem I.3. (3 pts) The following graphs show the rejection regions and pdfs of the null and alternative hypotheses for two different hypothesis tests. Which graph shows the test with the higher power?

Your answer should be 'left graph' or 'right graph'. Give a short explanation.



Problem I.4. (3 pts) You find a coin on the street, with some unknown probability θ of landing heads when tossed. Circle the only reasonable prior for θ . (No explanation needed.)

- (i) Uniform($[0, 0.5]$) (ii) Beta(2, 2) (iii) N(0.5, 0.25).

Problem I.5. (4 pts: 2,2) For each of the following: Is the prior conjugate to the given likelihood? In each case, a and b are parameters for the priors.

	hypothesis	data	prior	likelihood
(a)	$\theta \in [0, 1]$	x	$c_1 \theta^a (1 - \theta)^b$	$\binom{10}{x} \theta^x (1 - \theta)^{10-x}$
(b)	$\lambda \in [0, \infty)$	x	$c_1 \lambda^{a-1} e^{-b\lambda}$	$\lambda e^{-\lambda x}$

Part II: Problems

Answer the Part II questions **on the paper provided**. You must show your work for credit.

Problem II.1. (12 pts.)

The gamma distribution with shape parameter 3 and unknown rate parameter β has range $(0, \infty)$ and pdf

$$f(x) = \frac{\beta^3 x^2}{2} e^{-\beta x}.$$

Suppose the data

$$1, 1, 2, 3, 5$$

was drawn independently from such a distribution. Find the maximum likelihood estimate (MLE) of β .

(Note: we shouldn't expect to get integer values for the data. So either our measurements were quite crude or we didn't want you to have to do arithmetic with fractions.)

Problem II.2. (15 pts)

A random process produces outcomes labeled A , B and C with probabilities $\theta/2$, $\theta/2$, $1 - \theta$ respectively. Here θ is an unknown parameter with value between 0 and 1. You want to know the value of θ .

Before running any experiments you have a prior pdf for θ of $f(\theta) = 3\theta^2$. You then run the process five times producing data A, B, C, A, B .

Find the posterior probability density for θ .

Problem II.3. (12 pts.) It is the year 2122 and a small percentage of children are born with an array of superpowers. The usual super strength and ability to make bad jokes in the direst of dire situations won't manifest themselves till puberty. The one superpower that manifests at age 7 is known as Bayesian intelligence. They can think clearly about statistics and can answer virtually any stats question.

So, a screening test was developed that asks 7 year-olds to compute the posterior odds that a 7 year-old who correctly answers the screening question has superpowers. The test is quite accurate, but some ordinary children do answer correctly

Suppose the odds that a random 7 year-old has superpowers is $1/100$. Suppose the screening test has a 100% true positive rate and a 10% false positive rate. If a randomly chosen child correctly answers the question, what are the posterior odds that they have superpowers?

Problem II.4. (15 pts: 3,3,3,3,3)

You collect data from an experiment and do a one-sided Z -test with the rejection region in the right tail and significance level 0.1. You find the Z -value is 2.

(a) Which R code computes the critical value for the rejection region?

- (i) `pnorm(0.1, 0, 1)` (ii) `pnorm(0.9, 0, 1)` (iii) `pnorm(0.95, 0, 1)`
(iv) `pnorm(2.0, 0, 1)` (v) `1 - pnorm(2.0, 0, 1)` (vi) `qnorm(0.05, 0, 1)`
(vii) `qnorm(0.1, 0, 1)` (viii) `qnorm(0.9, 0, 1)`
(ix) `qnorm(0.95, 0, 1)`

(b) Using the tables at the end of the exam, compute this critical value.

(c) Which R code computes the p -value for this experiment?

- (i) `pnorm(0.1, 0, 1)` (ii) `pnorm(0.9, 0, 1)` (iii) `pnorm(0.95, 0, 1)`
(iv) `pnorm(2.0, 0, 1)` (v) `1 - pnorm(2.0, 0, 1)` (vi) `qnorm(0.05, 0, 1)`
(vii) `qnorm(0.1, 0, 1)` (viii) `qnorm(0.9, 0, 1)`
(ix) `qnorm(0.95, 0, 1)`

(d) Using the tables at the end of the exam, compute the p -value.

(e) Should you reject the null hypothesis?

Problem II.5. (15 pts)

Adult onset diabetes is known to be highly genetically determined. A study was done comparing frequencies of a particular allele in a sample of diabetics and a sample of nondiabetics. The data are shown in the following table. (We adjusted the data to make hand calculation easier.)

	Diabetic	Normal
Bb or bb	15	5
BB	35	45

Do a significance test for whether the frequencies of the alleles is different in the two groups at a significance level of 0.05.

Extra Paper

Standard normal table of left tail probabilities.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.00	0.0000	-2.00	0.0228	0.00	0.5000	2.00	0.9772
-3.95	0.0000	-1.95	0.0256	0.05	0.5199	2.05	0.9798
-3.90	0.0000	-1.90	0.0287	0.10	0.5398	2.10	0.9821
-3.85	0.0001	-1.85	0.0322	0.15	0.5596	2.15	0.9842
-3.80	0.0001	-1.80	0.0359	0.20	0.5793	2.20	0.9861
-3.75	0.0001	-1.75	0.0401	0.25	0.5987	2.25	0.9878
-3.70	0.0001	-1.70	0.0446	0.30	0.6179	2.30	0.9893
-3.65	0.0001	-1.65	0.0495	0.35	0.6368	2.35	0.9906
-3.60	0.0002	-1.60	0.0548	0.40	0.6554	2.40	0.9918
-3.55	0.0002	-1.55	0.0606	0.45	0.6736	2.45	0.9929
-3.50	0.0002	-1.50	0.0668	0.50	0.6915	2.50	0.9938
-3.45	0.0003	-1.45	0.0735	0.55	0.7088	2.55	0.9946
-3.40	0.0003	-1.40	0.0808	0.60	0.7257	2.60	0.9953
-3.35	0.0004	-1.35	0.0885	0.65	0.7422	2.65	0.9960
-3.30	0.0005	-1.30	0.0968	0.70	0.7580	2.70	0.9965
-3.25	0.0006	-1.25	0.1056	0.75	0.7734	2.75	0.9970
-3.20	0.0007	-1.20	0.1151	0.80	0.7881	2.80	0.9974
-3.15	0.0008	-1.15	0.1251	0.85	0.8023	2.85	0.9978
-3.10	0.0010	-1.10	0.1357	0.90	0.8159	2.90	0.9981
-3.05	0.0011	-1.05	0.1469	0.95	0.8289	2.95	0.9984
-3.00	0.0013	-1.00	0.1587	1.00	0.8413	3.00	0.9987
-2.95	0.0016	-0.95	0.1711	1.05	0.8531	3.05	0.9989
-2.90	0.0019	-0.90	0.1841	1.10	0.8643	3.10	0.9990
-2.85	0.0022	-0.85	0.1977	1.15	0.8749	3.15	0.9992
-2.80	0.0026	-0.80	0.2119	1.20	0.8849	3.20	0.9993
-2.75	0.0030	-0.75	0.2266	1.25	0.8944	3.25	0.9994
-2.70	0.0035	-0.70	0.2420	1.30	0.9032	3.30	0.9995
-2.65	0.0040	-0.65	0.2578	1.35	0.9115	3.35	0.9996
-2.60	0.0047	-0.60	0.2743	1.40	0.9192	3.40	0.9997
-2.55	0.0054	-0.55	0.2912	1.45	0.9265	3.45	0.9997
-2.50	0.0062	-0.50	0.3085	1.50	0.9332	3.50	0.9998
-2.45	0.0071	-0.45	0.3264	1.55	0.9394	3.55	0.9998
-2.40	0.0082	-0.40	0.3446	1.60	0.9452	3.60	0.9998
-2.35	0.0094	-0.35	0.3632	1.65	0.9505	3.65	0.9999
-2.30	0.0107	-0.30	0.3821	1.70	0.9554	3.70	0.9999
-2.25	0.0122	-0.25	0.4013	1.75	0.9599	3.75	0.9999
-2.20	0.0139	-0.20	0.4207	1.80	0.9641	3.80	0.9999
-2.15	0.0158	-0.15	0.4404	1.85	0.9678	3.85	0.9999
-2.10	0.0179	-0.10	0.4602	1.90	0.9713	3.90	1.0000
-2.05	0.0202	-0.05	0.4801	1.95	0.9744	3.95	1.0000

$\Phi(z) = P(Z \leq z)$ for $N(0, 1)$.

(Use interpolation to estimate z values to a 3rd decimal place.)

Table of Student t critical values (right-tail)

The table shows $t_{df,p}$ = the $1 - p$ quantile of $t(df)$.

We only give values for $p \leq 0.5$. Use symmetry to find the values for $p > 0.5$, e.g.

$$t_{5,0.975} = -t_{5,0.025}$$

In R notation $t_{df,p} = \text{qt}(1-p, df)$.

df \ p	0.005	0.010	0.015	0.020	0.025	0.030	0.040	0.050	0.100	0.200	0.300	0.400	0.500
1	63.66	31.82	21.20	15.89	12.71	10.58	7.92	6.31	3.08	1.38	0.73	0.32	0.00
2	9.92	6.96	5.64	4.85	4.30	3.90	3.32	2.92	1.89	1.06	0.62	0.29	0.00
3	5.84	4.54	3.90	3.48	3.18	2.95	2.61	2.35	1.64	0.98	0.58	0.28	0.00
4	4.60	3.75	3.30	3.00	2.78	2.60	2.33	2.13	1.53	0.94	0.57	0.27	0.00
5	4.03	3.36	3.00	2.76	2.57	2.42	2.19	2.02	1.48	0.92	0.56	0.27	0.00
6	3.71	3.14	2.83	2.61	2.45	2.31	2.10	1.94	1.44	0.91	0.55	0.26	0.00
7	3.50	3.00	2.71	2.52	2.36	2.24	2.05	1.89	1.41	0.90	0.55	0.26	0.00
8	3.36	2.90	2.63	2.45	2.31	2.19	2.00	1.86	1.40	0.89	0.55	0.26	0.00
9	3.25	2.82	2.57	2.40	2.26	2.15	1.97	1.83	1.38	0.88	0.54	0.26	0.00
10	3.17	2.76	2.53	2.36	2.23	2.12	1.95	1.81	1.37	0.88	0.54	0.26	0.00
16	2.92	2.58	2.38	2.24	2.12	2.02	1.87	1.75	1.34	0.86	0.54	0.26	0.00
17	2.90	2.57	2.37	2.22	2.11	2.02	1.86	1.74	1.33	0.86	0.53	0.26	0.00
18	2.88	2.55	2.36	2.21	2.10	2.01	1.86	1.73	1.33	0.86	0.53	0.26	0.00
19	2.86	2.54	2.35	2.20	2.09	2.00	1.85	1.73	1.33	0.86	0.53	0.26	0.00
20	2.85	2.53	2.34	2.20	2.09	1.99	1.84	1.72	1.33	0.86	0.53	0.26	0.00
21	2.83	2.52	2.33	2.19	2.08	1.99	1.84	1.72	1.32	0.86	0.53	0.26	0.00
22	2.82	2.51	2.32	2.18	2.07	1.98	1.84	1.72	1.32	0.86	0.53	0.26	0.00
23	2.81	2.50	2.31	2.18	2.07	1.98	1.83	1.71	1.32	0.86	0.53	0.26	0.00
24	2.80	2.49	2.31	2.17	2.06	1.97	1.83	1.71	1.32	0.86	0.53	0.26	0.00
25	2.79	2.49	2.30	2.17	2.06	1.97	1.82	1.71	1.32	0.86	0.53	0.26	0.00
30	2.75	2.46	2.28	2.15	2.04	1.95	1.81	1.70	1.31	0.85	0.53	0.26	0.00
31	2.74	2.45	2.27	2.14	2.04	1.95	1.81	1.70	1.31	0.85	0.53	0.26	0.00
32	2.74	2.45	2.27	2.14	2.04	1.95	1.81	1.69	1.31	0.85	0.53	0.26	0.00
33	2.73	2.44	2.27	2.14	2.03	1.95	1.81	1.69	1.31	0.85	0.53	0.26	0.00
34	2.73	2.44	2.27	2.14	2.03	1.95	1.80	1.69	1.31	0.85	0.53	0.26	0.00
35	2.72	2.44	2.26	2.13	2.03	1.94	1.80	1.69	1.31	0.85	0.53	0.26	0.00
40	2.70	2.42	2.25	2.12	2.02	1.94	1.80	1.68	1.30	0.85	0.53	0.26	0.00
41	2.70	2.42	2.25	2.12	2.02	1.93	1.80	1.68	1.30	0.85	0.53	0.25	0.00
42	2.70	2.42	2.25	2.12	2.02	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
43	2.70	2.42	2.24	2.12	2.02	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
44	2.69	2.41	2.24	2.12	2.02	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
45	2.69	2.41	2.24	2.12	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
46	2.69	2.41	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
47	2.68	2.41	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
48	2.68	2.41	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
49	2.68	2.40	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00

Table of χ^2 critical values (right-tail)

The table shows $c_{df,p}$ = the $1 - p$ quantile of $\chi^2(df)$.

In R notation $c_{df,p} = \text{qchisq}(1-p, df)$.

df\p	0.010	0.025	0.050	0.100	0.200	0.300	0.500	0.700	0.800	0.900	0.950	0.975	0.990
1	6.63	5.02	3.84	2.71	1.64	1.07	0.45	0.15	0.06	0.02	0.00	0.00	0.00
2	9.21	7.38	5.99	4.61	3.22	2.41	1.39	0.71	0.45	0.21	0.10	0.05	0.02
3	11.34	9.35	7.81	6.25	4.64	3.66	2.37	1.42	1.01	0.58	0.35	0.22	0.11
4	13.28	11.14	9.49	7.78	5.99	4.88	3.36	2.19	1.65	1.06	0.71	0.48	0.30
5	15.09	12.83	11.07	9.24	7.29	6.06	4.35	3.00	2.34	1.61	1.15	0.83	0.55
6	16.81	14.45	12.59	10.64	8.56	7.23	5.35	3.83	3.07	2.20	1.64	1.24	0.87
7	18.48	16.01	14.07	12.02	9.80	8.38	6.35	4.67	3.82	2.83	2.17	1.69	1.24
8	20.09	17.53	15.51	13.36	11.03	9.52	7.34	5.53	4.59	3.49	2.73	2.18	1.65
9	21.67	19.02	16.92	14.68	12.24	10.66	8.34	6.39	5.38	4.17	3.33	2.70	2.09
10	23.21	20.48	18.31	15.99	13.44	11.78	9.34	7.27	6.18	4.87	3.94	3.25	2.56
16	32.00	28.85	26.30	23.54	20.47	18.42	15.34	12.62	11.15	9.31	7.96	6.91	5.81
17	33.41	30.19	27.59	24.77	21.61	19.51	16.34	13.53	12.00	10.09	8.67	7.56	6.41
18	34.81	31.53	28.87	25.99	22.76	20.60	17.34	14.44	12.86	10.86	9.39	8.23	7.01
19	36.19	32.85	30.14	27.20	23.90	21.69	18.34	15.35	13.72	11.65	10.12	8.91	7.63
20	37.57	34.17	31.41	28.41	25.04	22.77	19.34	16.27	14.58	12.44	10.85	9.59	8.26
21	38.93	35.48	32.67	29.62	26.17	23.86	20.34	17.18	15.44	13.24	11.59	10.28	8.90
22	40.29	36.78	33.92	30.81	27.30	24.94	21.34	18.10	16.31	14.04	12.34	10.98	9.54
23	41.64	38.08	35.17	32.01	28.43	26.02	22.34	19.02	17.19	14.85	13.09	11.69	10.20
24	42.98	39.36	36.42	33.20	29.55	27.10	23.34	19.94	18.06	15.66	13.85	12.40	10.86
25	44.31	40.65	37.65	34.38	30.68	28.17	24.34	20.87	18.94	16.47	14.61	13.12	11.52
30	50.89	46.98	43.77	40.26	36.25	33.53	29.34	25.51	23.36	20.60	18.49	16.79	14.95
31	52.19	48.23	44.99	41.42	37.36	34.60	30.34	26.44	24.26	21.43	19.28	17.54	15.66
32	53.49	49.48	46.19	42.58	38.47	35.66	31.34	27.37	25.15	22.27	20.07	18.29	16.36
33	54.78	50.73	47.40	43.75	39.57	36.73	32.34	28.31	26.04	23.11	20.87	19.05	17.07
34	56.06	51.97	48.60	44.90	40.68	37.80	33.34	29.24	26.94	23.95	21.66	19.81	17.79
35	57.34	53.20	49.80	46.06	41.78	38.86	34.34	30.18	27.84	24.80	22.47	20.57	18.51
40	63.69	59.34	55.76	51.81	47.27	44.16	39.34	34.87	32.34	29.05	26.51	24.43	22.16
41	64.95	60.56	56.94	52.95	48.36	45.22	40.34	35.81	33.25	29.91	27.33	25.21	22.91
42	66.21	61.78	58.12	54.09	49.46	46.28	41.34	36.75	34.16	30.77	28.14	26.00	23.65
43	67.46	62.99	59.30	55.23	50.55	47.34	42.34	37.70	35.07	31.63	28.96	26.79	24.40
44	68.71	64.20	60.48	56.37	51.64	48.40	43.34	38.64	35.97	32.49	29.79	27.57	25.15
45	69.96	65.41	61.66	57.51	52.73	49.45	44.34	39.58	36.88	33.35	30.61	28.37	25.90
46	71.20	66.62	62.83	58.64	53.82	50.51	45.34	40.53	37.80	34.22	31.44	29.16	26.66
47	72.44	67.82	64.00	59.77	54.91	51.56	46.34	41.47	38.71	35.08	32.27	29.96	27.42
48	73.68	69.02	65.17	60.91	55.99	52.62	47.34	42.42	39.62	35.95	33.10	30.75	28.18
49	74.92	70.22	66.34	62.04	57.08	53.67	48.33	43.37	40.53	36.82	33.93	31.55	28.94

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18.05 Introduction to Probability and Statistics

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