Name ____________________

18.05 Final Exam Spring 2022

No calculators. You may bring one 8×11 cheat sheet.

Do your work directly on the exam.

Number of problems
18 concept questions, 13 problems

Test format
The test is divided into two parts. The first part is a series of concept questions. You don't need to show any work on this part. The second part consists of standard problems. You need to show your work on these.

Show your work
For the part II problems you must show your reasoning to get credit. If you need extra paper, we can provide some. Indicate clearly that your solution is continued on a separate page and write your name on the extra page.

Simplifying expressions
Unless asked to explicitly, you don't need to simplify complicated expressions. For example, you can leave $\frac{1}{4} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{5}$ exactly as is. Likewise for expressions like $\frac{20!}{18!2!}$.

Tables
There are z, t and χ^2 tables at the end of the exam.

Good luck!

Scores	
CQ.	______/70
1.	______/15
2.	______/16
3.	______/25
4.	______/20
5.	______/10
6.	______/15
7.	______/30
8.	______/20
9.	______/20
10.	______/20
11.	______/10
12.	______/20
13.	______/15
	______ /306

Part I: Concept questions (70 points)

These questions are all multiple choice or short answer. You don't have to show any work. Work through them quickly.

Concept 1. (4 pts.) Which of the following is a valid probability table?

(i)

outcome	1	1.5	2	2.5	3	3.5
probability	1/5	1/5	1/5	1/5	1/5	1/5

(ii)

outcome	red	blue	green	cyan	yellow
probability	4/10	2/10	0	1/10	3/10

Circle the best choice:

(i) (ii) (i) and (ii) neither (i) nor (ii)

Concept 2. (4 pts.) Suppose $P(A) + P(B) > 1$. Consider the following statements.

(i) $P(A \cup B) = 1$. (ii) $P(A \cap B) > 0$.

Which **must** be true? Circle the best choice below:

(i) (ii) (i) and (ii) neither (i) nor (ii).

Concept 3. (6 pts.) Circle **True** or **False** for each of the following.

(a) If A and B are independent then we must have $P(A \cap B) = P(A)P(B)$. **True** **False**

(b) If A and B are independent then we must have $P(A \cap B) = P(A) + P(B)$. **True** **False**

(c) If A and B are disjoint then A and B must be independent. **True** **False**

Concept 4. (4 pts.) You believe the MBTA subway arrives late by X hours, where X follows an exponential distribution with unknown parameter λ. To test your hypothesis, you record the lateness of 5 subway trains and get data $x_1, x_2, \dots, x_5$. Which of the following are statistics? Circle the correct answers.

(a) The expected value of a sample, namely $1/\lambda$.

(b) The sample average, $\overline{x} = (x_1 + x_2 + x_3 + x_4 + x_5)/5$.

(c) The difference between $\overline{x}$ and $1/\lambda$.

(d) The sample standard deviation.

Concept 5. (3 pts.) For each of the following, circle it if it is used in Bayesian inference.

(a) Likelihood function **(b)** prior odds **(c)** p-value

Concept 6. (4 pts.) Suppose $X \sim \text{Bernoulli}(\theta)$, where θ is unknown. Complete the following sentence using the words, "discrete," "continuous," or "neither discrete nor continuous."

The random variable is ____________________, the space of hypotheses is ____________________.

Concept 7. (2 pts.) A casino is considering installing a new slot machine. A player who wins is paid \$2 on a \$1 bet. The manufacturer claims that the probability of winning on any play of the slot machine is $p = 0.48$. Before using the machine the casino wants to make sure it will make them money. So they hire you to test the slot machine. Which of the following hypotheses would you use?

(i) $H_0 : p = 0.48$ vs $H_A : p \neq 0.48$

(ii) $H_0 : p = 0.48$ vs $H_A : p > 0.48$

(iii) $H_0 : p = 0.48$ vs $H_A : p < 0.48$

Circle the best answer: (i) (ii) (iii) Not enough information.

Concept 8. (6 pts.) The following are hypotheses considered in the previous problem. For each hypothesis circle all that apply.

(a) $H_0 : p = 0.48$ **Simple** **Composite** **Two-sided** **One-sided**

(b) $H_A : p \neq 0.48$ **Simple** **Composite** **Two-sided** **One-sided**

(c) $H_A : p > 0.48$ **Simple** **Composite** **Two-sided** **One-sided**

Concept 9. (5 pts.) Which of the following are **true** about p-values? Circle all that apply.

(a) The p-value gives the probability of making a type 1 error.

(b) The p-value is a measure of how extreme the observed data is.

(c) A p-value below the significance level allows us to conclude with certainty that the null hypothesis is false.

(d) The p-value is a frequentist concept.

(e) If the null hypothesis is true, then the p-value will always be larger than the significance level.

Concept 10. (8 pts.) A two-sample t-test for equal means of two populations has a p-value of 0.08.

Circle True or False for each of the following.

(a) For a significance level of 0.05, the null hypothesis of equal means should be rejected.

True **False**

(b) A 90% confidence interval for the difference of the means for the two populations includes 0.

True **False**

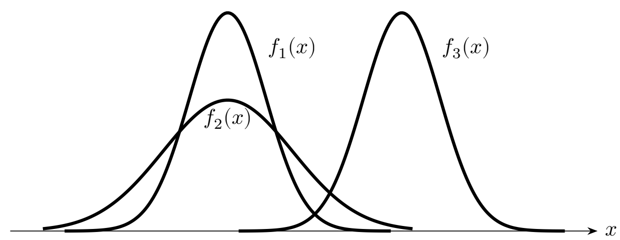
(c) A 95% confidence interval for the difference of the means for the two populations includes 0.

True **False**

(d) With probability 95% the actual value of the difference of the means is within the 95% t-confidence interval for the difference.

True **False**

Concept 11. (2 pts.) Data is drawn from a $\mathrm{N}(1, \sigma^2)$ distribution. Let $\overline{X}_5$ be the average of 5 data points and $\overline{X}_{10}$ the average of 10 data points. Three densities, f_1, f_2, f_3 are shown below. One is the pdf of $\overline{X}_5$, one is the pdf of $\overline{X}_{10}$ and one is another pdf. Circle the one that is the pdf of $\overline{X}_{10}$.



Concept 12. (4 pts.) Finish the following sentences with "a type I error", "a type II error", or "neither type of error".

(a) The rejection of a false null hypothesis is ____________

(b) The rejection of a true null hypothesis is ____________

Concept 13. (8 pts.) Suppose that the data x_1, x_2, ..., x_n are drawn from independent, identically distributed, random variables X_i with mean μ and standard deviation σ. Write $\overline{x} = (x_1 + \cdots + x_n)/n$ for the sample mean. The Central Limit Theorem states that: (circle all that apply)

(a) The distribution of each random variable X_i is approximately symmetric around the average μ.

(b) For large n, the distribution of the sample mean is approximately symmetric around the average μ.

(c) For large n, the average $\overline{x}$ approximately follows a normal distribution.

(d) For large n, $(\overline{x} - \mu)/\sigma$ approximately follows a standard normal distribution.

Concept 14. (2 pts.) Suppose for a certain endeavor θ is the probability of success. Suppose also that our prior for θ is Beta(5,5). We collect data from 30 trials, obtaining 20 successes and 10 failures. What is our posterior pdf $f(\theta|x)$? (Circle the best answer.)

(i) Binomial(30, 2/3)
(ii) Beta(25, 15)
(iii) Beta(20, 10)
(iv) None of the above

Concept 15. (2 pts.) Circle **True** or **False**.

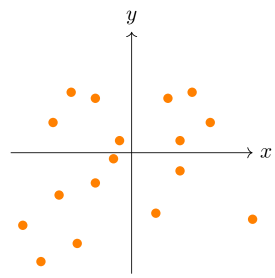
Let s be a statistic. If the theoretical distribution of the statistic is hard to compute, then it is not advisable to use bootstrapping to compute confidence intervals for the statistic.

True **False**

Concept 16. (2 pts.) Suppose we run a two-sample t-test for equal means with significance level $\alpha = 0.05$. If the data implies we should reject the null hypothesis, then the odds that the two samples come from distributions with the same mean are (circle the best answer)

(a) 19/1 **(b)** 1/19 **(c)** 20/1 **(d)** 1/20 **(e)** unknown

Concept 17. (2 pts.) For the bivariate data in the following scatter plot, is the correlation between x and y positive or negative? Circle your choice: **positive** **negative**



Concept 18. (2 pts.) Circle **True** or **False**

Linear regression can fit curves other than lines to given data.

True **False**

Part II: Problems (236 points)

Problem 1. (15: 5,5,5)
You roll a fair six-sided die 8 times.

(a) What is the probability that none of the 8 rolls is a six?

(b) What is the probability that exactly one of the 8 rolls is a six?

(c) What is the expected number of sixes in the 8 rolls?

Remember: you can leave numerical expressions unevaluated

Problem 2. (16: 4,4,4,4)
Suppose X is a random variable with values in [1,2] and density

$$f(x) = \begin{cases} kx^2 & \text{for } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a fixed constant.

(a) What is k?

(b) Find the cdf of X.

(c) Find $P(X < 3/2)$.

(d) Find $E[X]$

Problem 3. (25: 5,5,5,5,5)
Suppose random variables X and Y have units of dollars and:

$$E[X] = 5, \quad \mathrm{Var}(X) = 6^2, \qquad \text{and} \qquad E[Y] = 10, \quad \mathrm{Var}(Y) = 7^2.$$

Define $W = X + Y$.

(a) Find $E[W]$. What are the units of $E[W]$?

(b) Assume X and Y are independent. Find $\mathrm{Var}(W)$.

(c) Assume $\mathrm{Cov}(X, Y) = 21$. Find $\mathrm{Var}(W)$. (Note: this assumption differs from that in part (b), so $\mathrm{Var}(W)$ can also be different.)

(d) Assume $\mathrm{Cov}(X, Y) = 21$. Compute $\mathrm{Cor}(X, Y)$. What are the units of $\mathrm{Cor}(X, Y)$?

(e) Define $Z = \dfrac{(X - E[X])}{\sqrt{\mathrm{Var}(X)}} = (X - 5)/6$. Find $E[Z]$ and $\mathrm{Var}(Z)$.

Problem 4. (20: 10,5,5)
You roll two fair (6-sided) dice. If the sum of the dice is greater than 9, you win \$100. If the sum is 9 or less you get to roll again. On the second roll, if your sum of dice is greater than 9, you win \$50, otherwise you win nothing. Let X be the random variable of how much money you win by playing this game.

(a) Construct a probability model for X, i.e. make a probability table.

(b) What is the expected amount you will win playing the game?

(c) What would you be willing to pay to play this game? (Justify your answer).

Problem 5. (10)
The Pareto distribution is used in economics modeling. To keep it simple we'll use the Pareto distribution that takes values $x \geq 1$ and has pdf

$$f(x \mid \theta) = \theta x^{-\theta-1} \quad \text{for } x \geq 1.$$

It's defined whenever $\theta > 0$. Assume $x_1, \dots, x_n$ are n independent samples from a Pareto(θ) distribution, find the maximum likelihood estimate of θ.

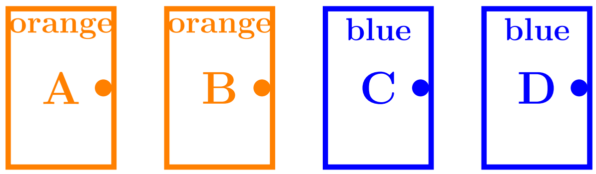
Problem 6. (15: 5,10)
A certain medical condition exists in 1% of the population. A screening test for the condition has a 4% false positive rate and a 0% false negative rate.

(a) What are the odds that a random person has the condition?

(b) Suppose a random person tests positive for the condition. What are the odds they have the condition?

Problem 7. (30: 5,10,5,10)
This question is about a robot, Bayz-E, who tries to figure out where it is located using Bayesian updating. Bayz-E is randomly placed in front of one of four doors, A, B, C, or D. At every time step, Bayz-E scans the color of the door in front of it. The outcome will be either orange of blue. However, its color scanner is not entirely accurate. Bayz-E scans the door's color correctly with a probability 0.7, and scans the color incorrectly with a probability 0.3.

Note. To avoid confusion for you, the color of the door is written above its letter. (Bayz-E has not yet learned to read, though its machine learning algorithm is working on it.)



(a) Bayz-E is placed in front of door A, B, C, or D at random (i.e. the probability of each door is the same). What is the prior probability it is in front of door B?

(b) Bayz-E scans the door color and detects "orange." What is the posterior probability it is in front of door B?

(c) After computing the posterior probabilities, Bayz-E moves to the next door to the right. (if it was at door D, it moves to door A). What is the (new) prior probability it is now in front of door C? (That is, after detecting orange in part (b) and after moving one to door to the right.)

(d) What is the predictive probability that the color scan of this door (after the move in part (c)) will detect "blue"?

Problem 8. (20)
According to the Mars website, each packet of Milk Chocolate M&M's should contain 20% blue, 20% brown, 20% green, 15% orange, 15% red, and 10% yellow M&M's.

Alessandre decides to test this claim. She buys 20 packets of Milk Chocolate M&M's. Each packet has 50 M&M's, so Alessandre has a total of 1000 M&M's. She counts each color and observes the following counts of M&M's.

	blue	brown	green	orange	red	yellow	total
Observed count	180	190	185	160	165	120	1000

Run a hypothesis test at the 0.05 significance level to test whether the published Mars color distribution is correct. Carefully state what you are testing and your conclusion.

(Write down the full numerical expression for your test statistic. In order to use the tables you will need to estimate the value of the test statistic. There is no need to compute it in full precision.)

Problem 9. (20: 5,10,5)
Jerry wants to brag to his non-MIT colleagues about how smart MIT students are. To give himself credibility, he decides to run a statistical test comparing the IQ scores of MIT students and Harvard students.

He collects IQ scores from 11 MIT students. The data has a sample mean of 115, with a sample standard deviation of 8.

He then collects IQ scores from 11 Harvard students. Their scores have a sample mean of 110, with a sample standard deviation of 6.

(a) Which test should he run to compare the IQ scores from the two schools? What assumptions will he need to make? What are the null and alternative hypotheses?

(b) Run the test with a significance level of $\alpha = 0.05$. Should Jerry reject the null hypothesis or not?

(In this problem there is some arithmetic. You will want to use $\sqrt{100/11} \approx \sqrt{9} = 3$.)

(c) Estimate the 95% confidence interval for the IQ of MIT students.

(Your answer should be a numerical expression. There is no need to work it out all the way to a decimal answer.)

Problem 10. (20: 10,10)
MIT has decided to form a new Department of Statistics and Probability. In a vote for the new head of this department, suppose 50% of the MIT population supports Sarah, 20% supports So Hee, and the remaining 30% is split evenly among Jerry, Jen, Alessandre and Gabe.

(a) A poll asks 100 random people whom they support. Estimate the probability that at least 45% of those polled support Sarah.

(b) A poll of n people reports that 53% $\pm$ 5% support Sarah at the 95% confidence level. What is the value of n?

Problem 11. (10)
You independently draw 100 data points from a $\mathrm{N}(\mu, 1)$ distribution, where μ is unknown. Suppose you test the null hypothesis $H_0 : \mu = 0$ against the alternative hypothesis $H_A : \mu \neq 0$ using a significance level of $\alpha = 0.05$. What is the power of the test for the alternative $H_A : \mu = 0.4$?

Problem 12. (20: 5,10,5)
Bivariate data $(4,1),(-2,1.5),(0,0.5)$ is assumed to arise from the model $y_i = b|x_i - 2| + e_i$, where b is a constant and e_i are independent random variables.

(a) What assumptions are needed on e_i so that it makes sense to do a least squares fit of a curve $y = b|x-2|$ to the data?

(b) Given the above data and the assumptions from part (a), determine the least squares estimate for b.

(c) Make a graph showing the data points and your least squares fit curve.

Problem 13. (15)
Data is collected on the time between trades at a stock exchange. We collect a data set of size 36 with sample mean $\bar{x} = 7.0$ and sample standard deviation $s = 0.84$.

Make no assumptions about the distribution of the data. By bootstrapping, we generate 500 bootstrap means $\overline{x}^*$. The smallest 50 and largest 50 are written in non-decreasing order below, e.g. the 12th smallest value is 6.672.

Use this data to find an 90% percentile bootstrap confidence interval for μ.

1- 10	6.466	6.506	6.509	6.515	6.578	6.597	6.618	6.635	6.653	6.664
11- 20	6.670	6.672	6.685	6.696	6.703	6.707	6.713	6.721	6.727	6.727
21- 30	6.729	6.731	6.738	6.738	6.740	6.743	6.744	6.745	6.751	6.752
31- 40	6.759	6.760	6.768	6.774	6.775	6.777	6.778	6.780	6.784	6.784
41- 50	6.787	6.789	6.789	6.790	6.791	6.791	6.792	6.796	6.798	6.800
451- 460	7.170	7.172	7.172	7.175	7.178	7.179	7.180	7.181	7.182	7.182
461- 470	7.182	7.186	7.195	7.202	7.202	7.205	7.206	7.210	7.216	7.219
471- 480	7.220	7.220	7.221	7.222	7.224	7.225	7.232	7.232	7.236	7.236
481- 490	7.243	7.244	7.245	7.251	7.253	7.258	7.261	7.263	7.266	7.273
491- 500	7.274	7.288	7.288	7.291	7.307	7.312	7.314	7.316	7.348	7.488

Extra Paper

Standard normal table of left tail probabilities.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.00	0.0000	-2.00	0.0228	0.00	0.5000	2.00	0.9772
-3.95	0.0000	-1.95	0.0256	0.05	0.5199	2.05	0.9798
-3.90	0.0000	-1.90	0.0287	0.10	0.5398	2.10	0.9821
-3.85	0.0001	-1.85	0.0322	0.15	0.5596	2.15	0.9842
-3.80	0.0001	-1.80	0.0359	0.20	0.5793	2.20	0.9861
-3.75	0.0001	-1.75	0.0401	0.25	0.5987	2.25	0.9878
-3.70	0.0001	-1.70	0.0446	0.30	0.6179	2.30	0.9893
-3.65	0.0001	-1.65	0.0495	0.35	0.6368	2.35	0.9906
-3.60	0.0002	-1.60	0.0548	0.40	0.6554	2.40	0.9918
-3.55	0.0002	-1.55	0.0606	0.45	0.6736	2.45	0.9929
-3.50	0.0002	-1.50	0.0668	0.50	0.6915	2.50	0.9938
-3.45	0.0003	-1.45	0.0735	0.55	0.7088	2.55	0.9946
-3.40	0.0003	-1.40	0.0808	0.60	0.7257	2.60	0.9953
-3.35	0.0004	-1.35	0.0885	0.65	0.7422	2.65	0.9960
-3.30	0.0005	-1.30	0.0968	0.70	0.7580	2.70	0.9965
-3.25	0.0006	-1.25	0.1056	0.75	0.7734	2.75	0.9970
-3.20	0.0007	-1.20	0.1151	0.80	0.7881	2.80	0.9974
-3.15	0.0008	-1.15	0.1251	0.85	0.8023	2.85	0.9978
-3.10	0.0010	-1.10	0.1357	0.90	0.8159	2.90	0.9981
-3.05	0.0011	-1.05	0.1469	0.95	0.8289	2.95	0.9984
-3.00	0.0013	-1.00	0.1587	1.00	0.8413	3.00	0.9987
-2.95	0.0016	-0.95	0.1711	1.05	0.8531	3.05	0.9989
-2.90	0.0019	-0.90	0.1841	1.10	0.8643	3.10	0.9990
-2.85	0.0022	-0.85	0.1977	1.15	0.8749	3.15	0.9992
-2.80	0.0026	-0.80	0.2119	1.20	0.8849	3.20	0.9993
-2.75	0.0030	-0.75	0.2266	1.25	0.8944	3.25	0.9994
-2.70	0.0035	-0.70	0.2420	1.30	0.9032	3.30	0.9995
-2.65	0.0040	-0.65	0.2578	1.35	0.9115	3.35	0.9996
-2.60	0.0047	-0.60	0.2743	1.40	0.9192	3.40	0.9997
-2.55	0.0054	-0.55	0.2912	1.45	0.9265	3.45	0.9997
-2.50	0.0062	-0.50	0.3085	1.50	0.9332	3.50	0.9998
-2.45	0.0071	-0.45	0.3264	1.55	0.9394	3.55	0.9998
-2.40	0.0082	-0.40	0.3446	1.60	0.9452	3.60	0.9998
-2.35	0.0094	-0.35	0.3632	1.65	0.9505	3.65	0.9999
-2.30	0.0107	-0.30	0.3821	1.70	0.9554	3.70	0.9999
-2.25	0.0122	-0.25	0.4013	1.75	0.9599	3.75	0.9999
-2.20	0.0139	-0.20	0.4207	1.80	0.9641	3.80	0.9999
-2.15	0.0158	-0.15	0.4404	1.85	0.9678	3.85	0.9999
-2.10	0.0179	-0.10	0.4602	1.90	0.9713	3.90	1.0000
-2.05	0.0202	-0.05	0.4801	1.95	0.9744	3.95	1.0000

$\Phi(z) = P(Z \leq z)$ for $N(0,1)$.

(Use interpolation to estimate z values to a 3rd decimal place.)

Table of Student t critical values (right-tail)

The table shows $t_{df,p}$ = the $1-p$ quantile of $t(df)$.

We only give values for $p \leq 0.5$. Use symmetry to find the values for $p > 0.5$, e.g.

$$t_{5,\,0.975} = -t_{5,\,0.025}$$

In R notation $t_{df,p}$ = `qt(1-p, df)`.

df\p	0.005	0.010	0.015	0.020	0.025	0.030	0.040	0.050	0.100	0.200	0.300	0.400	0.500
1	63.66	31.82	21.20	15.89	12.71	10.58	7.92	6.31	3.08	1.38	0.73	0.32	0.00
2	9.92	6.96	5.64	4.85	4.30	3.90	3.32	2.92	1.89	1.06	0.62	0.29	0.00
3	5.84	4.54	3.90	3.48	3.18	2.95	2.61	2.35	1.64	0.98	0.58	0.28	0.00
4	4.60	3.75	3.30	3.00	2.78	2.60	2.33	2.13	1.53	0.94	0.57	0.27	0.00
5	4.03	3.36	3.00	2.76	2.57	2.42	2.19	2.02	1.48	0.92	0.56	0.27	0.00
6	3.71	3.14	2.83	2.61	2.45	2.31	2.10	1.94	1.44	0.91	0.55	0.26	0.00
7	3.50	3.00	2.71	2.52	2.36	2.24	2.05	1.89	1.41	0.90	0.55	0.26	0.00
8	3.36	2.90	2.63	2.45	2.31	2.19	2.00	1.86	1.40	0.89	0.55	0.26	0.00
9	3.25	2.82	2.57	2.40	2.26	2.15	1.97	1.83	1.38	0.88	0.54	0.26	0.00
10	3.17	2.76	2.53	2.36	2.23	2.12	1.95	1.81	1.37	0.88	0.54	0.26	0.00
16	2.92	2.58	2.38	2.24	2.12	2.02	1.87	1.75	1.34	0.86	0.54	0.26	0.00
17	2.90	2.57	2.37	2.22	2.11	2.02	1.86	1.74	1.33	0.86	0.53	0.26	0.00
18	2.88	2.55	2.36	2.21	2.10	2.01	1.86	1.73	1.33	0.86	0.53	0.26	0.00
19	2.86	2.54	2.35	2.20	2.09	2.00	1.85	1.73	1.33	0.86	0.53	0.26	0.00
20	2.85	2.53	2.34	2.20	2.09	1.99	1.84	1.72	1.33	0.86	0.53	0.26	0.00
21	2.83	2.52	2.33	2.19	2.08	1.99	1.84	1.72	1.32	0.86	0.53	0.26	0.00
22	2.82	2.51	2.32	2.18	2.07	1.98	1.84	1.72	1.32	0.86	0.53	0.26	0.00
23	2.81	2.50	2.31	2.18	2.07	1.98	1.83	1.71	1.32	0.86	0.53	0.26	0.00
24	2.80	2.49	2.31	2.17	2.06	1.97	1.83	1.71	1.32	0.86	0.53	0.26	0.00
25	2.79	2.49	2.30	2.17	2.06	1.97	1.82	1.71	1.32	0.86	0.53	0.26	0.00
30	2.75	2.46	2.28	2.15	2.04	1.95	1.81	1.70	1.31	0.85	0.53	0.26	0.00
31	2.74	2.45	2.27	2.14	2.04	1.95	1.81	1.70	1.31	0.85	0.53	0.26	0.00
32	2.74	2.45	2.27	2.14	2.04	1.95	1.81	1.69	1.31	0.85	0.53	0.26	0.00
33	2.73	2.44	2.27	2.14	2.03	1.95	1.81	1.69	1.31	0.85	0.53	0.26	0.00
34	2.73	2.44	2.27	2.14	2.03	1.95	1.80	1.69	1.31	0.85	0.53	0.26	0.00
35	2.72	2.44	2.26	2.13	2.03	1.94	1.80	1.69	1.31	0.85	0.53	0.26	0.00
40	2.70	2.42	2.25	2.12	2.02	1.94	1.80	1.68	1.30	0.85	0.53	0.26	0.00
41	2.70	2.42	2.25	2.12	2.02	1.93	1.80	1.68	1.30	0.85	0.53	0.25	0.00
42	2.70	2.42	2.25	2.12	2.02	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
43	2.70	2.42	2.24	2.12	2.02	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
44	2.69	2.41	2.24	2.12	2.02	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
45	2.69	2.41	2.24	2.12	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
46	2.69	2.41	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
47	2.68	2.41	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
48	2.68	2.41	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00
49	2.68	2.40	2.24	2.11	2.01	1.93	1.79	1.68	1.30	0.85	0.53	0.25	0.00

Table of χ^2 critical values (right-tail)

The table shows $c_{df,\,p}$ = the $1-p$ quantile of $\chi^2(df)$.

In R notation $c_{df,\,p}$ = `qchisq(1-p, df)`.

df\p	0.010	0.025	0.050	0.100	0.200	0.300	0.500	0.700	0.800	0.900	0.950	0.975	0.990
1	6.63	5.02	3.84	2.71	1.64	1.07	0.45	0.15	0.06	0.02	0.00	0.00	0.00
2	9.21	7.38	5.99	4.61	3.22	2.41	1.39	0.71	0.45	0.21	0.10	0.05	0.02
3	11.34	9.35	7.81	6.25	4.64	3.66	2.37	1.42	1.01	0.58	0.35	0.22	0.11
4	13.28	11.14	9.49	7.78	5.99	4.88	3.36	2.19	1.65	1.06	0.71	0.48	0.30
5	15.09	12.83	11.07	9.24	7.29	6.06	4.35	3.00	2.34	1.61	1.15	0.83	0.55
6	16.81	14.45	12.59	10.64	8.56	7.23	5.35	3.83	3.07	2.20	1.64	1.24	0.87
7	18.48	16.01	14.07	12.02	9.80	8.38	6.35	4.67	3.82	2.83	2.17	1.69	1.24
8	20.09	17.53	15.51	13.36	11.03	9.52	7.34	5.53	4.59	3.49	2.73	2.18	1.65
9	21.67	19.02	16.92	14.68	12.24	10.66	8.34	6.39	5.38	4.17	3.33	2.70	2.09
10	23.21	20.48	18.31	15.99	13.44	11.78	9.34	7.27	6.18	4.87	3.94	3.25	2.56
16	32.00	28.85	26.30	23.54	20.47	18.42	15.34	12.62	11.15	9.31	7.96	6.91	5.81
17	33.41	30.19	27.59	24.77	21.61	19.51	16.34	13.53	12.00	10.09	8.67	7.56	6.41
18	34.81	31.53	28.87	25.99	22.76	20.60	17.34	14.44	12.86	10.86	9.39	8.23	7.01
19	36.19	32.85	30.14	27.20	23.90	21.69	18.34	15.35	13.72	11.65	10.12	8.91	7.63
20	37.57	34.17	31.41	28.41	25.04	22.77	19.34	16.27	14.58	12.44	10.85	9.59	8.26
21	38.93	35.48	32.67	29.62	26.17	23.86	20.34	17.18	15.44	13.24	11.59	10.28	8.90
22	40.29	36.78	33.92	30.81	27.30	24.94	21.34	18.10	16.31	14.04	12.34	10.98	9.54
23	41.64	38.08	35.17	32.01	28.43	26.02	22.34	19.02	17.19	14.85	13.09	11.69	10.20
24	42.98	39.36	36.42	33.20	29.55	27.10	23.34	19.94	18.06	15.66	13.85	12.40	10.86
25	44.31	40.65	37.65	34.38	30.68	28.17	24.34	20.87	18.94	16.47	14.61	13.12	11.52
30	50.89	46.98	43.77	40.26	36.25	33.53	29.34	25.51	23.36	20.60	18.49	16.79	14.95
31	52.19	48.23	44.99	41.42	37.36	34.60	30.34	26.44	24.26	21.43	19.28	17.54	15.66
32	53.49	49.48	46.19	42.58	38.47	35.66	31.34	27.37	25.15	22.27	20.07	18.29	16.36
33	54.78	50.73	47.40	43.75	39.57	36.73	32.34	28.31	26.04	23.11	20.87	19.05	17.07
34	56.06	51.97	48.60	44.90	40.68	37.80	33.34	29.24	26.94	23.95	21.66	19.81	17.79
35	57.34	53.20	49.80	46.06	41.78	38.86	34.34	30.18	27.84	24.80	22.47	20.57	18.51
40	63.69	59.34	55.76	51.81	47.27	44.16	39.34	34.87	32.34	29.05	26.51	24.43	22.16
41	64.95	60.56	56.94	52.95	48.36	45.22	40.34	35.81	33.25	29.91	27.33	25.21	22.91
42	66.21	61.78	58.12	54.09	49.46	46.28	41.34	36.75	34.16	30.77	28.14	26.00	23.65
43	67.46	62.99	59.30	55.23	50.55	47.34	42.34	37.70	35.07	31.63	28.96	26.79	24.40
44	68.71	64.20	60.48	56.37	51.64	48.40	43.34	38.64	35.97	32.49	29.79	27.57	25.15
45	69.96	65.41	61.66	57.51	52.73	49.45	44.34	39.58	36.88	33.35	30.61	28.37	25.90
46	71.20	66.62	62.83	58.64	53.82	50.51	45.34	40.53	37.80	34.22	31.44	29.16	26.66
47	72.44	67.82	64.00	59.77	54.91	51.56	46.34	41.47	38.71	35.08	32.27	29.96	27.42
48	73.68	69.02	65.17	60.91	55.99	52.62	47.34	42.42	39.62	35.95	33.10	30.75	28.18
49	74.92	70.22	66.34	62.04	57.08	53.67	48.33	43.37	40.53	36.82	33.93	31.55	28.94