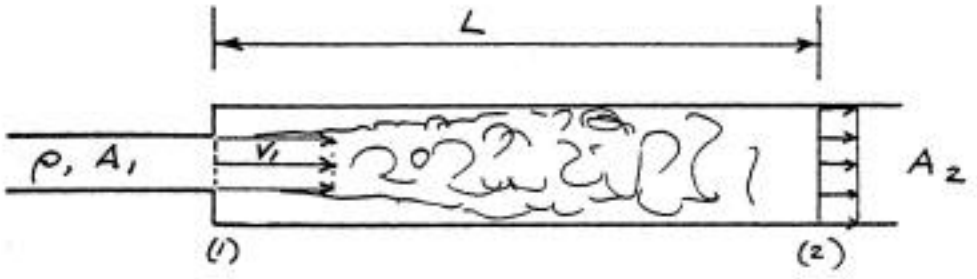
Problem 5.5

Sudden expansion in pipe



An incompressible fluid passes through a sudden expansion in a pipe, from area $A_1 = \pi R_1^2$ to $A_2 = \pi R_2^2$. The flow just downstream of the expansion looks like a jet of radius R_1 that emerges into an almost stagnant, re-circulating zone that surrounds it. Further downstream, the jet mixes in a turbulent, viscous manner with the fluid surrounding it, and eventually, at some station (2), the velocity is approximately uniform over the larger cross-section.

Assume that the velocity is uniform just upstream of the expansion as well as at station (2), and that, although viscous forces acting within the flow are important and in fact cause the velocity at (2) to be uniform, the net shear force exerted by the fluid on the wall between (1) and (2) is negligible.

(a) Find the rise in static pressure between (1) and (2),

HINT ANSWER

(b) Find the loss in stagnation pressure (total head) between (1) and (2). Check your results by showing that in the limit R_1/R_2 0, they reduce to the forms that apply to a jet issuing into an infinite reservoir.

ANSWER

(c) Provide a criterion for when the shear stress is indeed negligible, as we assume in the problem introduction. Express the criterion as an inequality between the shear average stress and some combination of the given quantities and the distance *L* between stations (1) and (2).

The given quantities are the upstream flow speed V_1, ρ, R_1 and R_2. Neglect gravitational effects.

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