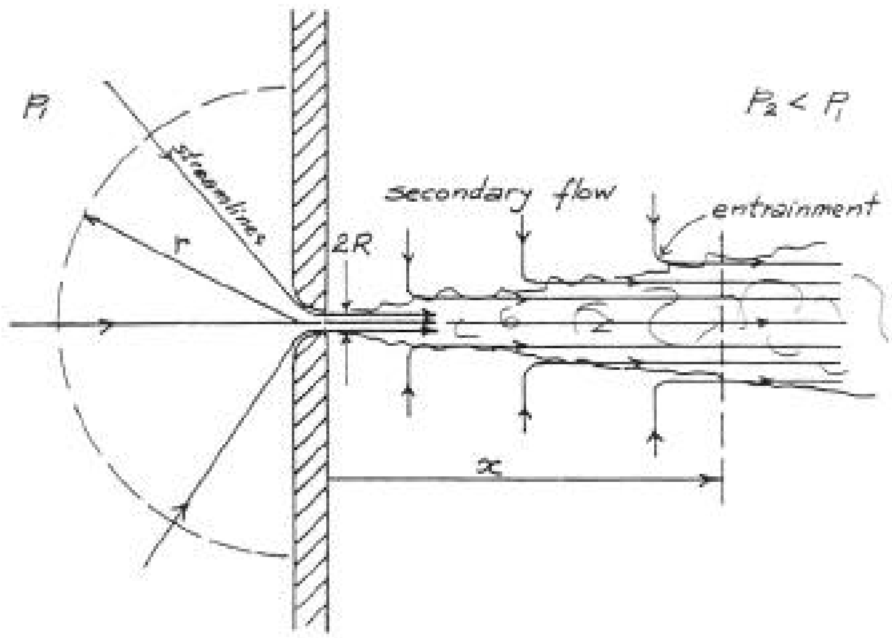
Problem 5.10

Difference between inflow and outflow in orifice flow



A wall in which there is a small nozzle, or hole, of radius *R*, separates two large compartments. The pressure p_1 in the left-hand compartment far from the nozzle is greater than the pressure p_2 in the right-hand compartment, and a steady volume flow *Q* takes place from 1 to 2.

The flow through the nozzle is an incompressible, high Reynolds number flow typical of the ones termed "inviscid". However, as in all such flows, viscous forces are responsible for the phenomenon of flow separation, which gives rise to a profound difference between the inflow and outflow regions of the nozzle flow field.

On the inflow side, in compartment 1, the flow is directed radially inward toward the nozzle entrance until one gets close to the nozzle, and is essentially inviscid. The pressure decreases and the velocity increases as one approaches the nozzle.

In compartment 2, on the other hand, the flow separates from the wall and emerges as a jet with approximately horizontal streamlines. This is due to boundary layer separation, the momentum-induced inability of real viscous flows to follow sharply curved walls like the lip of the nozzle at the exit plane. Inside compartment 2, viscous forces slow the jet and drag the surrounding fluid with it, and at high Reynolds numbers the combined flow becomes a turbulent jet which gradually broadens and slows down with distance x. The process whereby the jet drags some of the ambient fluid along with it is call "entrainment," and gives rise in compartment 2 to a secondary bulk flow that is directed approximately radially inward toward the axis of the jet, as sketched (the radial referring now to a cylindrical coordinate system). The velocities associated with this secondary flow are small compared with the jet velocity, however, and the pressure in compartment 2 can be taken as being essentially uniform, inside the jet as well as outside it.

(a) Consider a disc-shaped portion of the wall extending a radial distance *r* from the nozzle centerline. Using a control volume whose left side is a hemisphere of radius *r*, where $r >> R$, show that the x-component of force exerted by the flow on this portion of the wall is given by

$$F = (p_1 - p_2)\pi r^2\left[1 - 2(R/r)^2 + (1/2)(R/r)^4\right]$$

Gravity is negligible in this problem.

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(b) Consider the integrals of (i) mass flux and (ii) x-direction momentum flux across a plane at station x to the right of the nozzle exit. Do these integrals grow, decrease, or remain constant as the distance x from the nozzle exit plane increases?

HINT ANSWER