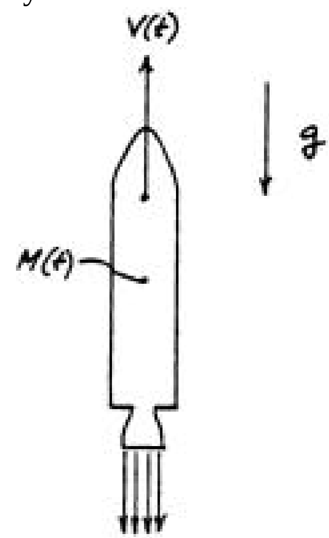
Problem 5.17

Rocket accelerating against gravity



After its second booster has been fired, a space vehicle finds itself outside the earth's atmosphere moving vertically upward against gravity $\vec{g}$ at speed V_0. Its total mass at that point is M_0. At t = 0, the vehicle's third stage is turned on and the rocket burns propellant at a mass rate of $\dot{m}_R$ kg/s. Show that if the gravitational acceleration remains essentially constant at the rocket during the firing, the rocket's velocity *V(t)* after time *t* will be given by

$$V(t) = V_0 + V_e \ln\left(\frac{M_0}{M(t)}\right) - \frac{g[M_0 - M(t)]}{\dot{m}_R}$$

where *M(t)* is the mass of the system at time *t*. Note that the pressure of the gas at the rocket exit plane, p_e, will not be zero as it is in the ambient space, since the rocket exhaust is supersonic and hence the pressure at the exit is in balance with the ambient pressure. The answer given above neglects the effect of the exit plane pressure on thrust.