Problem 5.18

Rocket firing on test bed

The figure shows a rocket burning solid propellant, the system being mounted on a stationary test bed. The rocket body has an inside area $A_1$ where the fuel is, and tapers to a smaller area $A_2$ at the exit plane. The combustible solid propellant has a density $\rho_f$ and burns at a constant rate $\dot{m}_f \text{ kgm}^{-2}\text{s}^{-1}$. The combusted gases leave the solid interface with a density $\rho_s < \rho_f$; the flow to the exit plane is essentially incompressible. (This is a good assumption only if Mach number at the exit is small.)

Given: $A_1$, $A_2$, $\dot{m}_f$, $\rho_f$, $\rho_s$.

(a) Derive an expression for the gas velocity $V_e$ at the exit plane.

(b) Derive an expression for the leftward thrust $F_x$ exerted by the rocket on its support.

(c) Assuming that the gas flow is inviscid and incompressible, obtain an expression for the gauge pressure in the gas region to the right of the solid propellant.

(d) What is the compressive stress $\sigma_s$ inside the solid propellant?

(e) Your answers will show that all of the above quantities are zero when $\rho_s = \rho_f$. Explain.