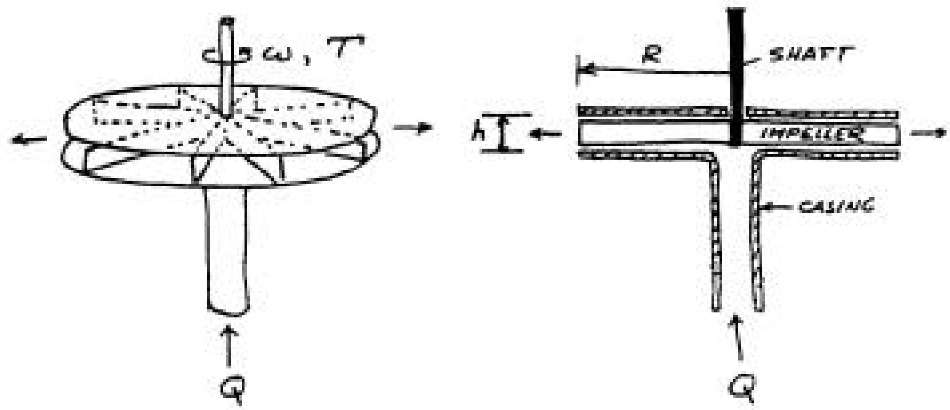
Problem 5.21

Radial outflow spinning up



The sketch shows an impeller with radial vanes in a flat-sided casing of radius R and height h. The impeller is mounted on bearings and is free to rotate without friction, but the casing is held stationary. Fluid of density ρ enters the impeller via a pipe which has a radius small compared with R. The velocity in the pipe is purely axial, and the volume flow rate is steady at a value Q.

(a) Show that if a torque T is applied to the impeller shaft, and frictional forces on the casing can be neglected, the angular velocity ω of the impeller obeys the equation

$$I\frac{d\omega}{dt} + \rho R^2 Q\omega = T$$

where

$$I = \frac{\pi}{2}\rho R^4 h.$$

(b) If the impeller is stationary at $t = 0$, and a constant torque T is applied at $t \geq 0$, obtain $\omega(t)$. Sketch $\omega(t)$ and compare with the corresponding solution for Problem 5.20. Why is a final steady state reached in this problem, and not in Problem 5.20?

ANSWER