Problem 5.27

Centrifugal pump/turbine

The sketch shows a centrifugal turbine or pump with an impeller in a flat-sided casing of width \( b \). A liquid of density \( \rho \) flows through the pump at a volume flow rate \( Q \). Assuming that (i) the liquid enters with a purely axial velocity and (ii) exits at radius \( r_2 \) with a velocity that follows the (moving) blade surfaces, and (iii) neglecting frictional effects,

(a) derive an expression for the counterclockwise external torque, \( T_S \), exerted on the shaft at an angular speed \( \omega \). Express it in terms of \( Q, \rho, b, r_1, r_2, \beta_1, \beta_2 \), and \( \omega \).

(b) What is the system’s run-away speed \( \omega_0 \), that is, the (counterclockwise) speed of the shaft running unrestrained? What is the torque \( T_0 \) at zero rotation?

(c) Show that your result in (a) reduces to the universal form

\[
T_S/T_{S0} = f(\omega/\omega_0)
\]

and plot this function.
(d) Derive the power output to the shaft,

\[ \dot{W}_s = T_s \omega , \]

as a function of \( \omega \). At what range of values of \( \beta \) will the power output be positive (that is, the system be a pump), if \( \beta > 0 \)? At what value(s) of \( \omega/\omega_0 \) is the power output maximized? Provide expressions for the maximum power and the angular frequency at which it is attained, expressed in terms of \( Q, \rho, b, r, r', \beta, \) and \( \beta_s \).