

CHAPTER 1 INTRODUCTION TO CALCULUS

Section 1.1 Velocity and Distance (page 6)

1 $v = 30, 0, -30; v = -10, 20$ **3** $v(t) = \begin{cases} 2 & \text{for } 0 < t < 10 \\ 1 & \text{for } 10 < t < 20 \\ -3 & \text{for } 20 < t < 30 \end{cases}$ $v(t) = \begin{cases} 0 & \text{for } 0 < t < T \\ \frac{1}{T} & \text{for } T < t < 2T \\ 0 & \text{for } 2T < t < 3T \end{cases}$
5 $25; 22; t + 10$ **7** $6; -30$ **9** $v(t) = \begin{cases} 20 & \text{for } t < .2 \\ 0 & \text{for } t > .2 \end{cases}$ $f(t) = \begin{cases} 20t & \text{for } t \leq .2 \\ 4 & \text{for } t \geq .2 \end{cases}$ **11** $10\%; 12\frac{1}{2}\%$
13 $f(t) = 0, 30(t-1), 30; f(t) = -30t, -60, 30(t-6)$ **15** Average 8, 20 **17** $40t - 80$ for $1 \leq t \leq 2.5$
21 $0 \leq t \leq 3, -40 \leq f \leq 20; 0 \leq t \leq 3T, 0 \leq f \leq 60T$ **23** $3 - 7t$ **25** $6t - 2$ **27** $3t + 7$
29 Slope $-2; 1 \leq f \leq 9$ **31** $v(t) = \begin{cases} 8 & \text{for } 0 < t < T \\ -2 & \text{for } T < t < 5T \end{cases}$ $f(t) = \begin{cases} 8t & \text{for } 0 \leq t \leq T \\ 10T - 2t & \text{for } T \leq t \leq 5T \end{cases}$
33 $\frac{9}{5}C + 32; \text{slope } \frac{9}{5}$ **35** $f(w) = \frac{w}{1000}; \text{slope} = \text{conversion factor}$ **37** $1 \leq t \leq 5, 0 \leq f \leq 2$
39 $0 \leq t \leq 5, 0 \leq f \leq 4$ **41** $0 \leq t \leq 5, 1 \leq t \leq 32$ **43** $\frac{1}{2}t + 4; \frac{1}{2}t + \frac{7}{2}; 2t + 12; 2t + 3$
45 Domains $-1 \leq t \leq 1$: ranges $0 \leq 2t + 2 \leq 4, -3 \leq t - 2 \leq -1, -2 \leq -f(t) \leq 0, 0 \leq f(-t) \leq 2$
47 $\frac{3}{2}V; \frac{3}{2}V$ **49** input * input $\rightarrow A$ input * input $\rightarrow A$ $B * B \rightarrow C$ input + 1 $\rightarrow A$
 input + $A \rightarrow$ output input + $A \rightarrow B$ $B + C \rightarrow$ output $A * A \rightarrow B$
 $A + B \rightarrow$ output
51 $3t + 5, 3t + 1, 6t - 2, 6t - 1, -3t - 1, 9t - 4$; slopes 3, 3, 6, 6, -3, 9
53 The graph goes up and down *twice*. $f(f(t)) = \begin{cases} 2(2t) & 0 \leq t \leq 1.5 \\ 12 - 4t & 1.5 \leq t \leq 3 \end{cases}$ $\begin{cases} 12 - 2(12 - 2t) & 3 \leq t \leq 4.5 \\ 2(12 - 2t) & 4.5 \leq t \leq 6 \end{cases}$

Section 1.2 Calculus Without Limits (page 14)

1 $2 + 5 + 3 = 10$; $f = 1, 3, 8, 11$; 10 **3** $f = 3, 4, 6, 7, 7, 6$; $\max f$ at $v = 0$ or at break from $v = 1$ to -1
5 $1.1, -2, 5$; $f(6) = 6.6, -11, 4$; $f(7) = 7.7, -13, 9$ **7** $f(t) = 2t$ for $t \leq 5$, $10 + 3(t - 5)$ for $t \geq 5$; $f(10) = 25$
9 $7, 28, 8t + 4$; multiply slopes **11** $f(8) = 8.8, -15, 14$; $\frac{\Delta f}{\Delta t} = 1.1, -2, 5$
13 $f(x) = 3052.50 + .28(x - 20, 350)$; then $11, 158.50$ is $f(49, 300)$ **15** $19\frac{1}{4}\%$
17 Credit subtracts 1,000, deduction only subtracts 15% of 1000 **19** All $v_j = 2$; $v_j = (-1)^{j-1}$; $v_j = (\frac{1}{2})^j$
21 L's have area 1,3,5,7 **23** $f_j = j$; sum $j^2 + j$; sum $\frac{j^2}{2} + \frac{j}{2}$ **25** $(101^2 - 99^2)/2 = \frac{400}{2}$ **27** $v_j = 2j$ **29** $f_{31} = 5$
31 $a_j = -f_j$ **33** $0; 1; .1$ **35** $v = 2, 6, 18, 54$; $2 \cdot 3^{j-1}$ **37** $\frac{\Delta f}{\Delta t} = 1, .7177, .6956, .6934 \rightarrow \ln 2 = .6931$ in Chapter 6
39 $v_j = -(\frac{1}{2})^j$ **41** $v_j = 2(-1)^j$, sum is $f_j - 1$ **45** $v = 1000, t = 10/V$
47 M, N **51** $\sqrt{9} < 2 \cdot 9 < 9^2 < 2^9$; $(\frac{1}{9})^2 < 2(\frac{1}{9}) < \sqrt{1/9} < 2^{1/9}$

Section 1.3 The Velocity at an Instant (page 21)

1 6, 6, $\frac{13}{2}a$, -12, 0, 13 **3** 4, 3.1, $3+h$, 2.9 **5** Velocity at $t = 1$ is 3 **7** Area $f = t + t^2$, slope of f is $1 + 2t$
9 F; F; F; T **11** $2; 2t$ **13** $12 + 10t^2; 2 + 10t^2$ **15** Time 2, height 1, stays above $\frac{3}{4}$ from $t = \frac{1}{2}$ to $\frac{3}{2}$
17 $f(6) = 18$ **21** $v(t) = -2t$ then $2t$ **23** Average to $t = 5$ is 2; $v(5) = 7$ **25** $4v(4t)$ **27** $v_{\text{ave}} = t, v(t) = 2t$

Section 1.4 Circular Motion (page 28)

1 $10\pi, (0, -1), (-1, 0)$ **3** $(4 \cos t, 4 \sin t); 4$ and $4t; 4 \cos t$ and $-4 \sin t$
5 $3t; (\cos 3t, \sin 3t); -3 \sin 3t$ and $3 \cos 3t$ **7** $x = \cos t; \sqrt{2}/2; -\sqrt{2}/2$ **9** $2\pi/3; 1; 2\pi$
11 Clockwise starting at $(1, 0)$ **13** Speed $\frac{2}{\pi}$ **15** Area 2 **17** Area 0

- 19 4 from speed, 4 from angle 21 $\frac{1}{4}$ from radius times 4 from angle gives 1 in velocity
 23 Slope $\frac{1}{2}$; average $(1 - \frac{\sqrt{3}}{2})/(\pi/6) = \frac{3(2-\sqrt{3})}{\pi} = .256$ 25 Clockwise with radius 1 from (1,0), speed 3
 27 Clockwise with radius 5 from (0,5), speed 10 29 Counterclockwise with radius 1 from $(\cos 1, \sin 1)$, speed 1
 31 Left and right from (1,0) to $(-1,0)$, $v = -\sin t$ 33 Up and down between 2 and -2 ; start $2 \sin \theta$, $v = 2 \cos(t+\theta)$
 35 Up and down from $(0, -2)$ to $(0, 2)$; $v = \sin \frac{1}{2}t$ 37 $x = \cos \frac{2\pi t}{360}$, $y = \sin \frac{2\pi t}{360}$, speed $\frac{2\pi}{360}$, $v_{up} = \cos \frac{2\pi t}{360}$

Section 1.5 A Review of Trigonometry (page 33)

- 1 Connect corner to midpoint of opposite side, producing 30° angle 3 π 7 $\frac{\theta}{2\pi} \rightarrow \text{area } \frac{1}{2}r^2\theta$
 9 $d = 1$, distance around hexagon < distance around circle 11 T; T; F; F
 13 $\cos(2t + t) = \cos 2t \cos t - \sin 2t \sin t = 4 \cos^3 t - 3 \cos t$
 15 $\frac{1}{2} \cos(s - t) + \frac{1}{2} \cos(s + t)$; $\frac{1}{2} \cos(s - t) - \frac{1}{2} \cos(s + t)$ 17 $\cos \theta = \sec \theta = \pm 1$ at $\theta = n\pi$
 19 Use $\cos(\frac{\pi}{2} - s - t) = \cos(\frac{\pi}{2} - s) \cos t + \sin(\frac{\pi}{2} - s) \sin t$ 23 $\theta = \frac{3\pi}{2} + \text{multiple of } 2\pi$
 25 $\theta = \frac{\pi}{4} + \text{multiple of } \pi$ 27 No θ 29 $\phi = \frac{\pi}{4}$ 31 $|OP| = a$, $|OQ| = b$

CHAPTER 2 DERIVATIVES

Section 2.1 The Derivative of a Function (page 49)

- 1 (b) and (c) 3 $12 + 3h$; $13 + 3h$; 3 ; 3 5 $f(x) + 1$ 7 -6 9 $2x + \Delta x + 1$; $2x + 1$
 11 $\frac{4}{t+\Delta t} - \frac{4}{t} = \frac{-4}{t(t+\Delta t)} \rightarrow \frac{-4}{t^2}$ 13 7; 9; corner 15 $A = 1$, $B = -1$ 17 F; F; T; F
 19 $b = B$; m and M ; m or undefined 21 Average $x_2 + x_1 \rightarrow 2x_1$
 25 $\frac{1}{2}$; no limit (one-sided limits 1, -1); 1; 1 if $t \neq 0$, -1 if $t = 0$ 27 $f'(3)$; $f(4) - f(3)$
 29 $2x^4(4x^3) = 8x^7$ 31 $\frac{du}{dx} = \frac{1}{2u} = \frac{1}{2\sqrt{x}}$ 33 $\frac{\Delta f}{\Delta x} = -\frac{1}{2}$; $f'(2)$ doesn't exist 35 $2f \frac{df}{dx} = 4u^3 \frac{du}{dx}$

Section 2.2 Powers and Polynomials (page 56)

- 1 $6x^5$; $30x^4$; $f'''''' = 720 = 6!$ 3 $2x + 7$ 5 $1 + 2x + 3x^2 + 4x^3$ 7 $nx^{n-1} - nx^{-n-1}$
 9 $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$ 11 $-\frac{1}{x}$, $(-\frac{1}{x}) + 5$ 13 $x^{-2/3}$; $x^{-4/3}$; $-\frac{1}{9}x^{-4/3}$
 15 $3x^2 - 1 = 0$ at $x = \frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$ 17 8 ft/sec; -8 ft/sec; 0 19 Decreases for $-1 < x < \frac{1}{3}$
 21 $\frac{(x+h)-x}{h(\sqrt{x+h}-\sqrt{x})} \rightarrow \frac{1}{2\sqrt{x}}$ 23 1 5 10 10 5 1 adds to $(1+1)^5$ ($x = h = 1$)
 25 $3x^2$; $2h$ is difference of x 's 27 $\frac{\Delta f}{\Delta x} = 2x + \Delta x + 3x^2 + 3x\Delta x + (\Delta x)^2 \rightarrow 2x + 3x^2 = \text{sum of separate derivatives}$
 29 $7x^6$; $7(x+1)^6$ 31 $\frac{1}{24}x^4$ plus any cubic 33 $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + C$ 35 $\frac{1}{24}x^4$, $\frac{1}{120}x^5$
 37 F; F; F; T; T 39 $\frac{y}{x} = .12$ so $\frac{\Delta y}{\Delta x} = \frac{1}{2}(.12)$; six cents 41 $\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x}(\frac{c}{x+\Delta x} - \frac{c}{x})$, $\frac{dy}{dx} = -\frac{c}{x^2}$
 43 $E = \frac{2x}{2x+3}$ 45 t to $\sqrt[3]{2}t$ 47 $\frac{1}{10}x^{10}$; $\frac{1}{n+1}x^{n+1}$; divide by $n+1 = 0$
 49 .7913, -3.7913 , 1.618 , $-.618$; 0 , 1.266 , -2.766

Section 2.3 The Slope and the Tangent Line (page 63)

- 1 $\frac{-12}{x^3}$; $y - 6 = -3(x - 2)$; $y - 6 = \frac{1}{3}(x - 2)$; $y - 6 = -\frac{3}{2}(x - 2)$ 3 $y + 1 = 3(x - 1)$; $y = 3x - 4$
 5 $y = x$; $(3, 3)$ 7 $y - a^2 = (c + a)(x - a)$; $y - a^2 = 2a(x - a)$ 9 $y = \frac{1}{5}x^2 + 2$; $y - 7 = -\frac{1}{2}(x - 5)$
 11 $y = 1$; $x = \frac{\pi}{2}$ 13 $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$; $y = \frac{2}{a}$, $x = 2a$; 2 15 $c = 4$, tangent at $x = 2$

- 17 $(-3, 19)$ and $(\frac{1}{3}, \frac{13}{27})$ 19 $c = 4, y = 3 - x$ tangent at $x = 1$
 21 $(1+h)^3; 3h + 3h^2 + h^3; 3 + 3h + h^2; 3$ 23 Tangents parallel, *same* normal
 25 $y = 2ax - a^2, Q = (0, -a^2)$; distance $a^2 + \frac{1}{4}$; angle of incidence = angle of reflection
 27 $x = 2p$; focus has $y = \frac{x^2}{4p} = p$ 29 $y - \frac{1}{\sqrt{2}} = x + \frac{1}{\sqrt{2}}; x = -\frac{2}{\sqrt{2}} = -\sqrt{2}$
 31 $y - a^2 = -\frac{1}{2a}(x - a); y = a^2 + \frac{1}{2}; a = \frac{\sqrt{3}}{2}$ 33 $(\frac{1}{x^2})(1000) = 10$ at $x = 10$ hours 35 $a = 2$
 37 1.01004512; $1 + 10(.001) = 1.01$ 39 $(2 + \Delta x)^3 - (8 + 6\Delta x) = 6(\Delta x)^2 + (\Delta x)^3$ 41 $x_1 = \frac{5}{4}; x_2 = \frac{41}{40}$
 43 $T = 8$ sec; $f(T) = 96$ meters 45 $a > \frac{4}{5}$ meters/sec²

Section 2.4 The Derivative of the Sine and Cosine (page 70)

- 1 (a) and (b) 3 $0; 1; 5; \frac{1}{5}$ 5 $\sin(x + 2\pi); (\sin h)/h \rightarrow 1; 2\pi$ 7 $\cos^2 \theta \approx 1 - \theta^2 + \frac{1}{4}\theta^4; \frac{1}{4}\theta^4$ is small
 9 $\sin \frac{1}{2}\theta \approx \frac{1}{2}\theta$ 11 $\frac{3}{2}; 4$ 13 $PS = \sin h$; area $OPR = \frac{1}{2} \sin h < \text{curved area } \frac{1}{2}h$
 15 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ 17 $\frac{1}{2h}(\cos(x+h) - \cos(x-h)) = \frac{1}{h}(-\sin x \sin h) \rightarrow -\sin x$
 19 $y' = \cos x - \sin x = 0$ at $x = \frac{\pi}{4} + n\pi$ 21 $(\tan h)/h = \sin h/h \cos h < \frac{1}{\cos h} \rightarrow 1$
 23 Slope $\frac{1}{2} \cos \frac{1}{2}x = \frac{1}{2}, 0, -\frac{1}{2}, \frac{1}{2}$; no 25 $y = 2 \cos x + \sin x; y'' = -y$ 27 $y = -\frac{1}{3} \cos 3x; y = \frac{1}{3} \sin 3x$
 29 In degrees $(\sin h)/h \rightarrow 2\pi/360 = .01745$ 31 $2 \sin x \cos x + 2 \cos x(-\sin x) = 0$

Section 2.5 The Product and Quotient and Power Rules (page 77)

- 1 $2x$ 3 $\frac{-1}{(1+x)^2} - \frac{\cos x}{(1+\sin x)^2}$ 5 $(x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2)$
 7 $-x^2 \sin x + 4x \cos x + 2 \sin x$ 9 $2x - 1 - \frac{1}{\sin^2 x}$ 11 $2\sqrt{x} \sin x \cos x + \frac{1}{2}x^{-1/2} \sin^2 x + \frac{1}{2}(\sin x)^{-1/2} \cos x$
 13 $4x^3 \cos x - x^4 \sin x + \cos^4 x - 4x \cos^3 x \sin x$ 15 $\frac{1}{2}x^2 \cos x + 2x \sin x$ 17 0 19 $-\frac{8}{3}(x-5)^{-5/3} + \frac{8}{3}(5-x)^{-5/3} (= 0?)$
 21 $3(\sin x \cos x)^2(\cos^2 x - \sin^2 x) + 2 \cos 2x$ 23 $u'vwz + v'u wz + w'uvw + z'uvw$ 25 $-\csc^2 x - \sec^2 x$
 27 $V = \frac{t \cos t}{1+t}, V' = \frac{\cos t - t \sin t - t^2 \sin t}{(1+t)^2}$ $A = 2(\frac{t}{t+1} + t \cos t + \frac{\cos t}{t+1})$ $A' = 2(\cos t - t \sin t + \frac{1 - \cos t}{(t+1)^2} - \frac{\sin t}{t+1})$
 29 $10t$ for $t < 10, \frac{50}{\sqrt{t-10}}$ for $t > 10$ 31 $\frac{2t^3+3t^2}{(1+t)^2}; \frac{2t^3+6t^2+6t}{(1+t)^3}$
 33 $u''v + 2u'v' + uv''; u'''v + 3u''v' + 3u'v'' + v'''$ 35 $\frac{1}{2} \sin^2 t; \frac{1}{2} \tan^2 t; \frac{2}{3}[(1+t)^{3/2} - 1]$
 39 T; F; F; T; F 41 degree $2n - 1$ / degree $2n$ 43 $v(t) = \cos t - t \sin t (t \leq \frac{\pi}{2}); v(t) = -\frac{\pi}{2} (t \geq \frac{\pi}{2})$
 45 $y = \frac{2hx^3}{L^3} + \frac{3hx^2}{L^2}$ has $\frac{dy}{dx} = 0$ at $x = 0$ (no crash) and at $x = -L$ (no dive). Then $\frac{dy}{dx} = \frac{6Vh}{L}(\frac{x^2}{L^2} + \frac{x}{L})$ and $\frac{d^2y}{dx^2} = \frac{6V^2h}{L^2}(\frac{2x}{L} + 1)$.

Section 2.6 Limits (page 84)

- 1 $\frac{1}{4}, L = 0$, after $N = 10; \frac{25}{24}, \infty$, no $N; \frac{1}{4}, 0$, after 5; 1.1111, $\frac{10}{9}$, all $n; \sqrt{2}, 1$, after 38; $\sqrt{20} - 4, \frac{1}{2}$, all n ;
 $\frac{625}{256}, e = 2.718\dots$, after $N = 12$. 3 (c) and (d)
 5 Outside any interval around zero there are only a finite number of a 's 7 $\frac{5}{2}$ 9 $\frac{f(h)-f(0)}{h}$ 11 1
 13 1 15 $\sin 1$ 17 No limit 19 $\frac{1}{2}$ 21 Zero if $f(x)$ is continuous at a 23 2
 25 .001, .0001, .005, .1 27 $|f(x) - L|; \frac{4x}{1+x}$ 29 0; $X = 100$ 33 4; $\infty; 7; 7$ 35 3; no limit; 0; 1
 37 $\frac{1}{1-r}$ if $|r| < 1$; no limit if $|r| \geq 1$ 39 .0001; after $N = 7$ (or 8?) 41 $\frac{1}{2}$
 43 $9; 8\frac{1}{2}; a_n - 8 = \frac{1}{2}(a_{n-1} - 8) \rightarrow 0$
 45 $a_n - L \leq b_n - L \leq c_n - L$ so $|b_n - L| < \epsilon$ if $|a_n - L| < \epsilon$ and $|c_n - L| < \epsilon$

Section 2.7 Continuous Functions (page 89)

- 1 $c = \sin 1$; no c 3 Any c ; $c = 0$ 5 $c = 0$ or 1 ; no c 7 $c = 1$; no c 9 no c ; no c
 11 $c = \frac{1}{64}$; $c = \frac{1}{64}$ 13 $c = -1$; $c = -1$ 15 $c = 1$; $c = 1$ 17 $c = -1$; $c = -1$
 19 $c = 2, 1, 0, -1, \dots$; same c 21 $f(x) = 0$ except at $x = 1$ 23 $\sqrt{x-1}$ 25 $-\frac{2x}{|x|}$ 27 $\frac{5}{x-1}$
 29 One; two; two 31 No; yes; no 33 $xf(x), (f(x))^2, x, f(x), 2(f(x)-x), f(x)+2x$ 35 F; F; F; T
 37 Step; $f(x) = \sin \frac{1}{x}$ with $f(0) = 0$ 39 Yes; no; no; yes ($f_4(0) = 1$)
 41 $g(\frac{1}{2}) = f(1) - f(\frac{1}{2}) = f(0) - f(\frac{1}{2}) = -g(0)$; zero is an intermediate value between $g(0)$ and $g(\frac{1}{2})$
 43 $f(x) - x$ is ≥ 0 at $x = 0$ and ≤ 0 at $x = 1$

CHAPTER 3 APPLICATIONS OF THE DERIVATIVE

Section 3.1 Linear Approximation (page 95)

- 1 $Y = x$ 3 $Y = 1 + 2(x - \frac{\pi}{4})$ 5 $Y = 2\pi(x - 2\pi)$ 7 $2^6 + 6 \cdot 2^5 \cdot .001$ 9 1
 11 $1 - 1(-.02) = 1.02$ 13 Error .000301 vs. $\frac{1}{2}(.0001)6$ 15 $.0001 - \frac{1}{3}10^{-8}$ vs. $\frac{1}{2}(.0001)(2)$
 17 Error .59 vs. $\frac{1}{2}(.01)(90)$ 19 $\frac{d}{dx}\sqrt{1-x} = \frac{-1}{2\sqrt{1-x}} = -\frac{1}{2}$ at $x = 0$
 21 $\frac{d}{du}\sqrt{c^2+u} = \frac{1}{2\sqrt{c^2+u}} = \frac{1}{2c}$ at $u = 0, c + \frac{u}{2c} = c + \frac{u}{2c}$ 23 $dV = 3(10)^2(.1)$
 25 $A = 4\pi r^2, dA = 8\pi r dr$ 27 $V = \pi r^2 h, dV = 2\pi r h dr$ (plus $\pi r^2 dh$) 29 $1 + \frac{1}{2}x$ 31 32nd root

Section 3.2 Maximum and Minimum Problems (page 103)

- 1 $x = -2$: abs min 3 $x = -1$: rel max, $x = 0$: abs min, $x = 4$: abs max
 5 $x = -1$: abs max, $x = 0, 1$: abs min, $x = \frac{1}{2}$: rel max 7 $x = -3$: abs min, $x = 0$: rel max, $x = 1$: rel min
 9 $x = 1, 9$: abs min, $x = 5$: abs max 11 $x = \frac{1}{3}$: rel max, $x = 1$: rel min, $x = 0$: stationary (not min or max)
 13 $x = 0, 1, 2, \dots$: abs min, $x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$: abs max 15 $|x| \leq 1$: all min, $x = -3$ abs max, $x = 2$ rel max
 17 $x = 0$: rel min, $x = \frac{1}{3}$: abs max, $x = 4$: abs min
 19 $x = 0$: abs min, $x = \pi$: stationary (not min or max), $x = 2\pi$: abs max
 21 $\theta = 0$: rel min, $\tan \theta = -\frac{4}{3}$ ($\sin \theta = \frac{4}{5}$ and $\cos \theta = -\frac{3}{5}$ abs max, $\sin \theta = -\frac{4}{5}$ and $\cos \theta = \frac{3}{5}$ abs min),
 $\theta = 2\pi$: rel max
 23 $h = \frac{1}{3}(62'' \text{ or } 158 \text{ cm})$; cube 25 $\frac{v}{av^2+b}$; $2\sqrt{ab}$ gallons/mile, $\frac{1}{2\sqrt{ab}}$ miles/gallon at $v = \sqrt{\frac{b}{a}}$
 27 (b) $\theta = \frac{3\pi}{8} = 67.5^\circ$ 29 $x = \frac{a}{\sqrt{3}}$; compare Example 7; $\frac{a}{b} = \sqrt{3}$
 31 $R(x) - C(x)$; $\frac{R(x)-C(x)}{x}$; $\frac{dR}{dx} - \frac{dC}{dx}$; profit 33 $x = \frac{d-a}{2(b-e)}$; zero 35 $x = 2$
 37 $V = x(6 - \frac{3x}{2})(12 - 2x)$; $x \approx 1.6$ 39 $A = \pi r^2 + x^2, x = \frac{1}{4}(4 - 2\pi r)$; $r_{\min} = \frac{2}{2+\pi}$
 41 max area 2500 vs $\frac{10000}{\pi} = 3185$ 43 $x = 2, y = 3$ 45 $P(x) = 12 - x$; thin rectangle up y axis
 47 $h = \frac{H}{3}, r = \frac{2R}{3}, V = \frac{4\pi R^2 H}{27} = \frac{4}{9}$ of cone volume
 49 $r = \frac{HR}{2(H-R)}$; best cylinder has no height, area $2\pi R^2$ from top and bottom (?)
 51 $r = 2, h = 4$ 53 25 and 0 55 8 and $-\infty$
 57 $\sqrt{r^2+x^2} + \sqrt{q^2+(s-x)^2}$; $\frac{d}{dx} = \frac{x}{\sqrt{r^2+x^2}} - \frac{s-x}{\sqrt{q^2+(s-x)^2}} = 0$ when $\sin a = \sin c$
 59 $y = x^2 = \frac{3}{2}$ 61 $(1, -1), (\frac{13}{5}, -\frac{1}{5})$ 63 $m = 1$ gives nearest line 65 $m = \frac{1}{3}$ 67 equal; $x = \frac{1}{2}$
 69 $\frac{1}{x}x^2$ 71 True (use sign change of f'')
 73 Radius R , swim $2R \cos \theta$, run $2R\theta$, time $\frac{2R \cos \theta}{v} + \frac{2R\theta}{10v}$; max when $\sin \theta = \frac{1}{10}$, min all run

Section 3.3 Second Derivatives: Bending and Acceleration (page 110)

- 3** $y = -1 - x^2$; no ... **5** False **7** True **9** True (f' has 8 zeros, f'' has 7)
11 $x = 3$ is min: $f''(3) = 2$ **13** $x = 0$ not max or min; $x = \frac{9}{2}$ is min: $f''(\frac{9}{2}) = 81$
15 $x = \frac{3\pi}{4}$ is max: $f''(\frac{3\pi}{4}) = -\sqrt{2}$; $x = \frac{7\pi}{4}$ is min: $f''(\frac{7\pi}{4}) = \sqrt{2}$
17 Concave down for $x > \frac{1}{3}$ (inflection point)
19 $x = 3$ is max: $f''(3) = -4$; $x = 2, 4$ are min but $f'' = 0$ **21** $f(\Delta x) = f(-\Delta x)$ **23** $1 + x - \frac{x^2}{2}$
25 $1 - \frac{x^2}{6}$ **27** $1 - \frac{1}{2}x - \frac{1}{8}x^2$ **29** Error $\frac{1}{2}f''(x)\Delta x$ **31** Error $0\Delta x + \frac{1}{3}f'''(x)(\Delta x)^2$
37 $\frac{1}{.99} = 1.010101$; $\frac{1}{1.1} = .90909$ **39** Inflection **41** 18 vs. 17 **43** Concave up; below

Section 3.4 Graphs (page 119)

- 1** 120; 150; $\frac{60}{x}$ **3** Odd; $x = 0, y = x$ **5** Even; $x = 1, x = -1, y = 0$ **7** Even; $y = 1$ **9** Even
11 Even; $x = 1, x = -1, y = 0$ **13** $x = 0, x = -1, y = 0$ **15** $x = 1, y = 1$ **17** Odd **19** $\frac{2x}{x-1}$
21 $x + \frac{1}{x-4}$ **23** $\sqrt{x^2 + 1}$ **25** Of the same degree **27** Have degree $P < \text{degree } Q$; none
29 $x = 1$ and $y = 3x + C$ if f is a polynomial; but $f(x) = (x-1)^{1/3} + 3x$ has no asymptote $x = 1$
31 $(x-3)^2$ **39** $x = \sqrt{2}, x = -\sqrt{2}, y = x$ **41** $Y = 100 \sin \frac{2\pi X}{360}$ **45** $c = 3, d = 10$; $c = 4, d = 20$
47 $x^* = \sqrt{5} = 2.236$ **49** $y = x - 2$; $Y = X$; $y = 2x$ **51** $x_{\max} = .281, x_{\min} = 6.339$; $x_{\text{infl}} = 4.724$
53 $x_{\min} = .393, x_{\max} = 1.53, x_{\min} = 3.33$; $x_{\text{infl}} = .896, 2.604$
55 $x_{\min} = -.7398, x_{\max} = .8135$; $x_{\text{infl}} = .04738$; $x_{\text{blowup}} = \pm 2.38$ **57** 8 digits

Section 3.5 Parabolas, Ellipses, and Hyperbolas (page 128)

- 1** $dy/dx = 0$ at $\frac{-b}{2a}$ **3** $V = (1, -4), F = (1, -3.75)$ **5** $V = (0, 0), F = (0, -1)$ **7** $F = (1, 1)$
9 $V = (0, \pm 3)$; $F = (0, \pm\sqrt{8})$ **11** $V = (0, \pm 1)$; $F = (0, \pm\sqrt{\frac{5}{4}})$ **13** Two lines, $a = b = c = 0$; $V = F = (0, 0)$
15 $y = 5x^2 - 4x$ **17** $y + p = \sqrt{x^2 + (y-p)^2} \rightarrow 4py = x^2$; $F = (0, \frac{1}{12})$, $y = -\frac{1}{12}$; $(\pm\frac{\sqrt{11}}{6}, \frac{11}{12})$
19 $x = ay^2$ with $a > 0$; $y = \frac{(x+p)^2}{4p}$; $y = -ax^2 + ax$ with $a > 0$
21 $\frac{x^2}{4} + y^2 = 1$; $\frac{(x-1)^2}{4} + (y-1)^2 = 1$ **23** $\frac{x^2}{25} + \frac{y^2}{9} = 1$; $\frac{(x-3)^2}{36} + \frac{(y-1)^2}{32} = 1$; $x^2 + y^2 = 25$
25 Circle, hyperbola, ellipse, parabola **27** $\frac{dy}{dx} = -\frac{4}{5}$; $y = -\frac{4}{5}x + 5$ **29** $\frac{5}{4}$; $\frac{9}{40} = \frac{1}{2}(\frac{5}{4} - \frac{4}{5})$
31 Circle; $(3, 1)$; 2 ; $X = \frac{x-3}{2}$, $Y = \frac{y-1}{2}$ **33** $3x'^2 + y'^2 = 2$ **35** $y^2 - \frac{1}{3}x^2 = 1$; $\frac{y^2}{9} - \frac{4x^2}{9} = 1$; $y^2 - x^2 = 5$
37 $\frac{x^2}{25} - \frac{y^2}{39} = 1$ **39** $y^2 - 4y + 4, 2x^2 + 12x + 18$; $-14, (-3, 2)$, right-left
41 $F = (\pm\frac{\sqrt{5}}{2}, 0)$; $y = \pm\frac{x}{2}$ **43** $(x + y + 1)^2 = 0$
45 $(a^2 - 1)x^2 + 2abxy + (b^2 - 1)y^2 + 2acx + 2bcy + c^2 = 0$; $4(a^2 + b^2 - 1)$; if $a^2 + b^2 < 1$ then $B^2 - 4AC < 0$

Section 3.6 Iterations $x_{n+1} = F(x_n)$ (page 136)

- 1** $-.366; \infty$ **3** 1; 1 **5** $\frac{2}{3}; \pm\infty$ **7** $-2; -2$
9 $\frac{1-\sqrt{3}}{2}$ attracts, $\frac{1+\sqrt{3}}{2}$ repels; $\frac{1}{2}$ attracts, 0 repels; 1 attracts, 0 repels; 1 attracts; $\frac{2}{3}$ attracts, 0 repels; $\pm\sqrt{2}$ repel
11 Negative **13** .900 **15** .679 **17** $|a| < 1$ **19** Unstable $|F'| > 1$ **21** $x^* = \frac{e}{1-a}$; $|a| < 1$

- 23** \$2000; \$2000 **25** $x_0, b/x_0, x_0, b/x_0, \dots$ **27** $F' = -\frac{\sqrt{2}}{2}x^{-3/2} = -\frac{1}{2}$ at x^*
29 $F' = 1 - 2cx = 1 - 4c$ at $x^* = 2$; $0 < c < \frac{1}{2}$ succeeds
31 $F' = 1 - 9c(x-2)^8 = 1 - 9c$ at $x^* = 3$; $0 < c < \frac{2}{9}$ succeeds
33 $x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2}$; $x_{n+1} = x_n - \frac{\sin x_n - \frac{1}{2}}{\cos x_n}$ **35** $x^* = 4$ if $x_0 > 2.5$; $x^* = 1$ if $x_0 < 2.5$
37 $m = 1 + c$ at $x^* = 0$, $m = 1 - c$ at $x^* = 1$ (converges if $0 < c < 2$) **39** 0 **43** $F' = 1$ at $x^* = 0$

Section 3.7 Newton's Method and Chaos (page 145)

- 1** $x_{n+1} = x_n - \frac{x_n^3 - b}{3x_n^2} = \frac{2x_n}{3} + \frac{b}{3x_n^2}$ **5** $x_1 = x_0$; x_1 is not defined (∞) **7** $x^* = 1$ or 5 from $x_0 < 3$, $x_0 > 3$
11 $x_0 < \frac{1}{2}$ to $x^* = 0$; $x_0 > \frac{1}{2}$ to $x^* = 1$ **21** $x_{n+1} = x_n - \frac{x_n^k - 7}{kx_n^{k-1}}$ **23** $x_4 = \cot \pi = \infty$; $x_3 = \cot \frac{8\pi}{7} = \cot \frac{\pi}{7}$
25 π is not a fraction **27** $\frac{1}{4}x_n^2 + \frac{1}{2} + \frac{1}{4x_n^2} = \frac{(x_n^2 + 1)^2}{4x_n^2} = \frac{y_n^2}{4(y_n - 1)}$ **29** $16z - 80z^2 + 128z^3 - 64z^4$; 4; 2
31 $|x_0| < 1$ **33** $\Delta x = 1$, one-step convergence for quadratics **35** $\frac{\Delta f}{\Delta x} = \frac{5.25}{1.5}$; $x_2 = 1.86$
37 $1.75 < x^* < 2.5$; $1.75 < x^* < 2.125$ **39** 8; $3 < x^* < 4$ **41** Increases by 1; doubles for Newton
45 $x_1 = x_0 + \cot x_0 = x_0 + \pi$ gives $x_2 = x_1 + \cot x_1 = x_1 + \pi$ **49** $a = 2$, Y 's approach $\frac{1}{2}$

Section 3.8 The Mean Value Theorem and l'Hôpital's Rule (page 152)

- 1** $c = \sqrt{\frac{4}{3}}$ **3** No c **5** $c = 1$ **7** Corner at $\frac{1}{2}$ **9** Cusp at 0
11 $\sec^2 x - \tan^2 x = \text{constant}$ **13** 6 **15** -2 **17** -1 **19** n **21** $-\frac{1}{2}$ **23** Not $\frac{0}{0}$
25 -1 **27** 1; $\frac{1 - \sin x}{1 + \cos x}$ has no limit **29** $f'(c) = \frac{4^3 - 1^3}{4 - 1}$; $c = \sqrt{7}$
31 $0 = x^* - x_{n+1} + \frac{f''(c)}{2f'(x_n)}(x^* - x_n)^2$ gives $M \approx \frac{f''(x^*)}{2f'(x^*)}$ **33** $f'(0)$; $\frac{f'(x)}{1}$; singularity **35** $\frac{f(x)}{g(x)} \rightarrow \frac{3}{4}$ **37** 1

CHAPTER 4 DERIVATIVES BY THE CHAIN RULE

Section 4.1 The Chain Rule (page 158)

- 1** $z = y^3, y = x^2 - 3, z' = 6x(x^2 - 3)^2$ **3** $z = \cos y, y = x^3, z' = -3x^2 \sin x^3$
5 $z = \sqrt{y}, y = \sin x, z' = \cos x / 2\sqrt{\sin x}$ **7** $z = \tan y + (1/\tan x), y = 1/x, z' = (\frac{-1}{x^2}) \sec^2(\frac{1}{x}) - (\tan x)^{-2} \sec^2 x$
9 $z = \cos y, y = x^2 + x + 1, z' = -(2x + 1) \sin(x^2 + x + 1)$ **11** $17 \cos 17x$ **13** $\sin(\cos x) \sin x$
15 $x^2 \cos x + 2x \sin x$ **17** $(\cos \sqrt{x+1})^{\frac{1}{2}}(x+1)^{-1/2}$ **19** $\frac{1}{2}(1 + \sin x)^{-1/2}(\cos x)$ **21** $\cos(\frac{1}{\sin x})(\frac{-\cos x}{\sin^2 x})$
23 $8x^7 = 2(x^2)^2(2x^2)(2x)$ **25** $2(x+1) + \cos(x+\pi) = 2x+2 - \cos x$
27 $(x^2+1)^2+1$; $\sin U$ from 0 to $\sin 1$; $U(\sin x)$ is 1 and 0 with period 2π ; R from 0 to x ; $R(\sin x)$ is half-waves.
29 $g(x) = x+2, h(x) = x^2+2, k(x) = 3$ **31** $f'(f(x))f'(x)$; no; $(-1/(1/x)^2)(-1/x^2) = 1$ and $f(f(x)) = x$
33 $\frac{1}{2}(\frac{1}{2}x+8)+8; \frac{1}{8}x+14; \frac{1}{16}$ **35** $f(g(x)) = x, g(f(y)) = y$
37 $f(g(x)) = \frac{1}{1-x}, g(f(x)) = 1 - \frac{1}{x}, f(f(x)) = x = g(g(x)), g(f(g(x))) = \frac{x}{x-1} = f(g(f(x)))$
39 $f(y) = y-1, g(x) = 1$ **43** $2 \cos(x^2+1) - 4x^2 \sin(x^2+1); -(x^2-1)^{-3/2}; -(\cos \sqrt{x})/4x + (\sin \sqrt{x})/4x^{3/2}$
45 $f'(u(t))u'(t)$ **47** $(\cos^2 u(x) - \sin^2 u(x)) \frac{du}{dx}$ **49** $2xu(x) + x^2 \frac{du}{dx}$ **51** $1/4\sqrt{1-\sqrt{1-x}}\sqrt{1-x}$
53 df/dt **55** $f'(g(x))g'(x) = 4(x^3)^3 3x^2 = 12x^{11}$ **57** 3600; $\frac{1}{2}$; 18 **59** 3; $\frac{1}{3}$

Section 4.2 Implicit Differentiation and Related Rates (page 163)

- 1 $-x^{n-1}/y^{n-1}$ 3 $\frac{dy}{dx} = 1$ 5 $\frac{dy}{dx} = \frac{1}{F'(y)}$ 7 $(y^2 - 2xy)/(x^2 - 2xy)$ or 1 9 $\frac{1}{\sec^2 y}$ or $\frac{1}{1+x^2}$
 11 First $\frac{dy}{dx} = -\frac{y}{x}$, second $\frac{dy}{dx} = \frac{x}{y}$ 13 Faster, faster 15 $2zz' = 2yy' \rightarrow z' = \frac{y}{x}y' = y' \sin \theta$
 17 $\sec^2 \theta = \frac{c}{200\pi}$ 19 $500 \frac{df}{dx}; 500\sqrt{1 + (\frac{df}{dx})^2}$ 21 $\frac{dy}{dt} = -\frac{8}{3}; \frac{dy}{dt} = -2\sqrt{3}; \infty$ then 0
 23 $V = \pi r^2 h; \frac{dh}{dt} = \frac{1}{4\pi} \frac{dV}{dt} = -\frac{1}{4\pi}$ in/sec 25 $A = \frac{1}{2}ab \sin \theta, \frac{dA}{dt} = 7$ 27 1.6 m/sec; 9 m/sec; 12.8 m/sec
 29 $-\frac{7}{5}$ 31 $\frac{dz}{dt} = \frac{\sqrt{2}}{2} \frac{dy}{dt}; \frac{d\theta}{dt} = \frac{1}{10} \cos^2 \theta \frac{d\theta}{dt}; \theta'' = \frac{\cos \theta}{10} y'' - \frac{1}{50} \cos^3 \theta \sin \theta (y')^2$

Section 4.3 Inverse Functions and Their Derivatives (page 170)

- 1 $x = \frac{y+6}{3}$ 3 $x = \sqrt{y+1}$ (x unrestricted $\rightarrow$ no inverse) 5 $x = \frac{1}{y-1}$ 7 $x = (1+y)^{1/3}$
 9 (x unrestricted $\rightarrow$ no inverse) 11 $y = \frac{1}{x-a}$ 13 $2 < f^{-1}(x) < 3$ 15 f goes up and down
 17 $f(x)g(x)$ and $\frac{1}{f(x)}$ 19 $m \neq 0; m \geq 0; |m| \geq 1$ 21 $\frac{dy}{dx} = 5x^4, \frac{dx}{dy} = \frac{1}{5}y^{-4/5}$
 23 $\frac{dy}{dx} = 3x^2; \frac{dx}{dy} = \frac{1}{3}(1+y)^{-2/3}$ 25 $\frac{dy}{dx} = \frac{-1}{(x-1)^2}, \frac{dx}{dy} = \frac{-1}{(y-1)^2}$ 27 $y; \frac{1}{2}y^2 + C$
 29 $f(g(x)) = -1/3x^3; g^{-1}(y) = \frac{-1}{y}; g(g^{-1}(x)) = x$ 39 $2/\sqrt{3}$ 41 $1/6 \cos 9$
 43 Decreasing; $\frac{dx}{dy} = \frac{1}{dy/dx} < 0$ 45 F; T; F 47 $g(x) = x^m, f(y) = y^n, x = (z^{1/n})^{1/m}$
 49 $g(x) = x^3, f(y) = y + 6, x = (z-6)^{1/3}$ 51 $g(x) = 10^x, f(y) = \log y, x = \log(10^y) = y$
 53 $y = x^3, y'' = 6x, d^2x/dy^2 = -\frac{2}{9}y^{-5/3}; \text{m/sec}^2, \text{sec/m}^2$ 55 $p = \frac{1}{\sqrt{y}} - 1; 0 < y \leq 1$
 57 $\max = G = \frac{3}{8}y^{4/3}, G' = \frac{1}{2}y^{1/3}$ 59 $y^2/100$

Section 4.4 Inverses of Trigonometric Functions (page 175)

- 1 $0, \frac{\pi}{2}, 0$ 3 $\frac{\pi}{2}, 0, \frac{\pi}{4}$ 5 π is outside $[-\frac{\pi}{2}, \frac{\pi}{2}]$ 7 $y = -\sqrt{3}/2$ and $\sqrt{3}/2$
 9 $\sin x = \sqrt{1-y^2}; \sqrt{1-y^2}$ and 1 11 $\frac{d(\sin^{-1} y)}{dy} \cos x = 1 \rightarrow \frac{d(\sin^{-1} y)}{dy} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-y^2}}$
 13 $y = 0: 1, -1, 1; y = 1: 0, 0, \frac{1}{2}$ 15 F; F; T; T; F; F 17 $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ 19 $\frac{dz}{dx} = 3$
 21 $\frac{dz}{dx} = \frac{2\sin^{-1} x}{\sqrt{1-x^2}}$ 23 $1 - \frac{y \sin^{-1} y}{\sqrt{1-y^2}}$ 25 $\frac{dx}{dy} = \frac{1}{|y+1|\sqrt{y^2+2y}}$ 27 $u = 1$ so $\frac{du}{dy} = 0$ 31 $\sec x = \sqrt{y^2+1}$
 33 $\frac{1}{10}, 1, \frac{1}{2}$ 35 $-y/\sqrt{1-y^2}$ 37 $\frac{1}{2} \sec \frac{x}{2} \tan \frac{x}{2}$ 39 $\frac{nx^{n-1}}{|x^n|\sqrt{x^{2n}-1}}$ 41 $\frac{dy}{dx} = \frac{1}{1+x^2}$
 43 $\frac{dy}{dx} = \pm \frac{1}{1+x^2}$ 47 $u = 4 \sin^{-1} y$ 49 π 51 $-\pi/4$

CHAPTER 5 INTEGRALS

Section 5.1 The Idea of the Integral (page 181)

- 1 1, 3, 7, 15, 127 3 $-\frac{1}{2} - \frac{1}{4} - \frac{1}{8} = \frac{1}{8} - 1$ 5 $f_j - f_0 = \frac{r^j-1}{r-1}$ 7 $3x$ for $x \leq 7, 7x-4$ for $x \geq 1$
 9 $\frac{1}{52}, \frac{1}{\sqrt{52}}, \frac{2}{52}, \frac{1}{52}\sqrt{\frac{j}{52}}$ 11 Lower by 2 13 Up, down; rectangle 15 $\sqrt{x+\Delta x} - \sqrt{x}; \frac{df}{dx}; \sqrt{x}$
 17 6; 18; triangle 19 18 rectangles 21 $6x - \frac{1}{2}x^2 - 10; 6-x$ 23 $\frac{14}{27}$ 25 $x^2; x^2; \frac{1}{3}x^3$

Section 5.2 Antiderivatives (page 186)

- 1 $x^5 + \frac{2}{3}x^6; \frac{5}{3}$ 3 $2\sqrt{x}; 2$ 5 $\frac{3}{4}x^{4/3}(1+2^{1/3}); \frac{3}{4}(1+2^{1/3})$ 7 $-2\cos x - \frac{1}{2}\cos 2x; \frac{5}{2} - 2\cos 1 - \frac{1}{2}\cos 2$
 9 $x\sin x + \cos x; \sin 1 + \cos 1 - 1$ 11 $\frac{1}{2}\sin^2 x; \frac{1}{2}\sin^2 1$ 13 $f = C; 0$ 15 $f(b) - f(a); f_7 - f_2$
 17 $8 + \frac{8}{N}$ 19 $\frac{\pi}{3}(1+\sqrt{3}); \frac{\pi}{6}(3+\sqrt{3}); 2$ 21 $\frac{5}{2}; \frac{205}{36}; \infty$ 23 $f(x) = 2\sqrt{x}$ 25 $\frac{1}{2}$, below $-1; \frac{1}{4}, \frac{5}{4}$
 27 Increase - decrease; increase - decrease - increase
 29 Area under B - area under D ; time when $B = D$; time when $B - D$ is largest 33 T; F; F; T; F

Section 5.3 Summation Versus Integration (page 194)

- 1 $\frac{25}{12}; 16$ 3 $127; 2^{n+1} - 1$ 5 $\sum_{j=1}^{50} 2j = 2550; \sum_{i=1}^{100} (2j-1) = 10,000; \sum_{k=1}^4 (-1)^{k+1}/k = \frac{7}{12}$
 7 $\sum_{k=0}^n a_k x^k; \sum_{j=1}^n \sin \frac{2\pi j}{n}$ 9 5.18738; 7.48547 11 $2(a_i^2 + b_i^2)$ 13 $2^n - 1; \frac{1}{11} - \frac{1}{1}$ 15 F; T
 17 $\frac{df}{dx} + C; f_9 - f_8 - f_1 + f_0$ 19 $f_1 = 1; n^2 + (2n+1) = (n+1)^2$
 21 $a + b + c = 1, 2a + 4b + 8c = 5, 3a + 9b + 27c = 14$; sum of squares 23 $S_{400} = 80200; E_{400} = .0025 = \frac{1}{n}$
 25 $S_{100,1/3} \approx 350, E_{100,1/3} \approx .00587; S_{100,3} = 25502500, E_{100,3} = .0201$ 27 v_1 and v_2 have the same sign
 29 $v_1 = 9, v_2 = 12, \Sigma\Sigma = 21$ 31 At $N = 1, 2^{N-2}$ is not 1 33 0; $\frac{1}{n}(v_1 + \cdots + v_n)$
 35 $\Delta x \sum_{j=1}^n v(j\Delta x)$ 37 $f(1) - f(0) = \int_0^1 \frac{df}{dx} dx$

Section 5.4 Indefinite Integrals and Substitutions (page 200)

- 1 $\frac{2}{3}(2+x)^{3/2} + C$ 3 $(x+1)^{n+1}/(n+1) + C (n \neq -1)$ 5 $\frac{1}{12}(x^2+1)^6 + C$ 7 $-\frac{1}{4}\cos^4 x + C$
 9 $-\frac{1}{8}\cos^4 2x + C$ 11 $\sin^{-1} t + C$ 13 $\frac{1}{3}(1+t^2)^{3/2} - (1+t^2)^{1/2} + C$ 15 $2\sqrt{x} + x + C$
 17 $\sec x + C$ 19 $-\cos x + C$ 21 $\frac{1}{3}x^3 + \frac{2}{3}x^{3/2}$ 23 $-\frac{1}{3}(1-2x)^{3/2}$ 25 $y = \sqrt{2x}$
 27 $\frac{1}{2}x^2$ 29 $a\sin x + b\cos x$ 31 $\frac{4}{15}x^{5/2}$ 33 F; F; F; F 35 $f(x-1); 2f(\frac{x}{2})$
 37 $x - \tan^{-1} x$ 39 $\int \frac{1}{u} du$ 41 $4.9t^2 + C_1 t + C_2$ 43 $f(t+3); f(t) + 3t; 3f(t); \frac{1}{3}f(3t)$

Section 5.5 The Definite Integral (page 205)

- 1 $C = -f(2)$ 3 $C = f(3)$ 5 $f(t)$ is wrong 7 $C = 0$ 9 $C = f(-a) - f(-b)$
 11 $u = x^2 + 1; \int_1^2 u^{10} \frac{du}{2} = \frac{u^{11}}{22} \Big|_1^2 = \frac{2^{11}-1}{22}$ 13 $u = \tan x; \int_0^1 u \, du = \frac{1}{2}$
 15 $u = \sec x; \int_1^{\sqrt{2}} u \, du = \frac{1}{2}$ (same as 13) 17 $u = \frac{1}{x}, x = \frac{1}{u}, dx = -\frac{du}{u^2}; \int_1^{1/2} -\frac{du}{u^2}$
 19 $S = \frac{1}{2}(\frac{1}{4} + 1)^4 + \frac{1}{2}(1 + 1)^4; s = \frac{1}{2}(0) + \frac{1}{2}(\frac{1}{4} + 1)^4$
 21 $S = \frac{1}{2}[(\frac{1}{2})^3 + 1^3 + (\frac{3}{2})^3 + 2^3]; s = \frac{1}{2}[0^3 + (\frac{1}{2})^3 + 1^3 + (\frac{3}{2})^3]$
 23 $S = \frac{1}{4}[(\frac{17}{16})^4 + (\frac{5}{4})^4 + (\frac{25}{16})^4 + 2^4]$ 25 Last rectangle minus first rectangle
 27 $S = .07$ since 7 intervals have points where $W = 1$. The integral of $W(x)$ exists and equals zero.
 29 M is increasing so Problem 25 gives $S - s = \Delta x(1 - 0)$; area from graph up to $y = 1$ is $\frac{1}{2} \cdot 1 + \frac{1}{4} \cdot \frac{1}{2} + \cdots = \frac{1}{2}(1 + \frac{1}{4} + \frac{1}{16} + \cdots) = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$; area under graph is $\frac{1}{3}$.
 31 $f(x) = 3 + \int_0^x v(x) dx; f(x) = \int_3^x v(x) dx$ 33 T; F; T; F; T; F; T

Section 5.6 Properties of the Integral and Average Value (page 212)

- 1 $\bar{v} = \frac{1}{2} \int_{-1}^1 x^4 dx = \frac{1}{5}$ equals c^4 at $c = \pm(\frac{1}{5})^{1/4}$ 3 $\bar{v} = \frac{1}{\pi} \int_0^\pi \cos^2 x dx = \frac{1}{2}$ equals $\cos^2 c$ at $c = \frac{\pi}{4}$ and $\frac{3\pi}{4}$
- 5 $\bar{v} = \int_1^2 \frac{dx}{x^2} = \frac{1}{2}$ equals $\frac{1}{c^2}$ at $c = \sqrt{2}$ 7 $\int_3^5 v(x) dx$ 9 False, take $v(x) < 0$
- 11 True; $\frac{1}{3} \int_0^1 v(x) dx + \frac{2}{3} \cdot \frac{1}{2} \int_1^3 v(x) dx = \frac{1}{3} \int_0^3 v(x) dx$ 13 False; when $v(x) = x^2$ the function $x^2 - \frac{1}{3}$ is even
- 15 False; take $v(x) = 1$; factor $\frac{1}{2}$ is missing 17 $\bar{v} = \frac{1}{b-a} \int_a^b v(x) dx$ 19 0 and $\frac{2}{\pi}$
- 21 $v(x) = Cx^2$; $v(x) = C$. This is "constant elasticity" in economics (Section 2.2) 23 $\bar{V} \rightarrow 0$; $\bar{V} \rightarrow 1$
- 25 $\frac{1}{2} \int_0^2 (a-x) dx = a+1$ if $a > 2$; $\frac{1}{2} \int_0^2 |a-x| dx = \frac{1}{2}$ area = $\frac{a^2}{2} - a + 1$ if $a < 2$; distance = absolute value
- 27 Small interval where $y = \sin \theta$ has probability $\frac{d\theta}{\pi}$; the average y is $\int_0^\pi \frac{\sin \theta d\theta}{\pi} = \frac{2}{\pi}$
- 29 Area under $\cos \theta$ is 1. Rectangle $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq y \leq 1$ has area $\frac{\pi}{2}$. Chance of falling across a crack is $\frac{1}{\pi/2} = \frac{2}{\pi}$.
- 31 $\frac{1}{6^3}, \frac{3}{6^3}, \dots, \frac{1}{6^3}; 10.5$ 33 $\frac{1}{t} \int_0^t 220 \cos \frac{2\pi t}{60} dt = \frac{1}{t} \cdot 220 \cdot \frac{60}{2\pi} \sin \frac{2\pi t}{60} = V_{\text{ave}}$
- 35 Any $v(x) = v_{\text{even}}(x) + v_{\text{odd}}(x)$; $(x+1)^3 = (3x^2+1) + (x^3+3x)$; $\frac{1}{x+1} = \frac{1}{1-x^2} - \frac{x}{1-x^2}$
- 37 16 per class; $\frac{6}{64}$; $E(x) = \frac{1800}{64} = \frac{225}{8}$ 39 F; F; T; T
- 41 $f(x) = \begin{cases} \frac{1}{2}(x-2)^2 & x \geq 2 \\ -\frac{1}{2}(x-2)^2 & x \leq 2 \end{cases} + C$; $f(5) - f(0) = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$

Section 5.7 The Fundamental Theorem and Its Applications (page 219)

- 1 $\cos^2 x$ 3 0 5 $(x^2)^3(2x) = 2x^7$ 7 $v(x+1) - v(x)$ 9 $\frac{\sin^2 x}{x} - \frac{1}{x^2} \int_0^x \sin^2 t dt$
- 11 $\int_0^x v(u) du$ 13 0 15 $2 \sin x^2$ 17 $u(x)v(x)$ 19 $\sin^{-1}(\sin x) \cos x = x \cos x$
- 21 F; F; F; T 23 Taking derivatives $v(x) = (x \cos x)' = \cos x - x \sin x$
- 25 Taking derivatives $-v(-x)(-1) = v(x)$ so v is even 27 F; T; T; F
- 29 $\int_1^x v(t) dt = \int_0^x v(t) dt - \int_0^1 v(t) dt = \frac{x}{x+2} - \frac{1}{1+2}$ (in revised printing)
- 31 $V = s^3$; $A = 3s^2$; half of hollow cube; $\Delta V \approx 3s^2 dS$; $3s^2$ (which is A)
- 33 $dH/dr = 2\pi^2 r^3$ 35 Wedge has length $r \approx$ height of triangle; $\int_0^{\pi/2} \frac{1}{2} r^2 d\theta = \frac{\pi r^2}{4}$
- 37 $r = \frac{1}{\cos \theta}$; $\frac{d\theta}{2 \cos^2 \theta}$; $\int_0^{\pi/4} \frac{d\theta}{2 \cos^2 \theta} = \frac{\tan \theta}{2} \Big|_0^{\pi/4} = \frac{1}{2}$
- 39 $x = y^2$; $\int_0^2 y^2 dy = \frac{y^3}{3} \Big|_0^2 = \frac{8}{3}$; vertical strips have length $2 - \sqrt{x}$
- 41 Length $\sqrt{2}a$; width $\frac{da}{\sqrt{2}}$; $\int_0^1 ada = \frac{1}{2}$ 43 The differences of the sums $f_j = v_1 + v_2 + \dots + v_j$ are $f_j - f_{j-1} = v_j$
- 45 No, $\int_0^x a(t) dt = \frac{df}{dx}(x) - \frac{df}{dx}(0)$ and $\int_0^1 (\int_0^x a(t) dt) dx = f(1) - f(0) - \frac{df}{dx}(0)$

Section 5.8 Numerical Integration (page 226)

- 1 $\frac{1}{2} \Delta x (v_0 + v_n)$ 3 1, .5625, .3025; 0, .0625, .2025 5 $L_8 \approx .1427$, $T_8 \approx .2052$, $S_8 \approx .2000$
- 7 $p = 2$: for $y = x^2$, $\frac{1}{4} \cdot 0^2 + \frac{1}{2} \cdot (\frac{1}{2})^2 + \frac{1}{4} \cdot 1^2 \neq \frac{1}{3}$ 9 For $y = x^2$, error $\frac{1}{6}(\Delta x)^2$ from $\frac{1}{2} - \frac{1}{3}$, $y'_1 = 2\Delta x$
- 13 8 intervals give $\frac{(\Delta x)^2}{12} [-\frac{1}{b^3} + \frac{1}{a^3}] = \frac{1}{1024} < .001$ 15 $f'''(c)$ is $y'(c)$ 17 ∞ ; .683, .749, .772 $\rightarrow \frac{\pi}{4}$
- 19 $A + B + C = 1$, $\frac{1}{2}B + C = \frac{1}{2}$, $\frac{1}{4}B + C = \frac{1}{3}$; Simpson
- 21 $y = 1$ and x on $[0, 1]$: $L_n = 1$ and $\frac{1}{2} - \frac{1}{2n}$, $R_n = 1$ and $\frac{1}{2} + \frac{1}{2n}$, so only $\frac{1}{2}L_n + \frac{1}{2}R_n$ gives 1 and $\frac{1}{2}$
- 23 $T_{10} \approx 500,000,000$; $T_{100} \approx 50,000,000$; $25,000\pi$
- 25 $a = 4$, $b = 2$, $c = 1$; $\int_0^1 (4x^2 + 2x + 1) dx = \frac{19}{3}$; Simpson fits parabola 27 $c = \frac{1}{4320}$

CHAPTER 6 EXPONENTIALS AND LOGARITHMS

Section 6.1 An Overview (page 234)

- 1** $5; -5; -1; \frac{1}{5}; \frac{3}{2}; 2$ **5** $1; -10; 80; 1; 4; -1$ **7** $n \log_b x$ **9** $\frac{10}{3}; \frac{3}{10}$ **13** 10^5
15 $0; I_{SF} = 10^7 I_0; 8.3 + \log_{10} 4$ **17** $A = 7, b = 2.5$ **19** $A = 4, k = 1.5$
21 $\frac{1}{cx}; \frac{2}{cx}; \log 2$ **23** $y - 1 = cx; y - 10 = c(x - 1)$ **25** $(.1^{-h} - 1)/(-h) = (10^h - 1)/(-h)$
27 $y' = c^2 b^x; x'' = -1/cy^2$ **29** Logarithm

Section 6.2 The Exponential e^x (page 241)

- 1** $49e^{7x}$ **3** $8e^{8x}$ **5** $3^x \ln 3$ **7** $(\frac{2}{3})^x \ln \frac{2}{3}$ **9** $\frac{-e^x}{(1+e^x)^2}$ **11** 2 **13** xe^x **15** $\frac{4}{(e^x + e^{-x})^2}$
17 $e^{\sin x} \cos x + e^x \cos e^x$ **19** .1246, .0135, .0014 are close to $\frac{e}{2n}$ **21** $\frac{1}{e}; \frac{1}{e}$
23 $Y(h) = 1 + \frac{1}{10}; Y(1) = (1 + \frac{1}{10})^{10} = 2.59$ **25** $(1 + \frac{1}{x})^x < e < e^x < e^{3x/2} < e^{2x} < 10^x < x^x$
27 $\frac{e^{3x}}{3} + \frac{e^{7x}}{7}$ **29** $x + \frac{2^x}{\ln 2} + \frac{3^x}{\ln 3}$ **31** $\frac{(2e)^x}{\ln(2e)} + 2e^x$ **33** $\frac{e^{x^2}}{2} - \frac{e^{-x^2}}{2}$
35 $2e^{x/2} + \frac{e^{2x}}{2}$ **37** e^{-x} drops faster at $x = 0$ (slope -1); meet at $x = 1$; $e^{-x^2}/e^{-x} < e^{-9}/e^{-3} < \frac{1}{100}$ for $x > 3$
39 $y - e^a = e^a(x - a)$; need $-e^a = -ae^a$ or $a = 1$
41 $y' = x^x(\ln x + 1) = 0$ at $x_{\min} = \frac{1}{e}$; $y'' = x^x[(\ln x + 1)^2 + \frac{1}{x}] > 0$
43 $\frac{d}{dx}(e^{-x}y) = e^{-x}\frac{dy}{dx} - e^{-x}y = 0$ so $e^{-x}y = \text{Constant}$ or $y = Ce^x$
45 $\frac{e^{2x}}{2}|_0^1 = \frac{e^2 - 1}{2}$ **47** $\frac{2^x}{\ln 2}|_{-1}^1 = \frac{2 - \frac{1}{2}}{\ln 2} = \frac{3}{2 \ln 2}$ **49** $-e^{-x}|_0^\infty = 1$ **51** $e^{1+x}|_0^1 = e^2 - e$ **53** $\frac{2^{\sin x}}{\ln 2}|_0^\pi = 0$
55 $\int \frac{du}{e^u} dx = -e^{-u} + C$; $\int (e^u)^2 \frac{du}{dx} dx = \frac{1}{2}e^{2u} + C$ **57** $yy' = 1$ gives $\frac{1}{2}y^2 = x + C$ or $y = \sqrt{2x + 2C}$
59 $\frac{dF}{dx} = (n - x)x^{n-1}/e^x < 0$ for $x > n$; $F(2x) < \frac{\text{constant}}{e^x} \rightarrow 0$ **61** $\frac{6!}{\sqrt{12\pi}} \approx 117$; $(\frac{6}{e})^6 \approx 116$; 7 digits

Section 6.3 Growth and Decay in Science and Economics (page 250)

- 1** $t^2 + y_0$ **3** $y_0 e^{2t}$ **5** $10 e^{4t}; t = \frac{\ln 10}{4}$ **7** $\frac{1}{4} e^{4t} + 9.75; t = \frac{\ln 361}{4}$ **11** $c = \frac{\ln 2}{2}; t = \frac{\ln 10}{c}$
13 $\frac{5568}{-7} \ln(\frac{1}{5})$ **15** $c = \frac{\ln 2}{20}; t = \frac{1}{c} \ln(\frac{8}{5})$ **17** $t = \frac{\ln(1/240)}{\ln(.98)}$ **19** $e^c = 3$ so $y_0 = e^{-3c} 1000 = \frac{1000}{27}$
21 $p = 1013 e^{ch}$; $50 = 1013 e^{20c}$; $c = \frac{1}{20} \ln(\frac{50}{1013})$; $p(10) = 1013 e^{10c} = 1013 \sqrt{\frac{50}{1013}} = \sqrt{(1013)(50)}$
23 $c = \frac{\ln 2}{3}; (\frac{1}{2})^3 = \frac{1}{8}$ **25** $y = y_0 - at$ reaches y_1 at $t = \frac{y_0 - y_1}{a}$; then $y = Ae^{-at/y_1}$ **27** F; F; T; T
29 $A = \frac{1}{3}, B = -\frac{1}{3}$ **31** $e^t - 1$ **33** $1 - e^{-t}$ **35** $6; 6 + Ae^{-2t}; 6 - 6e^{-2t}; 6 + 4e^{-2t}; 6$
37 $4; 4 - \frac{1}{e}; 4$ **39** $ye^{-t}; y(t) = te^t$ **41** $A = 1, B = -1, C = -1$ **43** $e^{.0725} > .075$ **45** $s(e - 1); \frac{s(e-1)}{e}$
47 $(1.02)(1.03) \rightarrow 5.06\%$; 5% by Problem 27 **49** $20,000 e^{(20-T)(.05)} = 34,400$ (it grows for $20 - T$ years)
51 $s = -cy_0 e^{ct}/(e^{ct} - 1) = -(0.1)(1000)e^{.60}/(e^{.60} - 1)$ **53** $y_0 = \frac{100}{.005}(1 - e^{-.005(48)})$
55 $e^{4c} = 1.20$ so $c = \frac{\ln 1.20}{4}$ **57** $24e^{36.5} = ?$ **59** To $-\infty$; constant; to $+\infty$
61 $\frac{dY}{dT} = 60cY$; $\frac{dY}{dT} = 60(-Y + 5)$; still $Y_\infty = 5$
63 $y = 60e^{ct} + 20, 60 = 60e^{12c} + 20, c = \frac{1}{12} \ln(\frac{40}{60})$; $100 = 60e^{ct} + 20$ at $t = \frac{1}{c} \ln(\frac{80}{60})$ **65** 0

Section 6.4 Logarithms (page 258)

- 1** $\frac{1}{x}$ **3** $\frac{-1}{x(\ln x)^2}$ **5** $\ln x$ **7** $\frac{\cos x}{\sin x} = \cot x$ **9** $\frac{7}{x}$ **11** $\frac{1}{3} \ln t + C$ **13** $\ln \frac{4}{3}$
15 $\frac{1}{2} \ln 5$ **17** $-\ln(\ln 2)$ **19** $\ln(\sin x) + C$ **21** $-\frac{1}{3} \ln(\cos 3x) + C$ **23** $\frac{1}{3}(\ln x)^3 + C$
27 $\ln y = \frac{1}{2} \ln(x^2 + 1); \frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 1}}$ **29** $\frac{dy}{dx} = e^{\sin x} \cos x$
31 $\frac{dy}{dx} = e^x e^{e^x}$ **33** $\ln y = e^x \ln x; \frac{dy}{dx} = ye^x(\ln x + \frac{1}{x})$ **35** $\ln y = -1$ so $y = \frac{1}{e}, \frac{dy}{dx} = 0$ **37** 0
39 $-\frac{1}{x}$ **41** $\sec x$ **47** .1; .095; .095310179 **49** $-.01; -.01005; -.010050335$
51 l'Hôpital: 1 **53** $\frac{1}{\ln b}$ **55** $3 - 2 \ln 2$ **57** Rectangular area $\frac{1}{2} + \cdots + \frac{1}{n} < \int_1^n \frac{dt}{t} = \ln n$
59 Maximum at e **61** 0 **63** $\log_{10} e$ or $\frac{1}{\ln 10}$ **65** $1 - x; 1 + x \ln 2$
67 Fraction is $y = 1$ when $\ln(T + 2) - \ln 2 = 1$ or $T = 2e - 2$ **69** $y' = \frac{2}{(t+2)^2} \rightarrow y = 1 - \frac{2}{t+2}$ never equals 1
71 $\ln p = x \ln 2; \text{LD } 2^x \ln 2; \text{ED } p = e^{x \ln 2}, p' = \ln 2 e^{x \ln 2}$
75 $2^4 = 4^2; y \ln x = x \ln y \rightarrow \frac{\ln x}{x} = \frac{\ln y}{y}; \frac{\ln x}{x}$ decreases after $x = e$, and the only integers before e are 1 and 2.

Section 6.5 Separable Equations Including the Logistic Equation (page 266)

- 1** $7e^t - 5$ **3** $(\frac{3}{2}x^2 + 1)^{1/3}$ **5** x **7** $e^{1 - \cos t}$ **9** $(\frac{ct}{2} + \sqrt{y_0})^2$ **11** $y_\infty = 0; t = \frac{1}{by_0}$
15 $z = 1 + e^{-t}, y$ is in **13** **17** $ct = \ln 3, ct = \ln 9$
19 $b = 10^{-9}, c = 13 \cdot 10^{-3}; y_\infty = 13 \cdot 10^6$; at $y = \frac{c}{2b}$ (10) gives $\ln \frac{1}{b} = ct + \ln \frac{10^6}{c - 10^6 b}$ so $t = 1900 + \frac{\ln 12}{c} = 2091$
21 y^2 dips down and up (a valley) **23** $sc = 1 = sbr$ so $s = \frac{1}{c}, r = \frac{c}{b}$
25 $y = \frac{N}{1 + e^{-Nt/(N-1)}}; T = \frac{\ln(N-1)}{N} \rightarrow 0$ **27** Dividing cy by $y + K > 1$ slows down y'
29 $\frac{dR}{dy} = \frac{cK}{(y+K)^2} > 0, \frac{cy}{y+K} \rightarrow c$
31 $\frac{dY}{dT} = \frac{-Y}{Y+1}$; multiply $e^{Y/K} \frac{Y}{K} = e^{-ct/K} e^{y_0/K} (\frac{y_0}{K})$ by K and take the K th power to reach (19)
33 $y' = (3 - y)^2; \frac{1}{3-y} = t + \frac{1}{3}; y = 2$ at $t = \frac{2}{3}$
35 $Ae^t + D = Ae^t + B + Dt + t \rightarrow D = -1, B = -1; y_0 = A + B$ gives $A = 1$
37 $y \rightarrow 1$ from $y_0 > 0, y \rightarrow -\infty$ from $y_0 < 0; y \rightarrow 1$ from $y_0 > 0, y \rightarrow -1$ from $y_0 < 0$
39 $\int \frac{\cos y \, dy}{\sin y} = \int dt \rightarrow \ln(\sin y) = t + C = t + \ln \frac{1}{2}$. Then $\sin y = \frac{1}{2}e^t$ stops at 1 when $t = \ln 2$

Section 6.6 Powers Instead of Exponentials (page 276)

- 1** $1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$ **3** $1 \pm x + \frac{x^2}{2} \pm \frac{x^3}{6} + \cdots$ **5** 1050.62; 1050.95; 1051.25
7 $1 + n(\frac{-1}{n}) + \frac{n(n+1)}{2}(\frac{-1}{n})^2 \rightarrow 1 - 1 + \frac{1}{2}$ **9** square of $(1 + \frac{1}{n})^n$; set $N = 2n$
11 Increases; $\ln(1 + \frac{1}{x}) - \frac{1}{x+1} > 0$ **13** $y(3) = 8$ **15** $y(t) = 4(3^t)$ **17** $y(t) = t$
19 $y(t) = \frac{1}{2}(3^t - 1)$ **21** $s(\frac{a^t-1}{a-1})$ if $a \neq 1; st$ if $a = 1$ **23** $y_0 = 6$ **25** $y_0 = 3$
27 $-2, -10, -26 \rightarrow -\infty; -5, -\frac{17}{2}, -\frac{41}{4} \rightarrow -12$ **29** $P = \frac{b}{c+d}$ **31** 10.38% **33** $100(1.1)^{20} = \$673$
35 $\frac{100,000(.1/12)}{1 - (1.1/12)^{-240}} = 965$ **37** $\frac{1000}{.1}(1.1^{20} - 1) = 57,275$ **39** $y_\infty = 1500$ **41** $2; (\frac{53}{52})^{52} = 2.69; e$
43 $1.0142^{12} = 1.184 \rightarrow$ Visa charges 18.4%

Section 6.7 Hyperbolic Functions (page 280)

- 1** $e^x, e^{-x}, \frac{e^{2x} - e^{-2x}}{4} = \frac{1}{2} \sinh 2x$ **7** $\sinh nx$ **9** $3 \sinh(3x + 1)$ **11** $\frac{-\sinh x}{\cosh^3 x} = -\tanh x \operatorname{sech} x$
13 $4 \cosh x \sinh x$ **15** $\frac{x}{\sqrt{x^2 + 1}} (\operatorname{sech} \sqrt{x^2 + 1})^2$ **17** $6 \sinh^5 x \cosh x$
19 $\cosh(\ln x) = \frac{1}{2}(x + \frac{1}{x}) = 1$ at $x = 1$ **21** $\frac{5}{13}, \frac{13}{5}, -\frac{12}{5}, -\frac{13}{12}, -\frac{5}{12}$ **23** 0, 0, 1, ∞, ∞
25 $\frac{1}{2} \sinh(2x + 1)$ **27** $\frac{1}{3} \cosh^3 x$ **29** $\ln(1 + \cosh x)$ **31** e^x

- 33 $\int y \, dx = \int \sinh t (\sinh t \, dt); A = \frac{1}{2} \sinh t \cosh t - \int y \, dx; A' = \frac{1}{2}; A = 0$ at $t = 0$ so $A = \frac{1}{2}t$.
 41 $e^y = x + \sqrt{x^2 + 1}, y = \ln[x + \sqrt{x^2 + 1}]$ 47 $\frac{1}{4} \ln \left| \frac{2+x}{2-x} \right|$ 49 $\sinh^{-1} x$ (see 41) 51 $-\operatorname{sech}^{-1} x$
 53 $\frac{1}{2} \ln 3; \infty$ 55 $y(x) = \frac{1}{c} \cosh cx; \frac{1}{c} \cosh cL - \frac{1}{c}$
 57 $y'' = y - 3y^2; \frac{1}{2}(y')^2 = \frac{1}{2}y^2 - y^3$ is satisfied by $y = \frac{1}{2}\operatorname{sech}^2 \frac{x}{2}$

CHAPTER 7 TECHNIQUES OF INTEGRATION

Section 7.1 Integration by Parts (page 287)

- 1 $-x \cos x + \sin x + C$ 3 $-xe^{-x} - e^{-x} + C$ 5 $x^2 \sin x + 2x \cos x - 2 \sin x + C$
 7 $\frac{1}{2}(2x+1) \ln(2x+1) - x + C$ 9 $\frac{1}{2}e^x(\sin x - \cos x) + C$ 11 $\frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + C$
 13 $\frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + C$ 15 $x(\ln x)^2 - 2x \ln x + 2x + C$ 17 $x \sin^{-1} x + \sqrt{1-x^2} + C$
 19 $\frac{1}{2}(x^2+1) \tan^{-1} x - \frac{x}{2} + C$ 21 $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$
 23 $e^x(x^3 - 3x^2 + 6x - 6) + C$ 25 $x \tan x + \ln(\cos x) + C$ 27 -1 29 $-\frac{3}{4}e^{-2} + \frac{1}{4}$ 31 -2
 33 $3 \ln 10 - 6 + 2 \tan^{-1} 3$ 35 $u = x^n, v = e^x$ 37 $u = x^n, v = \sin x$ 39 $u = (\ln x)^n, v = x$
 41 $u = x \sin x, v = e^x \rightarrow \int e^x \sin x \, dx$ in 9 and $-\int x \cos x e^x dx$. Then $u = -x \cos x, v = e^x \rightarrow \int e^x \cos x \, dx$ in 10 and $-\int x \sin x e^x dx$ (move to left side): $\frac{e^x}{2}(x \sin x - x \cos x + \cos x)$. Also try $u = xe^x, v = -\cos x$.
 43 $\int \frac{1}{2} u \sin u \, du = \frac{1}{2}(\sin u - u \cos u) = \frac{1}{2}(\sin x^2 - x^2 \cos x^2)$; odd
 45 3· step function; $3e^x$ · step function 49 $0; x\delta(x) - \int \delta(x) dx = -1; v(x)\delta(x) - \int v(x)\delta(x) dx$
 51 $v(x) = \int_x^1 f(x) dx$
 53 $u(x) = \frac{1}{k} \int_0^x v(x) dx; \frac{1}{k}(\frac{x}{2} - \frac{x^3}{6}); \frac{x}{k}$ for $x \leq \frac{1}{2}, \frac{1}{k}(2x - x^2 - \frac{1}{4})$ for $x \geq \frac{1}{2}; \frac{x}{k}$ for $x \leq \frac{1}{2}, \frac{1}{2k}$ for $x \geq \frac{1}{2}$.
 55 $u = x^2, v = -\cos x \rightarrow -x^2 \cos x + (2x) \sin x - \int 2 \sin x \, dx$ 57 Compare 23
 59 $uw'|_0^1 - \int_0^1 u'w' - u'w|_0^1 + \int_0^1 u'w' = [uw' - u'w]_0^1$
 61 No mistake: $e^x \cosh x - e^x \sinh x = 1$ is part of the constant C

Section 7.2 Trigonometric Integrals (page 293)

- 1 $\int (1 - \cos^2 x) \sin x \, dx = -\cos x + \frac{1}{3} \cos^3 x + C$ 3 $\frac{1}{2} \sin^2 x + C$
 5 $\int (1 - u^2)^2 u^2 (-du) = -\frac{1}{3} \cos^3 x + \frac{2}{5} \cos^5 x - \frac{1}{7} \cos^7 x + C$ 7 $\frac{2}{3}(\sin x)^{3/2} + C$
 9 $\frac{1}{8} \int \sin^3 2x \, dx = \frac{1}{16}(-\cos 2x + \frac{1}{3} \cos^3 2x) + C$ 11 $\frac{\pi}{2}$ 13 $\frac{1}{3}(\frac{3x}{2} + \frac{\sin 6x}{4}) + C$
 15 $x + C$ 17 $\frac{1}{6} \cos^5 x \sin x + \frac{5}{6} \int \cos^4 x \, dx$; use equation (5)
 19 $\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \cos^{n-2} x \, dx = \dots = \frac{n-1}{n} \frac{n-3}{n-2} \dots \frac{1}{2} \int_0^{\pi/2} dx$
 21 $I = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \cos^2 x \, dx = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \, dx - (n-1)I$.
 So $nI = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \, dx$.
 23 $0, +, 0, 0, 0, -$ 25 $-\frac{2}{3} \cos^3 x, 0$ 27 $-\frac{1}{2}(\frac{\cos 2x}{2} + \frac{\cos 200x}{200}), 0$ 29 $\frac{1}{2}(\frac{\sin 200x}{200} + \frac{\sin 2x}{2}), 0$
 31 $-\frac{1}{2} \cos x, 0$ 33 $\int_0^\pi x \sin x \, dx = \int_0^\pi A \sin^2 x \, dx \rightarrow A = 2$ 35 Sum = zero = $\frac{1}{2}$ (left + right)
 37 p is even 39 $p - q$ is even 41 $\sec x + C$ 43 $\frac{1}{3} \tan^3 x + C$ 45 $\frac{1}{3} \sec^3 x + C$
 47 $\frac{1}{3} \tan^3 x - \tan x + x + C$ 49 $\ln |\sin x| + C$ 51 $\frac{1}{2 \cos^2 x} + C$ 53 $A = \sqrt{2}, -\sqrt{2} \sin(x + \frac{\pi}{4})$
 55 $4\sqrt{2}$ 57 $\frac{1000}{\sqrt{3}}$ 59 $\frac{1-\cos x + \sin x}{1+\cos x + \sin x} + C$ 61 p and q are 10 and 1

Section 7.3 Trigonometric Substitutions (page 299)

- 1** $x = 2 \sin \theta; \int d\theta = \sin^{-1} \frac{x}{2} + C$ **3** $x = 2 \sin \theta; \int 4 \cos^2 \theta d\theta = 2 \sin^{-1} \frac{x}{2} + x \sqrt{1 - \frac{x^2}{4}} + C$
5 $x = \sin \theta; \int \sin^2 \theta d\theta = \frac{1}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1 - x^2} + C$
7 $x = \tan \theta; \int \cos^2 \theta d\theta = \frac{1}{2} \tan^{-1} x + \frac{1}{2} \frac{x}{1+x^2} + C$
9 $x = 5 \sec \theta; \int 5(\sec^2 \theta - 1) d\theta = \sqrt{x^2 - 25} - 5 \sec^{-1} \frac{x}{5} + C$
11 $x = \sec \theta; \int \cos \theta d\theta = \frac{\sqrt{x^2-1}}{x} + C$ **13** $x = \tan \theta; \int \cos \theta d\theta = \frac{x}{\sqrt{1+x^2}} + C$
15 $x = 3 \sec \theta; \int \frac{\cos \theta d\theta}{9 \sin^3 \theta} = \frac{-1}{9 \sin \theta} + C = \frac{-x}{9\sqrt{x^2-9}} + C$
17 $x = \sec \theta; \int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln(\sec \theta + \tan \theta) + C = \frac{1}{2} x \sqrt{x^2 - 1} + \frac{1}{2} \ln(x + \sqrt{x^2 - 1}) + C$
19 $x = \tan \theta; \int \frac{\cos \theta d\theta}{\sin^3 \theta} = -\frac{1}{\sin \theta} + C = \frac{-\sqrt{x^2+1}}{x} + C$
21 $\int \frac{-\sin \theta d\theta}{\sin \theta} = -\theta + C = -\cos^{-1} x + C$; with $C = \frac{\pi}{2}$ this is $\sin^{-1} x$
23 $\int \frac{\tan \theta \sec^3 \theta d\theta}{\sec^2 \theta} = -\ln(\cos \theta) + C = \ln \sqrt{x^2 + 1} + C$ which is $\frac{1}{2} \ln(x^2 + 1) + C$
25 $x = a \sin \theta; \int_{-\pi/2}^{\pi/2} a^2 \cos^2 \theta d\theta = \frac{a^2 \pi}{2} = \text{area of semicircle}$ **27** $\sin^{-1} x \Big|_{-1/5}^1 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$
29 Like Example 6: $x = \sin \theta$ with $\theta = \frac{\pi}{2}$ when $x = \infty, \theta = \frac{\pi}{3}$ when $x = 2, \int_{\pi/3}^{\pi/2} \frac{\cos \theta d\theta}{\sin^2 \theta} = -1 + \frac{2}{\sqrt{3}}$
31 $x = 3 \tan \theta; \int_{-\pi/2}^{\pi/2} \frac{3 \sec^2 \theta d\theta}{9 \sec^2 \theta} = \frac{\theta}{3} \Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{3}$ **33** $\int \frac{x^{n+1} + x^{n-1}}{x^2 + 1} dx = \int x^{n-1} dx = \frac{x^n}{n}$
35 $x = \sec \theta; \frac{1}{2}(e^f + e^{-f}) = \frac{1}{2}(x + \sqrt{x^2 - 1} + \frac{1}{x + \sqrt{x^2 - 1}}) = \frac{1}{2}(x + \sqrt{x^2 - 1} + x - \sqrt{x^2 - 1}) = x$
37 $x = \cosh \theta; \int d\theta = \cosh^{-1} x + C$
39 $x = \cosh \theta; \int \sinh^2 \theta d\theta = \frac{1}{2}(\sinh \theta \cosh \theta - \theta) + C = \frac{1}{2} x \sqrt{x^2 - 1} - \frac{1}{2} \ln(x + \sqrt{x^2 - 1}) + C$
41 $x = \tanh \theta; \int d\theta = \tanh^{-1} x + C$ **43** $(x-2)^2 + 4$ **45** $(x-3)^2 - 9$ **47** $(x+1)^2$
49 $u = x - 2, \int \frac{du}{u^2 + 4} = \frac{1}{2} \tan^{-1} \frac{u}{2} = \frac{1}{2} \tan^{-1}(\frac{x-2}{2}) + C; u = x - 3, \int \frac{du}{u^2 - 9} = \frac{1}{6} \ln \frac{u-3}{u+3} = \frac{1}{6} \ln \frac{x-6}{x} + C;$
 $u = x + 1, \int \frac{du}{u^2} = \frac{-1}{u} = \frac{-1}{x+1} + C$
51 $u = x + b; \int \frac{du}{u^2 - b^2 + c}$ uses $u = a \sec \theta$ if $b^2 > c, u = a \tan \theta$ if $b^2 < c$, equals $-\frac{1}{u} = \frac{-1}{x+b}$ if $b^2 = c$
53 $\cos \theta$ is negative ($-\sqrt{1-x^2}$) from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$; then $\int_0^1 -\int_1^{-1} + \int_{-1}^0 \sqrt{1-x^2} dx = \pi = \text{area of unit circle}$
55 Divide y by 4, multiply dx by 4, same $\int y dx$
57 No $\sin^{-1} x$ for $x > 1$; the square root is imaginary. All correct with complex numbers.

Section 7.4 Partial Fractions (page 304)

- 1** $A = -1, B = 1, -\ln x + \ln(x-1) + C$ **3** $\frac{1}{x-3} - \frac{1}{x-2}$ **5** $\frac{1}{2x} - \frac{2}{x+1} + \frac{5/2}{x+2}$
7 $\frac{3}{x} + \frac{1}{x^2}$ **9** $3 - \frac{3}{x^2+1}$ **11** $-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1}$ **13** $-\frac{1/6}{x} + \frac{1/2}{x-1} - \frac{1/2}{x-2} + \frac{1/6}{x-3}$
15 $\frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}; A = -\frac{1}{4}, B = \frac{1}{4}, C = 0, D = -\frac{1}{2}$
17 Coefficients of $y: 0 = -Ab + B$; match constants $1 = Ac; A = \frac{1}{c}, B = \frac{b}{c}$
19 $A = 1$, then $B = 2$ and $C = 1; \int \frac{dx}{x-1} + \int \frac{(2x+1)dx}{x^2+x+1} =$
 $\ln(x-1) + \ln(x^2+x+1) = \ln(x-1)(x^2+x+1) = \ln(x^3-1)$
21 $u = e^x; \int \frac{du}{u^2-u} = \int \frac{du}{u-1} - \int \frac{du}{u} = \ln\left(\frac{u-1}{u}\right) + C = \ln\left(\frac{e^x-1}{e^x}\right) + C$
23 $u = \cos \theta; \int \frac{-du}{1-u^2} = -\frac{1}{2} \int \frac{du}{1-u} - \frac{1}{2} \int \frac{du}{1+u} = \frac{1}{2} \ln(1-u) - \frac{1}{2} \ln(1+u) = \frac{1}{2} \ln \frac{1-\cos \theta}{1+\cos \theta} + C$. We can reach
 $\frac{1}{2} \ln \frac{(1-\cos \theta)^2}{1-\cos^2 \theta} = \ln \frac{1-\cos \theta}{\sin \theta} = \ln(\csc \theta - \cot \theta)$ or a different way $\frac{1}{2} \ln \frac{1-\cos^2 \theta}{(1+\cos \theta)^2} = \ln \frac{\sin \theta}{1+\cos \theta} = -\ln \frac{1+\cos \theta}{\sin \theta} =$
 $-\ln(\csc \theta + \cot \theta)$
25 $u = e^x; du = e^x dx = u dx; \int \frac{1+u}{(1-u)u} du = \int \frac{2du}{1-u} + \int \frac{du}{u} = -2 \ln(1-e^x) + \ln e^x + C = -2 \ln(1-e^x) + x + C$

$$27 \quad x+1=u^2, dx=2u \, du; \int \frac{2u \, du}{1+u} = \int [2 - \frac{2}{1+u}] du = 2u - 2 \ln(1+u) + C = 2\sqrt{x+1} - 2 \ln(1+\sqrt{x+1}) + C$$

$$29 \quad \text{Note } Q(a) = 0. \text{ Then } \frac{x-a}{Q(x)} = \frac{x-a}{Q(x)-Q(a)} \rightarrow \frac{1}{Q'(a)} \text{ by definition of derivative. At a double root } Q'(a) = 0.$$

Section 7.5 Improper Integrals (page 309)

$$1 \quad \frac{x^{1-e}}{1-e} \Big|_1^\infty = \frac{1}{e-1} \quad 3 \quad -2(1-x)^{1/2} \Big|_0^1 = 2 \quad 5 \quad \tan^{-1} x \Big|_{-\pi/2}^0 = \frac{\pi}{2} \quad 7 \quad \frac{1}{2}(\ln x)^2 \Big|_0^1 = -\infty$$

$$9 \quad x \ln x - x \Big|_0^\infty = 0 \quad 11 \quad \ln(\ln(\ln x)) \Big|_{100}^\infty = \infty \quad 13 \quad \frac{1}{2}(x + \sin x \cos x) \Big|_0^\infty = \infty$$

$$15 \quad \frac{x^{1-p}}{1-p} \Big|_0^\infty \text{ diverges for every } p! \quad 17 \quad \text{Less than } \int_1^\infty \frac{dx}{x^5} = \frac{1}{5}$$

$$19 \quad \text{Less than } \int_0^1 \frac{dx}{x^2+1} + \int_1^\infty \frac{\sqrt{x} \, dx}{x^2} = \tan^{-1} x \Big|_0^1 - \frac{2}{\sqrt{x}} \Big|_1^\infty = \frac{\pi}{4} + 2$$

$$21 \quad \text{Less than } \int_1^\infty e^{-x} dx = \frac{1}{e}, \text{ greater than } -\frac{1}{e}$$

$$23 \quad \text{Less than } \int_0^1 e^2 dx + e \int_1^\infty e^{-(x-1)^2} dx = e^2 + e \int_1^\infty e^{-u^2} du = e^2 + \frac{e}{\sqrt{\pi}}$$

$$25 \quad \int_0^1 \frac{\sin^2 x \, dx}{x^2} + \int_1^\infty \frac{\sin^2 x \, dx}{x^2} \text{ less than } 1 + \int_1^\infty \frac{dx}{x^2} = 2 \quad 27 \quad p! = p \text{ times } (p-1)!; 1 = 1 \text{ times } 0!$$

$$29 \quad u = x, dv = x e^{-x^2} dx : -x \frac{e^{-x^2}}{2} \Big|_0^\infty + \int_0^\infty \frac{e^{-x^2}}{2} dx = \frac{1}{4} \sqrt{\pi} \quad 31 \quad \int_0^\infty 1000 e^{-.1t} dt = -10,000 e^{-.1t} \Big|_0^\infty = \$10,000$$

$$33 \quad W = \frac{-GMm}{x} \Big|_R^\infty = \frac{GMm}{R} = \frac{1}{2} m v_0^2 \text{ if } v_0 = \sqrt{\frac{2GM}{R}}$$

$$35 \quad \int_0^\infty \frac{dx}{2^x} = \int_0^\infty e^{-x \ln 2} dx = \frac{e^{-x \ln 2}}{-\ln 2} \Big|_0^\infty = \frac{1}{\ln 2}$$

$$37 \quad \int_0^{\pi/2} (\sec x - \tan x) dx = [\ln(\sec x + \tan x) + \ln(\cos x)]_0^{\pi/2} = [\ln(1 + \sin x)]_0^{\pi/2} = \ln 2.$$

The areas under $\sec x$ and $\tan x$ separately are infinite 39 Only $p = 0$

CHAPTER 8 APPLICATIONS OF THE INTEGRAL

Section 8.1 Areas and Volumes by Slices (page 318)

$$1 \quad x^2 - 3 = 1 \text{ gives } x = \pm 2; \int_{-2}^2 [(1 - (x^2 - 3))] dx = \frac{32}{3}$$

$$3 \quad y^2 = x = 9 \text{ gives } y = \pm 3; \int_{-3}^3 [9 - y^2] dy = 36$$

$$5 \quad x^4 - 2x^2 = 2x^2 \text{ gives } x = \pm 2 \text{ (or } x = 0); \int_{-2}^2 [2x^2 - (x^4 - 2x^2)] dx = \frac{128}{15}$$

$$7 \quad y = x^2 = -x^2 + 18x \text{ gives } x = 0, 9; \int_0^9 [(-x^2 + 18x) - x^2] dx = 243$$

$$9 \quad y = \cos x = \cos^2 x \text{ when } \cos x = 1 \text{ or } 0, x = 0 \text{ or } \frac{\pi}{2} \text{ or } \dots \quad \int_0^{\pi/2} (\cos x - \cos^2 x) dx = 1 - \frac{\pi}{4}$$

$$11 \quad e^x = e^{2x-1} \text{ gives } x = 1; \int_0^1 [e^x - e^{2x-1}] dx = (e-1) - \left(\frac{e-e^{-1}}{2}\right)$$

$$13 \quad \text{Intersections } (0,0), (1,3), (2,2); \int_0^1 [3-x] dx + \int_1^2 [4-x-x] dx = 2$$

$$15 \quad \text{Inside, since } 1 - x^2 < \sqrt{1-x^2}; \int_{-1}^1 [\sqrt{1-x^2} - (1-x^2)] dx = \frac{\pi}{2} - \frac{4}{3}$$

$$17 \quad V = \int_{-a}^a \pi y^2 dx = \int_{-a}^a \pi b^2 (1 - \frac{x^2}{a^2}) dx = \frac{4\pi b^2 a}{3}; \text{ around } y \text{ axis } V = \frac{4\pi a^2 b}{3}; \text{ rotating}$$

$x = 2, y = 0$ around y axis gives a circle not in the first football

$$19 \quad V = \int_0^\pi 2\pi x \sin x \, dx = 2\pi^2 \quad 21 \quad \int_0^8 \pi(8-x)^2 dx = \frac{512\pi}{3}; \int_0^8 2\pi x(8-x) dx = \frac{512\pi}{3} \text{ (same cone tipped over)}$$

$$23 \quad \int_0^1 \pi \cdot 1^2 dx - \int_0^1 \pi(x^4)^2 dx = \frac{8\pi}{9}; \int_0^1 2\pi(1-x^4)x \, dx = \frac{2\pi}{3}$$

$$25 \quad \int_{1/3}^2 \pi(3^2) dx - \int_{1/3}^2 \pi(\frac{1}{x})^2 dx = \frac{25\pi}{2}; \int_{1/3}^2 2\pi x(3 - \frac{1}{x}) dx = \frac{25\pi}{3}$$

$$27 \quad \int_0^1 \pi[(x^{2/3})^2 - (x^{3/2})^2] dx = \frac{5\pi}{28}; \int_0^1 2\pi x(x^{2/3} - x^{3/2}) dx = \frac{5\pi}{28} \text{ (notice } xy \text{ symmetry)}$$

$$29 \quad x^2 = R^2 - y^2, V = \int_{R-h}^R \pi(R^2 - y^2) dy = \pi(Rh^2 - \frac{h^3}{3})$$

$$31 \quad \int_{-a}^a (2\sqrt{a^2 - x^2})^2 dx = \frac{16}{3} a^3 \quad 33 \quad \int_0^1 (2\sqrt{1-y})^2 dy = 2 \quad 37 \quad \int A(x) dx \text{ or in this case } \int a(y) dy$$

$$39 \quad \text{Ellipse; } \sqrt{1-x^2} \tan \theta; \frac{1}{2}(1-x^2) \tan \theta; \frac{2}{3} \tan \theta$$

$$41 \quad \text{Half of } \pi r^2 h; \text{ rectangles} \quad 43 \quad \int_1^3 \pi(5^2 - 2^2) dx = 42\pi \quad 45 \quad \int_1^3 \pi(4^2 - 1^2) dx = 30\pi$$

- 47 $\int_0^{b-a} \pi((b-y)^2 - a^2) dy = \frac{\pi}{3}(b^3 - 3a^2b + 2a^3)$ 49 $\int_0^2 \pi(3-x)^2 dx; \int_0^1 2\pi y(2) dy + \int_1^3 2\pi y(3-y) dy$
 51 $\int_a^b \pi(\frac{y}{m})^2 dy = \frac{\pi(b^3 - a^3)}{3m^3}$ 53 960 π cm 55 $\frac{\pi}{2}$ 57 $\frac{2\pi}{3}$
 59 2π 61 $\int_0^4 2\pi y(2 - \sqrt{y}) dy = \frac{32\pi}{5}$ 63 $3\pi e$ 65 Height 1; $\int_0^a 2\pi x dx = \pi a^2$; cylinder
 67 Length of hole is $2\sqrt{b^2 - a^2} = 2$, so $b^2 - a^2 = 1$ and volume is $\frac{4\pi}{3}$ 69 F; T(?); F; T

Section 8.2 Length of a Plane Curve (page 324)

- 1 $\int_0^1 \sqrt{1 + (\frac{3}{2}x^{1/2})^2} dx = \frac{8}{27}[(\frac{13}{4})^{3/2} - 1] = \frac{13\sqrt{13}-8}{27}$ 3 $\int_0^1 \sqrt{1 + x^2(x^2 + 2)} dx = \int_0^1 (1 + x^2) dx = \frac{4}{3}$
 5 $\int_1^3 \sqrt{1 + (x^2 - \frac{1}{4x^2})^2} dx = \int_1^3 (x^2 + \frac{1}{4x^2}) dx = \frac{53}{6}$
 7 $\int_1^4 \sqrt{1 + (x^{1/2} - \frac{1}{4}x^{-1/2})^2} dx = \int_1^4 (x^{1/2} + \frac{1}{4}x^{-1/2}) dx = \frac{31}{6}$
 9 $\int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t} dt = \int_0^{\pi/2} 3\cos t \sin t dt = \frac{3}{2}$
 11 $\int_0^{\pi/2} \sqrt{\sin^2 t + (1 - \cos t)^2} dt = \int_0^{\pi/2} \sqrt{2 - 2\cos t} dt = \int_0^{\pi/2} 2\sin \frac{t}{2} dt = 4 - 2\sqrt{2}$
 13 $\int_0^1 \sqrt{t^2 + 2t + 1} dt = \int_0^1 (t + 1) dt = \frac{3}{2}$ 15 $\int_0^\pi \sqrt{1 + \cos^2 x} dx = 3.820$ 17 $\int_1^e \sqrt{1 + \frac{1}{x^2}} dx = 2.003$
 19 Graphs are flat toward (1,0) then steep up to (1,1); limiting length is 2
 21 $\frac{ds}{dt} = \sqrt{36\sin^2 3t + 36\cos^2 3t} = 6$ 23 $\int_0^1 \sqrt{26} dy = \sqrt{26}$
 25 $\int_{-1}^1 \sqrt{\frac{1}{4}(e^y - e^{-y})^2 + 1} dy = \int_{-1}^1 \frac{1}{2}(e^y + e^{-y}) dy = \frac{1}{2}(e^y - e^{-y})|_{-1}^1 = e - \frac{1}{e}$
 Using $x = \cosh y$ this is $\int \sqrt{1 + \sinh^2 y} dy = \int \cosh y dy = \sinh y|_{-1}^1 = 2\sinh 1$
 27 Ellipse; two y 's for the same x 29 Carpet length $2 \neq$ straight distance $\sqrt{2}$
 31 $(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2; ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2} dt;$
 $ds = \sqrt{\sin^2 t + \cos^2 t + 1} dt = \sqrt{2} dt; 2\pi\sqrt{2}$; curve = helix, shadow = circle
 33 $L = \int_0^1 \sqrt{1 + 4x^2} dx; \int_0^2 \sqrt{1 + x^2} dx = \int_0^1 \sqrt{1 + 4u^2} 2du = 2L$; stretch xy plane by 2 ($y = x^2$ becomes $\frac{y}{2} = (\frac{x}{2})^2$)

Section 8.3 Area of a Surface of Revolution (page 327)

- 1 $\int_2^6 2\pi x \sqrt{1 + (\frac{1}{2\sqrt{x}})^2} dx = \int_2^6 2\pi x \sqrt{x + \frac{1}{4}} dx = \frac{49\pi}{3}$ 3 $2 \int_0^1 2\pi(7x)\sqrt{50} dx = 14\pi\sqrt{50}$
 5 $\int_{-1}^1 2\pi\sqrt{4 - x^2} \sqrt{1 + \frac{x^2}{4 - x^2}} dx = \int_{-1}^1 4\pi dx = 8\pi$ 7 $\int_0^2 2\pi x \sqrt{1 + (2x)^2} dx = \frac{\pi}{6}(1 + 4x^2)^{3/2}|_0^2 = \frac{\pi}{6}[17^{3/2} - 1]$
 9 $\int_0^3 2\pi x \sqrt{2} dx = 9\pi\sqrt{2}$ 11 Figure shows radius s times angle $\theta = \arccos \frac{2\pi R}{s}$
 13 $2\pi r \Delta s = \pi(R + R')(s - s') = \pi R s - \pi R' s'$ because $R's - Rs' = 0$
 15 Radius a , center at $(0, b)$; $(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 = a^2$, surface area $\int_0^{2\pi} 2\pi(b + a \sin t)a dt = 4\pi^2 ab$
 17 $\int_1^2 2\pi x \sqrt{1 + \frac{(1-x)^2}{2x-x^2}} dx = \int_1^2 \frac{2\pi x dx}{\sqrt{2x-x^2}} = \pi^2 + 2\pi$ (write $2x - x^2 = 1 - (x - 1)^2$ and set $x - 1 = \sin \theta$)
 19 $\int_{1/2}^1 2\pi x \sqrt{1 + \frac{1}{x^4}} dx$ (can be done)
 21 Surface area $= \int_1^\infty 2\pi \frac{1}{x} \sqrt{1 + \frac{1}{x^4}} dx > \int_1^\infty \frac{2\pi dx}{x} = 2\pi \ln x|_1^\infty = \infty$ but volume $= \int_1^\infty \pi(\frac{1}{x})^2 dx = \pi$
 23 $\int_0^\pi 2\pi \sin t \sqrt{2\sin^2 t + \cos^2 t} dt = \int_0^\pi 2\pi \sin t \sqrt{2 - \cos^2 t} dt = \int_{-1}^1 2\pi \sqrt{2 - u^2} du =$
 $\pi u \sqrt{2 - u^2} + 2\pi \sin^{-1} \frac{u}{\sqrt{2}}|_{-1}^1 = 2\pi + \pi^2$

Section 8.4 Probability and Calculus (page 334)

- 1 $P(X < 4) = \frac{7}{8}, P(X = 4) = \frac{1}{16}, P(X > 4) = \frac{1}{16}$ 3 $\int_0^\infty p(x) dx$ is not 1; $p(x)$ is negative for large x
 5 $\int_2^\infty e^{-x} dx = \frac{1}{e^2}; \int_1^{1.01} e^{-x} dx \approx (.01)\frac{1}{e}$ 7 $p(x) = \frac{1}{\pi}; F(x) = \frac{x}{\pi}$ for $0 \leq x \leq \pi$ ($F = 1$ for $x > \pi$)

- 9 $\mu = \frac{1}{7} \cdot 1 + \frac{1}{7} \cdot 2 + \cdots + \frac{1}{7} \cdot 7 = 4$ 11 $\int_0^\infty \frac{2x dx}{\pi(1+x^2)} = \frac{1}{\pi} \ln(1+x^2)|_0^\infty = +\infty$
- 13 $\int_0^\infty axe^{-ax} dx = [-xe^{-ax}]_0^\infty + \int_0^\infty e^{-ax} dx = \frac{1}{a}$
- 15 $\int_0^x \frac{2dx}{\pi(1+x^2)} = \frac{2}{\pi} \tan^{-1} x$; $\int_0^x e^{-x} dx = 1 - e^{-x}$; $\int_0^x ae^{-ax} dx = 1 - e^{-ax}$ 17 $\int_{10}^\infty \frac{1}{10} e^{-x/10} dx = -e^{-x/10}|_{10}^\infty = \frac{1}{e}$
- 19 *Exponential* better than *Poisson*: 60 years $\rightarrow \int_0^{60} .01e^{-.01x} dx = 1 - e^{-.6} = .45$
- 21 $y = \frac{x-\mu}{\sigma}$; three areas $\approx \frac{1}{3}$ each because $\mu - \sigma$ to μ is the same as μ to $\mu + \sigma$ and areas add to 1
- 23 $-2\mu \int xp(x) dx + \mu^2 \int p(x) dx = -2\mu \cdot \mu + \mu^2 \cdot 1 = -\mu^2$
- 25 $\mu = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = 1$; $\sigma^2 = (0-1)^2 \cdot \frac{1}{3} + (1-1)^2 \cdot \frac{1}{3} + (2-1)^2 \cdot \frac{1}{3} = \frac{2}{3}$.
Also $\sum n^2 p_n - \mu^2 = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 4 \cdot \frac{1}{3} - 1 = \frac{2}{3}$
- 27 $\mu = \int_0^\infty \frac{xe^{-x/2} dx}{2} = 2$; $1 - \int_0^4 \frac{e^{-x/2} dx}{2} = 1 + [e^{-x/2}]_0^4 = e^{-2}$
- 29 Standard deviation (yes - no poll) $\leq \frac{1}{2\sqrt{N}} = \frac{1}{2\sqrt{900}} = \frac{1}{60}$ Poll showed $\frac{870}{900} = \frac{29}{30}$ peaceful.
95% confidence interval is from $\frac{29}{30} - \frac{2}{60}$ to $\frac{29}{30} + \frac{2}{60}$, or 93% to 100% peaceful.
- 31 95% confidence of unfair if more than $\frac{2\sigma}{\sqrt{N}} = \frac{1}{\sqrt{2500}} = 2\%$ away from 50% heads.
2% of 2500 = 50. So unfair if more than 1300 or less than 1200.
- 33 55 is 1.5σ below the mean, and the area up to $\mu - 1.5\sigma$ is about 8% so 24 students fail.
A grade of 57 is 1.3σ below the mean and the area up to $\mu - 1.3\sigma$ is about 10%.
- 35 .999; $.999^{1000} = (1 - \frac{1}{1000})^{1000} \approx \frac{1}{e}$ because $(1 - \frac{1}{n})^n \rightarrow \frac{1}{e}$.

Section 8.5 Masses and Moments (page 340)

- 1 $\bar{x} = \frac{10}{6}$ 3 $\bar{x} = \frac{4}{4}$ 5 $\bar{x} = \frac{3.5}{3}$ 7 $\bar{x} = \frac{2}{3} = \bar{y}$ 9 $\bar{x} = \frac{7/2}{7} = \bar{y}$ 11 $\bar{x} = \frac{1/3}{\pi/4} = \bar{y}$ 13 $\bar{x} = \frac{1/4}{1/2}, \bar{y} = \frac{1/8}{1/2}$
- 15 $\bar{x} = \frac{0}{3\pi} = \bar{y}$ 21 $I = \int x^2 \rho dx - 2t \int x \rho dx + t^2 \int \rho dx$; $\frac{dI}{dt} = -2 \int x \rho dx + 2t \int \rho dx = 0$ for $t = \bar{x}$
- 23 South Dakota 25 $2\pi^2 a^2 b$ 27 $M_x = 0, M_y = \frac{\pi}{2}$ 29 $\frac{2}{\pi}$ 31 Moment
- 33 $I = \sum m_n r_n^2$; $\frac{1}{2} \sum m_n r_n^2 \omega_n^2$; 0 35 $14\pi \ell \frac{r^2}{2}$; $14\pi \ell \frac{r^4}{4}$; $\frac{1}{2}$
- 37 $\frac{2}{3}$; solid ball, solid cylinder, hollow ball, hollow cylinder 39 No
- 41 $T \approx \sqrt{1+J}$ by Problem 40 so $T \approx \sqrt{1.4}, \sqrt{1.5}, \sqrt{5/3}, \sqrt{2}$

Section 8.6 Force, Work, and Energy (page 346)

- 1 2.4 ft lb; 2.424 ... ft lb 3 24000 lb/ft; $83\frac{1}{3}$ ft lb 5 $10x$ ft lb; $10x$ ft lb 7 25000 ft lb; 20000 ft lb
- 9 864,000 Nkm 11 $5.6 \cdot 10^7$ Nkm 13 $k = 10$ lb/ft; $W = 25$ ft lb 15 $\int 60wh dh = 48000w, 12000w$
- 17 $\frac{1}{2} wAH^2$; $\frac{3}{8} wAH^2$ 19 $9600w$ 21 $(1 - \frac{v^2}{c^2})^{-3/2}$ 23 (800) (9800) kg 25 $\pm$ force

CHAPTER 9 POLAR COORDINATES AND COMPLEX NUMBERS

Section 9.1 Polar Coordinates (page 350)

- 1 $(1, \frac{\pi}{2})$ 3 $(2, \frac{\pi}{4})$ 5 $(\sqrt{2}, \frac{5\pi}{4})$ 7 (0, 2) 9 $(\sqrt{10}, \sqrt{10})$ 11 $(\sqrt{3}, -1)$ 13 $2\sqrt{2}$
- 15 $\sqrt{r^2 + R^2 - 2rR \cos(\theta - \phi)}$
- 17 $0 < r < \infty, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$; $0 \leq r < \infty, \pi \leq \theta \leq 2\pi$; $\sqrt{4} < r < \sqrt{5}, 0 \leq \theta < 2\pi$; $0 \leq r < \infty, -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$
- 19 $y = x \tan \theta, r = x \sec \theta$ 21 $\theta = \frac{\pi}{4}$, all r ; $r = \frac{1}{\sin \theta + \cos \theta}$; $r = \cos \theta + \sin \theta$
- 23 $x^2 + y^2 = y$ 25 $x = r \sin \theta \cos \theta, y = r \sin^2 \theta, x^2 + y^2 = y$
- 27 $x^2 + y^2 = x + y, (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = (\frac{\sqrt{2}}{2})^2$ 29 $x = \frac{\cos \theta}{\cos \theta + \sin \theta}, y = \frac{\sin \theta}{\cos \theta + \sin \theta}$ 31 $(x^2 + y^2)^3 = x^4$

Section 9.2 Polar Equations and Graphs (page 355)

- 1 Line $y = 1$ 3 Circle $x^2 + y^2 = 2x$ 5 Ellipse $3x^2 + 4y^2 = 1 - 2x$ 7 x, y, r symmetries
 9 x symmetry only 11 No symmetry 13 x, y, r symmetries!
 15 $x^2 + y^2 = 6y + 8x \rightarrow (x - 4)^2 + (y - 3)^2 = 5^2$, center $(4, 3)$ 17 $(2, 0), (0, 0)$
 19 $r = 1 - \frac{\sqrt{2}}{2}, \theta = \frac{3\pi}{4}; r = 1 + \frac{\sqrt{2}}{2}, \theta = \frac{7\pi}{4}; (0, 0)$ 21 $r = 2, \theta = \pm \frac{\pi}{12}, \pm \frac{5\pi}{12}, \pm \frac{7\pi}{12}, \pm \frac{11\pi}{12}$
 23 $(x, y) = (1, 1)$ 25 $r = \cos 5\theta$ has 5 petals 27 $(x^2 + y^2 - x)^2 = x^2 + y^2$
 29 $(x^2 + y^2)^3 = (x^2 - y^2)^2$ 31 $\cos \theta = -\frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2} \rightarrow y = \frac{2\sqrt{3}}{3}, x = -\frac{2}{3}$ 33 $x = \frac{4}{3}, r = -\frac{5}{3}$ 35 .967

Section 9.3 Slope, Length, and Area for Polar Curves (page 359)

- 1 Area $\frac{3\pi}{2}$ 3 Area $\frac{9\pi}{2}$ 5 Area $\frac{\pi}{8}$ 7 Area $\frac{\pi}{8} - \frac{1}{4}$ 9 $\int_{-\pi/3}^{\pi/3} (\frac{9}{2} \cos^2 \theta - \frac{(1+\cos \theta)^2}{2}) d\theta = \pi$
 11 Area 8π 13 Only allow $r^2 > 0$, then $4 \int_0^{\pi/4} \frac{1}{2} \cos 2\theta d\theta = 1$ 15 $2 + \frac{\pi}{4}$
 17 $\theta = 0$; left points $r = \frac{1}{2}, \theta = \pm \frac{2\pi}{3}, x = -\frac{1}{4}, y = \pm \frac{\sqrt{3}}{4}$
 19 $\frac{r^2}{2c} \Big|_6^{14} = 40,000; \frac{1}{2c} [r\sqrt{r^2 + c^2} + c^2 \ln(r + \sqrt{r^2 + c^2})]_6^{14} = 40,000.001$
 21 $\tan \psi = \tan \theta$ 23 $x = 0, y = 1$ is on limaçon but not circle 25 $\frac{1}{2} \ln(2\pi + \sqrt{1 + 4\pi^2}) + \pi\sqrt{1 + 4\pi^2}$
 27 $\frac{3\pi}{2}$ 29 $\frac{1}{2} (\text{base})(\text{height}) \approx \frac{1}{2}(r\Delta\theta)r$ 31 $\frac{4\pi}{5}\sqrt{2}$ 33 $2\pi(2 - \sqrt{2})$ 35 $\frac{8\pi}{3}$ 39 $\sec \theta$

Section 9.4 Complex Numbers (page 364)

- 1 Sum = 4, product = 5 5 Angles $\frac{3\pi}{4}, \frac{3\pi}{2}, \frac{9\pi}{4}$ 7 Real axis; imaginary axis; $\frac{1}{2}$ axis $x \geq 0$; unit circle
 9 $cd = 5 + 10i, \frac{c}{d} = \frac{11-2i}{25}$ 11 $2 \cos \theta, 1; -1, 1$ 13 Sum = 0, product = -1 15 $r^4 e^{4i\theta}, \frac{1}{r} e^{-i\theta}, \frac{1}{r^4} e^{-4i\theta}$
 17 Evenly spaced on circle around origin 19 e^{it}, e^{-it} 21 e^t, e^{-t}, e^0 23 $\cos 7t, \sin 7t$
 29 $t = -\frac{2\pi}{\sqrt{3}}, y = -e^{\pi/\sqrt{3}}$ 31 F; T; at most 2; $\operatorname{Re} c < 0$ 33 $\frac{1}{r} e^{-i\theta}, x = \frac{1}{r} \cos \theta, y = -\frac{1}{r} \sin \theta; \pm \frac{1}{r} e^{-i\theta/2}$

CHAPTER 10 INFINITE SERIES**Section 10.1 The Geometric Series (page 373)**

- 1 Subtraction leaves $G - xG = 1$ or $G = \frac{1}{1-x}$ 3 $\frac{1}{2}; \frac{4}{5}; \frac{100}{11}; 3\frac{4}{99}$ 5 $2 \cdot 1 + 3 \cdot 2x + 4 \cdot 3x^2 + \cdots = \frac{2}{(1-x)^3}$
 7 .142857 repeats because the next step divides 7 into 1 again
 9 If q (prime, not 2 or 5) divides $10^N - 10^M$ then it divides $10^{N-M} - 1$ 11 This decimal does not repeat
 13 $\frac{87}{99}, \frac{123}{999}$ 15 $\frac{x}{1-x^2}$ 17 $\frac{x^3}{1-x^3}$ 19 $\frac{\ln x}{1-\ln x}$ 21 $\frac{1}{x-1}$ 23 $\tan^{-1}(\tan x) = x$
 25 $(1 + x + x^2 + x^3 \cdots)(1 - x + x^2 - x^3 \cdots) = 1 + x^2 + x^4 + \cdots$
 27 $2(.1234 \dots)$ is $2 \cdot \frac{1}{10} \cdot \frac{1}{(1-\frac{1}{10})^2} = \frac{20}{81}; 1 - .0123 \dots$ is $1 - \frac{1}{100} \frac{1}{(1-\frac{1}{10})^2} = \frac{80}{81}$ 29 $\frac{2}{3} \frac{1}{1-\frac{1}{3}} = 1$
 31 $-\ln(1 - .1) = -\ln .9$ 33 $\frac{1}{2} \ln \frac{1.1}{.9}$ 35 $(n+1)!$ 37 $y = \frac{b}{1-bx}$
 39 All products like $a_1 b_2$ are missed; $(1+1)(1+1) \neq 1+1$ 41 Take $x = \frac{1}{2}$ in (13): $\ln 3 = 1.0986$
 43 In 3 seconds the ball goes 78 feet 45 $\tan z = \frac{2}{3}; (18)$ is slower with $x = \frac{2}{3}$

Section 10.2 Convergence Tests: Positive Series (page 380)

- 1 $\frac{1}{2} + \frac{1}{4} + \cdots$ is smaller than $1 + \frac{1}{3} + \cdots$
 3 $a_n = s_n - s_{n-1} = \frac{1}{n^2 - n}$, $s = 1$; $a_n = 4$, $s = \infty$; $a_n = \ln \frac{2n}{n+1} - \ln \frac{2(n-1)}{n} = \ln \frac{n^2}{n-1}$, $s = \ln 2$
 5 No decision on $\sum b_n$ 7 Diverges: $\frac{1}{100}(1 + \frac{1}{2} + \cdots)$ 9 $\sum \frac{1}{100+n^2}$ converges: $\sum \frac{1}{n^2}$ is larger
 11 Converges: $\sum \frac{1}{n^2}$ is larger 13 Diverges: $\sum \frac{1}{2n}$ is smaller 15 Diverges: $\sum \frac{1}{2n}$ is smaller
 17 Converges: $\sum \frac{2}{n^2}$ is larger 19 Converges: $\sum \frac{3}{n^2}$ is larger 21 $L = 0$ 23 $L = 0$ 25 $L = \frac{1}{2}$
 27 root $(\frac{n-1}{n})^n \rightarrow L = \frac{1}{e}$ 29 $s = 1$ (only survivor) 31 If y decreases, $\sum_2^n y(i) \leq \int_1^n y(x) dx \leq \sum_1^{n-1} y(i)$
 33 $\sum_1^\infty e^{-n} \leq \int_0^\infty e^{-x} dx = 1$; $\frac{1}{e} + \frac{1}{e^2} + \cdots = \frac{1}{e-1}$ 35 Converges faster than $\int \frac{dx}{x^2+1}$
 37 Diverges because $\int_0^\infty \frac{x dx}{x^2+1} = \frac{1}{2} \ln(x^2+1) \Big|_0^\infty = \infty$ 39 Diverges because $\int_1^\infty x^{e-\pi} dx = \frac{x^{e-\pi+1}}{e-\pi+1} \Big|_0^\infty = \infty$
 41 Converges (geometric) because $\int_1^\infty (\frac{e}{\pi})^x dx < \infty$ 43 (b) $\int_n^{n+1} \frac{dx}{x} > (\text{base } 1) (\text{height } \frac{1}{n+1})$
 45 After adding we have $1 + \frac{1}{2} + \cdots + \frac{1}{2n}$ (close to $\ln 2n$); thus originally close to $\ln 2n - \ln n = \ln \frac{2n}{n} = \ln 2$
 47 $\int_{100}^{1000} \frac{dx}{x^2} = \frac{1}{100} - \frac{1}{1000} = .009$ 49 Comparison test: $\sin a_n < a_n$; if $a_n = \pi n$ then $\sin a_n = 0$ but $\sum a_n = \infty$
 51 $a_n = n^{-5/2}$ 53 $a_n = \frac{2^n}{n^n}$ 55 Ratios are $1, \frac{1}{2}, 1, \frac{1}{2}, \dots$ (no limit L); $(\frac{1}{2^k})^{1/2k} = \frac{1}{\sqrt{2}}$; yes
 57 Root test $\frac{1}{\ln n} \rightarrow L = 0$ 59 Root test $L = \frac{1}{10}$ 61 Divergence: N terms add to $\ln \frac{N+2}{2} \rightarrow \infty$
 63 Diverge (compare $\sum \frac{1}{n}$) 65 Root test $L = \frac{3}{4}$ 67 Beyond some point $\frac{a_n}{b_n} < 1$ or $a_n < b_n$

Section 10.3 Convergence Tests: All Series (page 384)

- 1 Terms don't approach zero 3 Absolutely 5 Conditionally not absolutely 7 No convergence
 9 Absolutely 11 No convergence 13 By comparison with $\sum |a_n|$
 15 Even sums $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots$ diverge; a_n 's are not decreasing 17 (b) If $a_n > 0$ then s_n is too large so $s - s_n < 0$
 19 $s = 1 - \frac{1}{e}$; below by less than $\frac{1}{5!}$
 21 Subtract $2(\frac{1}{2^2} + \frac{1}{4^2} + \cdots) = \frac{2}{4}(\frac{1}{1^2} + \frac{1}{2^2} + \cdots) = \frac{\pi^2}{12}$ from positive series to get alternating series
 23 Text proves: If $\sum |a_n|$ converges so does $\sum a_n$
 25 New series $= (\frac{1}{2}) - \frac{1}{4} + (\frac{1}{8}) - \frac{1}{8} \cdots = \frac{1}{2}(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots)$ 27 $\frac{3}{2} \ln 2$: add $\ln 2$ series to $\frac{1}{2}$ ($\ln 2$ series)
 29 Terms alternate and decrease to zero; partial sums are $1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n \rightarrow \gamma$
 31 .5403? 33 Hint + comparison test 35 Partial sums $a_n - a_0$; sum $-a_0$ if $a_n \rightarrow 0$
 37 $\frac{1}{1-\frac{1}{2}} \frac{1}{1-\frac{1}{3}} = 3$ but product is not $1 + \frac{2}{3} + \cdots$
 39 Write x to base 2, as in 1.0010 which keeps $1 + \frac{1}{8}$ and deletes $\frac{1}{2}, \frac{1}{4}, \dots$
 41 $\frac{1}{9} + \frac{1}{27} + \cdots$ adds to $\frac{1/9}{1-1/3} = \frac{1}{6}$ and can't cancel $\frac{1}{3}$
 43 $\frac{\sin 1}{1-\cos 1} = \cot \frac{1}{2}$ (trig identity) $= \tan(\frac{\pi}{2} - \frac{1}{2})$; $s = \sum \frac{e^{in}}{n} = -\log(1 - e^i)$ by 10a in Section 10.1;
 take imaginary part

Section 10.4 The Taylor Series for e^x , $\sin x$, and $\cos x$ (page 390)

- 1 $1 + 2x + \frac{(2x)^2}{2!} + \cdots$; derivatives 2^n ; $1 + 2 + \frac{2^2}{2!} + \cdots$ 3 Derivatives i^n ; $1 + ix + \cdots$
 5 Derivatives $2^n n!$; $1 + 2x + 4x^2 + \cdots$ 7 Derivatives $-(n-1)!$; $-x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$
 9 $y = 2 - e^x = 1 - x - \frac{x^2}{2!} - \cdots$ 11 $y = x - \frac{x^3}{6} + \cdots = \sin x$ 13 $y = xe^x = x + x^2 + \frac{x^3}{2!} + \cdots$
 15 $1 + 2x + x^2$; $4 + 4(x-1) + (x-1)^2$ 17 $-(x-1)^5$ 19 $1 - (x-1) + (x-1)^2 - \cdots$
 21 $(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots = \ln(1 + (x-1))$ 23 $e^{-1}e^{1-x} = e^{-1}(1 - (x-1) + \frac{(x-1)^2}{2!} - \cdots)$
 25 $x + 2x^2 + 2x^3$ 27 $\frac{1}{2} - \frac{x^2}{24} + \frac{x^4}{720}$ 29 $x - \frac{x^3}{18} + \frac{x^5}{600}$ 31 $1 + x^2 + \frac{x^4}{2}$ 33 $1 + x - \frac{x^3}{2}$

35 ∞ slope; $1 + \frac{1}{2}(x-1)$ **37** $x - \frac{x^3}{3} + \frac{x^5}{5}$ **39** $x + \frac{x^3}{3} + \frac{2x^5}{15}$ **41** $1 + x + \frac{x^2}{2}$ **43** $1 + 0x - x^2$
45 $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$, $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$ **47** 99th powers $-1, -i, e^{3\pi i/4}, -i$
49 $e^{-i\pi/3}$ and -1 ; sum zero, product -1 **53** $i\frac{\pi}{2}, i\frac{\pi}{2} + 2\pi i$ **55** $2e^x$

Section 10.5 Power Series (page 395)

1 $1 + 4x + (4x)^2 + \cdots$; $r = \frac{1}{4}$; $x = \frac{1}{4}$ **3** $e(1 - x + \frac{x^2}{2!} - \cdots)$; $r = \infty$
5 $\ln e + \ln(1 + \frac{x}{e}) = 1 + \frac{x}{e} - \frac{1}{2}(\frac{x}{e})^2 + \cdots$; $r = e$; $x = -e$
7 $|\frac{x-1}{2}| < 1$ or $(-1, 3)$; $\frac{2}{3-x}$ **9** $|x-a| < 1$; $-\ln(1 - (x-a))$
11 $1 + \frac{x}{2!} + \frac{x^2}{3!} + \cdots$; add to 1 at $x = 0$ **13** $a_1, a_3, \dots$ are all zero **15** $\frac{1 - (1 - \frac{1}{2}x^2 \cdots)}{x^2} \rightarrow \frac{1}{2}$
17 $f^{(8)}(c) = \cos c < 1$; alternating terms might not decrease (as required)
19 $f = \frac{1}{1-x}$, $|R_n| \leq \frac{x^{n+1}}{(1-c)^{n+1}}$; $R_n = \frac{x^{n+1}}{1-x}$; $(1-c)^4 = 1 - \frac{1}{2}$
21 $f^{(n+1)}(x) = \frac{n!}{(1-x)^{n+1}}$, $|R_n| \leq \frac{x^{n+1}}{(1-c)^{n+1}}(\frac{1}{n+1}) \rightarrow 0$ when $x = \frac{1}{2}$ and $1-c > \frac{1}{2}$
23 $R_2 = f(x) - f(a) - f'(a)(x-a) - \frac{1}{2}f''(a)(x-a)^2$ so $R_2 = R'_2 = R''_2 = 0$ at $x = a$, $R'_2 = f'''$;
 Generalized Mean Value Theorem in 3.8 gives $a < c < c_2 < c_1 < x$
25 $1 + \frac{1}{2}x^2 + \frac{3}{8}(x^2)^2$ **27** $(-1)^n$; $(-1)^n(n+1)$
29 (a) one friend k times, the other $n-k$ times, $0 \leq k \leq n$; **21** **33** $(16-1)^{1/4} \approx 1.968$
35 $(1+.1)^{1.1} = 1(1.1)(.1) + \frac{(1.1)(.1)}{2}(.1)^2 \approx 1.1105$ **37** $1 + \frac{x^2}{2} + \frac{5x^4}{24}$; $r = \frac{\pi}{2}$ **41** $x + x^2 + \frac{5}{6}x^3 + \frac{5}{6}x^4$
43 $x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6$ **45** $1 + \frac{x}{2} + \frac{3x}{8} + \frac{5x}{16}$ **47** .2727 **49** $-\frac{1}{6} - \frac{1}{3} = -\frac{1}{2}$ **51** $r = 1$, $r = \frac{\pi}{2} - 1$

CHAPTER 11 VECTORS AND MATRICES

Section 11.1 Vectors and Dot Products (page 405)

1 $(0, 0, 0)$; $(5, 5, 5)$; 3 ; -3 ; $\cos \theta = -1$ **3** $2\mathbf{i} - \mathbf{j} - \mathbf{k}$; $-\mathbf{i} - 7\mathbf{j} + 8\mathbf{k}$; 6 ; -3 ; $\cos \theta = -\frac{1}{2}$
5 $(v_2, -v_1)$; $(v_2, -v_1, 0)$, $(v_3, 0, -v_1)$ **7** $(0, 0)$; $(0, 0, 0)$ **9** Cosine of θ ; projection of $\mathbf{w}$ on $\mathbf{v}$
11 F; T; F **13** Zero; sum = 10 o'clock vector; sum = 8 o'clock vector times $\frac{1+\sqrt{3}}{2}$
15 45° **17** Circle $x^2 + y^2 = 4$; $(x-1)^2 + y^2 = 4$; vertical line $x = 2$; half-line $x \geq 0$
19 $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j}$, $\mathbf{w} = 2\mathbf{i} - \mathbf{j}$; $\mathbf{i} = 4\mathbf{v} - \mathbf{w}$ **21** $d = -6$; $C = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$
23 $\cos \theta = \frac{1}{\sqrt{3}}$; $\cos \theta = \frac{2}{\sqrt{6}}$; $\cos \theta = \frac{1}{3}$ **25** $\mathbf{A} \cdot (\mathbf{A} + \mathbf{B}) = 1 + \mathbf{A} \cdot \mathbf{B} = 1 + \mathbf{B} \cdot \mathbf{A} = \mathbf{B} \cdot (\mathbf{A} + \mathbf{B})$; equilateral, 60°
27 $\mathbf{a} = \mathbf{A} \cdot \mathbf{I}$, $\mathbf{b} = \mathbf{A} \cdot \mathbf{J}$ **29** $(\cos t, \sin t)$ and $(-\sin t, \cos t)$; $(\cos 2t, \sin 2t)$ and $(-2 \sin 2t, 2 \cos 2t)$
31 $\mathbf{C} = \mathbf{A} + \mathbf{B}$, $\mathbf{D} = \mathbf{A} - \mathbf{B}$; $\mathbf{C} \cdot \mathbf{D} = \mathbf{A} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{A} - \mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{B} = r^2 - r^2 = 0$
33 $\mathbf{U} + \mathbf{V} - \mathbf{W} = (2, 5, 8)$, $\mathbf{U} - \mathbf{V} + \mathbf{W} = (0, -1, -2)$, $-\mathbf{U} + \mathbf{V} + \mathbf{W} = (4, 3, 6)$
35 c and $\sqrt{a^2 + b^2}$; b/a and $\sqrt{a^2 + b^2 + c^2}$
37 $\mathbf{M}_1 = \frac{1}{2}\mathbf{A} + \mathbf{C}$, $\mathbf{M}_2 = \mathbf{A} + \frac{1}{2}\mathbf{B}$, $\mathbf{M}_3 = \mathbf{B} + \frac{1}{2}\mathbf{C}$; $\mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 = \frac{3}{2}(\mathbf{A} + \mathbf{B} + \mathbf{C}) = \mathbf{0}$
39 $8 \leq 3 \cdot 3$; $2\sqrt{xy} \leq x + y$ **41** Cancel a^2c^2 and b^2d^2 ; then $b^2c^2 + a^2d^2 \geq 2abcd$ because $(bc - ad)^2 \geq 0$
43 F; T; T; F **45** all $2\sqrt{2}$; $\cos \theta = -\frac{1}{3}$

Section 11.2 Planes and Projections (page 414)

1 $(0, 0, 0)$ and $(2, -1, 0)$; $\mathbf{N} = (1, 2, 3)$ **3** $(0, 5, 6)$ and $(0, 6, 7)$; $\mathbf{N} = (1, 0, 0)$
5 $(1, 1, 1)$ and $(1, 2, 2)$; $\mathbf{N} = (1, 1, -1)$ **7** $x + y = 3$ **9** $x + 2y + z = 2$

11 Parallel if $\mathbf{N} \cdot \mathbf{V} = 0$; perpendicular if $\mathbf{V} =$ multiple of $\mathbf{N}$

13 $\mathbf{i} + \mathbf{j} + \mathbf{k}$ (vector between points) is not perpendicular to $\mathbf{N}$; $\mathbf{V} \cdot \mathbf{N}$ is not zero; plane through first three is $x + y + z = 1$; $x + y - z = 3$ succeeds; right side must be zero

15 $ax + by + cz = 0$; $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ 17 $\cos \theta = \frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}, \frac{1}{3}$

19 $\frac{2}{36}\mathbf{A}$ has length $\frac{1}{3}$ 21 $\mathbf{P} = \frac{1}{2}\mathbf{A}$ has length $\frac{1}{2}|\mathbf{A}|$ 23 $\mathbf{P} = -\mathbf{A}$ has length $|\mathbf{A}|$ 25 $\mathbf{P} = \mathbf{O}$

27 Projection on $\mathbf{A} = (1, 2, 2)$ has length $\frac{5}{3}$; force down is 4; mass moves in the direction of $\mathbf{F}$

29 $|\mathbf{P}|_{\min} = \frac{5}{|\mathbf{N}|}$ = distance from plane to origin 31 Distances $\frac{1}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$ both reached at $(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$

33 $\mathbf{i} + \mathbf{j} + \mathbf{k}$; $t = -\frac{4}{3}; (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}); \frac{4}{\sqrt{3}}$

35 Same $\mathbf{N} = (2, -2, 1)$; for example $\mathbf{Q} = (0, 0, 1)$; then $\mathbf{Q} + \frac{2}{9}\mathbf{N} = (\frac{4}{9}, -\frac{4}{9}, \frac{11}{9})$ is on second plane; $\frac{2}{9}|\mathbf{N}| = \frac{2}{3}$

37 $3\mathbf{i} + 4\mathbf{j}$; $(3t, 4t)$ is on the line if $3(3t) + 4(4t) = 10$ or $t = \frac{10}{25}$; $P = (\frac{30}{25}, \frac{40}{25})$, $|P| = 2$

39 $2x + 2(\frac{10}{4} - \frac{3}{4}x)(-\frac{3}{4}) = 0$ so $x = \frac{30}{25} = \frac{6}{5}$; $3x + 4y = 10$ gives $y = \frac{8}{5}$

41 Use equations (8) and (9) with $\mathbf{N} = (a, b)$ and $\mathbf{Q} = (x_1, y_1)$ 43 $t = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}|^2}$; $\mathbf{B}$ onto $\mathbf{A}$

45 $aVL = \frac{1}{2}\mathbf{L}_I - \frac{1}{2}\mathbf{L}_{III}$; $aVF = \frac{1}{2}\mathbf{L}_{II} + \frac{1}{2}\mathbf{L}_{III}$

47 $\mathbf{V} \cdot \mathbf{L}_I = 2 - 1$; $\mathbf{V} \cdot \mathbf{L}_{II} = -3 - 1$, $\mathbf{V} \cdot \mathbf{L}_{III} = -3 - 2$; thus $\mathbf{V} \cdot 2\mathbf{i} = 1$, $\mathbf{V} \cdot (\mathbf{i} - \sqrt{3}\mathbf{j}) = -4$, and $\mathbf{V} = \frac{1}{2}\mathbf{i} + \frac{3\sqrt{3}}{2}\mathbf{j}$

Section 11.3 Cross Products and Determinants (page 423)

1 $\mathbf{O}$ 3 $3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ 5 $-2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ 7 $27\mathbf{i} + 12\mathbf{j} - 17\mathbf{k}$

9 $\mathbf{A}$ perpendicular to $\mathbf{B}$; $\mathbf{A}, \mathbf{B}, \mathbf{C}$ mutually perpendicular 11 $|\mathbf{A} \times \mathbf{B}| = \sqrt{2}$, $\mathbf{A} \times \mathbf{B} = \mathbf{j} - \mathbf{k}$ 13 $\mathbf{A} \times \mathbf{B} = \mathbf{O}$

15 $|\mathbf{A} \times \mathbf{B}|^2 = (a_1^2 + a_2^2)(b_1^2 + b_2^2) - (a_1b_1 + a_2b_2)^2 = (a_1b_2 - a_2b_1)^2$; $\mathbf{A} \times \mathbf{B} = (a_1b_2 - a_2b_1)\mathbf{k}$

17 $\mathbf{F}$; $\mathbf{T}$; $\mathbf{F}$; $\mathbf{T}$ 19 $\mathbf{N} = (2, 1, 0)$ or $2\mathbf{i} + \mathbf{j}$ 21 $x - y + z = 2$ so $\mathbf{N} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

23 $[(1, 2, 1) - (2, 1, 1)] \times [(1, 1, 2) - (2, 1, 1)] = \mathbf{N} = \mathbf{i} + \mathbf{j} + \mathbf{k}$; $x + y + z = 4$

25 $(1, 1, 1) \times (a, b, c) = \mathbf{N} = (c - b)\mathbf{i} + (a - c)\mathbf{j} + (b - a)\mathbf{k}$; points on a line if $a = b = c$ (many planes)

27 $\mathbf{N} = \mathbf{i} + \mathbf{j}$, plane $x + y = \text{constant}$ 29 $\mathbf{N} = \mathbf{k}$, plane $z = \text{constant}$

31 $\begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = x - y + z = 0$ 33 $\mathbf{i} - 3\mathbf{j}$; $-\mathbf{i} + 3\mathbf{j}$; $-3\mathbf{i} - \mathbf{j}$ 35 $-1, 4, -9$

39 $+c_1 \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - c_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + c_3 \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$

41 $\text{area}^2 = (\frac{1}{2}ab)^2 + (\frac{1}{2}ac)^2 + (\frac{1}{2}bc)^2 = (\frac{1}{2}|\mathbf{A} \times \mathbf{B}|)^2$ when $\mathbf{A} = a\mathbf{i} - b\mathbf{j}$, $\mathbf{B} = a\mathbf{i} - c\mathbf{k}$

43 $\mathbf{A} = \frac{1}{2}(2 \cdot 1 - (-1)1) = \frac{3}{2}$; fourth corner can be $(3, 3)$

45 $a_1\mathbf{i} + a_2\mathbf{j}$ and $b_1\mathbf{i} + b_2\mathbf{j}$; $|a_1b_2 - a_2b_1|$; $\mathbf{A} \times \mathbf{B} = \cdots + (a_1b_2 - a_2b_1)\mathbf{k}$

47 $\mathbf{A} \times \mathbf{B}$; from Eq. (6), $(\mathbf{A} \times \mathbf{B}) \times \mathbf{i} = -(a_3b_1 - a_1b_3)\mathbf{k} + (a_1b_2 - a_2b_1)\mathbf{j}$; $(\mathbf{A} \cdot \mathbf{i})\mathbf{B} - (\mathbf{B} \cdot \mathbf{i})\mathbf{A} = a_1(b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) - b_1(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k})$

49 $\mathbf{N} = (Q - P) \times (R - P) = \mathbf{i} + \mathbf{j} + \mathbf{k}$; area $\frac{1}{2}\sqrt{3}$; $x + y + z = 2$

Section 11.4 Matrices and Linear Equations (page 433)

1 $x = 5, y = 2, D = -2$, $\begin{bmatrix} 7 \\ 3 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ 3 $x = 3, y = 1$, $\begin{bmatrix} 8 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, $D = -8$

5 $x = 2y, y = \text{anything}, D = 0$, $2y \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} -4 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ 7 no solution, $D = 0$

9 $x = \frac{1}{D} \begin{vmatrix} 8 & -1 \\ 0 & -3 \end{vmatrix} = \frac{-24}{-8} = 3, y = \frac{1}{D} \begin{bmatrix} 3 & 8 \\ 1 & 0 \end{bmatrix} = \frac{-8}{-8} = 1$ 11 $\frac{0}{0}$

- 15** $ad - bc = -2$ so $A^{-1} = \begin{bmatrix} .5 & .5 \\ .5 & -.5 \end{bmatrix}$ **17** Are parallel; multiple; the same; infinite
19 Multiples of each other; in the same direction as the columns; infinite
21 $d_1 = .34, d_2 = 4.91$ **23** $.96x + .02y = .58, .04x + .98y = 4.92; D = .94, x = .5, y = 5$
25 $a = 1$ gives any $x = -y; a = -1$ gives any $x = y$
27 $D = -2, A^{-1} = -\frac{1}{2} \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}; D = -8, (2A)^{-1} = \frac{1}{2}A^{-1}; D = \frac{1}{-2}, (A^{-1})^{-1} = \text{original } A;$
 $D = -2$ (not $+2$), $(-A)^{-1} = -A^{-1}; D = 1, I^{-1} = I$
29 $AB = \begin{bmatrix} 7 & 5 \\ 5 & 1 \end{bmatrix}, BA = \begin{bmatrix} 5 & 11 \\ 3 & 3 \end{bmatrix}, BC = \begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix}, CB = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$
31 $AB = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}, \begin{bmatrix} aecf + aedh & bgc f + bgdh \\ -afce - afdg & -bhce - bhdg \end{bmatrix} = (ad - bc)(eh - fg)$
33 $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & \frac{1}{2} \end{bmatrix}, B^{-1} = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & 1 \end{bmatrix}$ **35** Identity; $B^{-1}A^{-1}$ **37** Perpendicular; $\mathbf{u} = \mathbf{v} \times \mathbf{w}$
39 Line 4 + t , errors $-1, 2, -1$ **41** $d_1 - 2d_2 + d_3 = 0$ **43** A^{-1} can't multiply $\mathbf{O}$ and produce $\mathbf{u}$

Section 11.5 Linear Algebra (page 443)

- 1** $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 5 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$ **3** $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
5 $\det A = 0$, add 3 equations $\rightarrow 0 = 1$ **7** $5\mathbf{a} + 1\mathbf{b} + 0\mathbf{c} = \mathbf{d}, A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$
9 $\mathbf{b} \times \mathbf{c}; \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = 0$; determinant is zero **11** 6, 2, 0; product of diagonal entries
13 $A^{-1} = \begin{bmatrix} 1 & -2 & 4 \\ 0 & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}, B^{-1} = \begin{bmatrix} 0 & 2 & -\frac{1}{2} \\ 0 & -3 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ **15** Zero; same plane; D is zero
17 $\mathbf{d} = (1, -1, 0); \mathbf{u} = (1, 0, 0)$ or $(7, 3, 1)$ **19** $AB = \begin{bmatrix} 8 & 4 & 1 \\ 40 & 26 & 0 \\ 18 & 12 & 0 \end{bmatrix}, \det AB = 12 = (\det A) \text{ times } (\det B)$
21 $A + C = \begin{bmatrix} 2 & 3 & -3 \\ -1 & 4 & 6 \\ 0 & -1 & 6 \end{bmatrix}, \det(A + C) \text{ is not } \det A + \det C$
23 $p = \frac{(2)(3) - (0)(6)}{6} = 1, q = \frac{-(4)(3) + (0)(0)}{6} = -2$ **25** $(A^{-1})^{-1}$ is always A
27 $-1, -1, 1, 1, (y, x, z), (z, y, x), (y, z, x), (z, x, y)$ **29** $2! = 2, 4! = 24$
31 $z = \frac{1}{2}, y = -\frac{3}{2}, x = 3; z = \frac{7}{2}, y = \frac{3}{2}, x = -\frac{1}{2}$
33 New second equation $3z = 0$ doesn't contain y ; exchange with third equation; there is a solution
35 Pivots 1, 2, 4, $D = 8$; pivots 1, -1, 2, $D = -2$ **37** $a_{12} = 1, a_{21} = 0, \sum a_{ij}b_{jk} = \text{row } i, \text{ column } k \text{ in } AB$
39 $a_{11}a_{22} - a_{12}a_{21} \neq 0; D = 0$

CHAPTER 12 MOTION ALONG A CURVE

Section 12.1 The Position Vector (page 452)

- 1** $\mathbf{v}(1) = \mathbf{i} + 3\mathbf{j}$; speed $\sqrt{10}$; **3** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t}$; tangent to circle is perpendicular to $\frac{\mathbf{x}}{y} = \frac{\cos t}{\sin t}$
5 $\mathbf{v} = e^t \mathbf{i} - e^{-t} \mathbf{j} = \mathbf{i} - \mathbf{j}; y - 1 = -(x - 1); xy = 1$

- 7 $\mathbf{R} = (1, 2, 4) + (4, 3, 0)t$; $\mathbf{R} = (1, 2, 4) + (8, 6, 0)t$; $\mathbf{R} = (5, 5, 4) + (8, 6, 0)t$
 9 $\mathbf{R} = (2 + t, 3, 4 - t)$; $\mathbf{R} = (2 + \frac{t^2}{2}, 3, 4 - \frac{t^2}{2})$; the same line
 11 Line; $y = 2 + 2t, z = 2 + 3t$; $y = 2 + 4t, z = 2 + 6t$
 13 Line; $\sqrt{36 + 9 + 4} = 7$; $(6, 3, 2)$; line segment 15 $\frac{\sqrt{2}}{2}; 1; \frac{\sqrt{2}}{2}$ 17 $x = t, y = mt + b$
 19 $\mathbf{v} = \mathbf{i} - \frac{1}{3}\mathbf{j}$; $|\mathbf{v}| = \sqrt{1 + \frac{1}{9}} = \frac{\sqrt{10}}{3}$; $\mathbf{T} = \mathbf{v}/|\mathbf{v}|$; $\mathbf{v} = (\cos t - t \sin t)\mathbf{i} + (\sin t + t \cos t)\mathbf{j}$; $|\mathbf{v}| = \sqrt{1 + t^2}$;
 $\mathbf{T} = \mathbf{v}/|\mathbf{v}|$; $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$; $|\mathbf{v}| = 3$; $\mathbf{T} = \frac{1}{3}\mathbf{v}$
 21 $\mathbf{R} = -\sin t \mathbf{i} + \cos t \mathbf{j} + \text{any } \mathbf{R}_0$; same $\mathbf{R}$ plus any $\mathbf{w}\mathbf{t}$
 23 $\mathbf{v} = (1 - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$; $|\mathbf{v}| = \sqrt{3 - 2\sin t - 2\cos t}$; $|\mathbf{v}|_{\min} = \sqrt{3 - 2\sqrt{2}}$; $|\mathbf{v}|_{\max} = \sqrt{3 + 2\sqrt{2}}$;
 $\mathbf{a} = -\cos t \mathbf{i} + \sin t \mathbf{j}$; $|\mathbf{a}| = 1$; center is on $x = t, y = t$
 25 Leaves at $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$; $\mathbf{v} = (-\sqrt{2}, \sqrt{2})$; $\mathbf{R} = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) + \mathbf{v}(t - \frac{\pi}{8})$
 27 $\mathbf{R} = \cos \frac{t}{\sqrt{2}}\mathbf{i} + \sin \frac{t}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}$
 29 $\mathbf{v} = \sec^2 t \mathbf{i} + \sec t \tan t \mathbf{j}$; $|\mathbf{v}| = \sec^2 t \sqrt{1 + \sin^2 t}$; $\mathbf{a} = 2 \sec^2 t \tan t \mathbf{i} + (\sec^3 t + \sec t \tan^2 t) \mathbf{j}$;
 curve is $y^2 - x^2 = 1$; hyperbola has asymptote $y = x$
 31 If $\mathbf{T} = \mathbf{v}$ then $|\mathbf{v}| = 1$; line $\mathbf{R} = t\mathbf{i}$ or helix in Problem 27
 33 $(x(t), y(t)) = \begin{matrix} (2t, 0) & 0 \leq t \leq \frac{1}{2} \\ (1, 2t - 1) & \frac{1}{2} \leq t \leq 1 \end{matrix}$ $\begin{matrix} (3 - 2t, 1) & 1 \leq t \leq \frac{3}{2} \\ (0, 4 - 2t) & \frac{3}{2} \leq t \leq 2 \end{matrix}$
 35 $x(t) = 4 \cos \frac{t}{2}, y(t) = 4 \sin \frac{t}{2}$ 37 F; F; T; T; F 39 $\frac{y}{x} = \tan \theta$ but $\frac{y}{x} \neq \tan t$
 41 $\mathbf{v}$ and $\mathbf{w}$; $\mathbf{v}$ and $\mathbf{w}$ and $\mathbf{u}$; $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v}$ and $\mathbf{w}$ and $\mathbf{u}$; not zero
 43 $\mathbf{u} = (8, 3, 2)$; projection perpendicular to $\mathbf{v} = (1, 2, 2)$ is $(6, -1, -2)$ which has length $\sqrt{41}$
 45 $x = G(t), y = F(t); y = x^{2/3}; t = 1$ and $t = -1$ give the same x so they would give the same $y; y = G(F^{-1}(x))$

Section 12.2 Plane Motion: Projectiles and Cycloids (page 457)

- 1 (a) $T = 16/g \text{ sec}, R = 128\sqrt{3}/g \text{ ft}, Y = 32/g \text{ ft}$ (b) $\frac{16\sqrt{2}}{g}; \frac{128\sqrt{3}}{g}, \frac{96}{g}$ (c) $\frac{32}{g}, 0, \frac{128}{g}$ 3 $x = 1.2$ or 33.5
 5 $y = x - \frac{1}{2}x^2 = 0$ at $x = 2$; $y = x \tan x - \frac{g}{2}(\frac{x}{v_0 \cos \alpha})^2 = 0$ at $x = R$ 7 $x = v_0 \sqrt{\frac{2h}{g}}$
 9 $v_0 \approx 11.2, \tan \alpha \approx 4.32$ 11 $v_0 = \sqrt{gR} = \sqrt{980} \text{ m/sec}$; larger 13 $v_0^2/2g = 40 \text{ meters}$
 15 Multiply R and H by 4; $dR = 2v_0^2 \cos 2\alpha d\alpha/g, dH = v_0^2 \sin \alpha \cos \alpha d\alpha/g$
 17 $t = \frac{12\sqrt{2}}{10} \text{ sec}$; $y = 12 - \frac{144g}{100} \approx -2.1 \text{ m}$; $+2.1 \text{ m}$ 19 $\mathbf{T} = \frac{(1 - \cos \theta)\mathbf{i} + \sin \theta \mathbf{j}}{\sqrt{2 - 2\cos \theta}}$
 21 Top of circle 25 $ca(1 - \cos \theta), ca \sin \theta; \theta = \pi, \frac{\pi}{2}$ 27 After $\theta = \pi: x = \pi a + v_0 t$ and $y = 2a - \frac{1}{2}gt^2$ 29 2; 3
 31 $\frac{64\pi a^2}{3}; 5\pi^2 a^3$ 33 $x = \cos \theta + \theta \sin \theta, y = \sin \theta - \theta \cos \theta$ 35 $(a = 4) 6\pi$
 37 $y = 2 \sin \theta - \sin 2\theta = 2 \sin \theta(1 - \cos \theta); x^2 + y^2 = 4(1 - \cos \theta)^2; r = 2(1 - \cos \theta)$

Section 12.3 Curvature and Normal Vector (page 463)

- 1 $\frac{e^x}{(1+e^{2x})^{3/2}}$ 3 $\frac{1}{2}$ 5 0 (line) 7 $\frac{2+t^2}{(1+t^2)^{3/2}}$ 9 $(-\sin t^2, \cos t^2); (-\cos t^2, -\sin t^2)$
 11 $(\cos t, \sin t); (-\sin t, -\cos t)$ 13 $(-\frac{3}{5} \sin t, \frac{3}{5} \cos t, \frac{4}{5})$; $|\mathbf{v}| = 5, \kappa = \frac{3}{25}; \frac{5}{3}$ longer; $\tan \theta = \frac{4}{3}$
 15 $\frac{1}{2\sqrt{2a}\sqrt{1 - \cos \theta}}$ 17 $\kappa = \frac{3}{16}, \mathbf{N} = \mathbf{i}$ 19 $(0, 0); (-3, 0)$ with $\frac{1}{\kappa} = 4; (-1, 2)$ with $\frac{1}{\kappa} = 2\sqrt{2}$
 21 Radius $\frac{1}{\kappa}$, center $(1, \pm\sqrt{\frac{1}{\kappa^2} - 1})$ for $\kappa \leq 1$ 23 $\mathbf{U} \cdot \mathbf{V}'$ 25 $\frac{1}{\sqrt{2}}(\sin t \mathbf{i} - \cos t \mathbf{j} + \mathbf{k})$ 27 $\frac{1}{2}$
 29 $\mathbf{N}$ in the plane, $\mathbf{B} = \mathbf{k}, \tau = 0$ 31 $\frac{d^2 y/dx^2}{1+(dy/dx)^2}$ 33 $\mathbf{a} = 0 \mathbf{T} + 5\omega^2 \mathbf{N}$ 35 $\mathbf{a} = \frac{t}{\sqrt{1+t^2}}\mathbf{T} + \frac{2+t^2}{\sqrt{1+t^2}}\mathbf{N}$
 37 $\mathbf{a} = \frac{4t}{\sqrt{1+4t^2}}\mathbf{T} + \frac{2}{\sqrt{1+4t^2}}\mathbf{N}$ 39 $|F^2 + 2(F')^2 - FF''|/(F^2 + F'^2)^{3/2}$

Section 12.4 Polar Coordinates and Planetary Motion (page 468)

- 1** $\mathbf{j}, -\mathbf{i}; \mathbf{i} + \mathbf{j} = \mathbf{u}_r - \mathbf{u}_\theta$ **3** $(2, -1); (1, 2)$ **5** $\mathbf{v} = 3e^3(\mathbf{u}_r + \mathbf{u}_\theta) = 3e^3(\cos 3 - \sin 3)\mathbf{i} + 3e^3(\sin 3 + \cos 3)\mathbf{j}$
7 $\mathbf{v} = -20 \sin 5t \mathbf{i} + 20 \cos 5t \mathbf{j} = 20 \mathbf{T} = 20 \mathbf{u}_\theta; \mathbf{a} = -100 \cos 5t \mathbf{i} - 100 \sin 5t \mathbf{j} = 100 \mathbf{N} = -100 \mathbf{u}_r$
9 $r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} = 0 = \frac{1}{r} \frac{d}{dt} (r^2 \frac{d\theta}{dt})$ **11** $\frac{d\theta}{dt} = .0004$ radians/sec; $h = r^2 \frac{d\theta}{dt} = 40,000$
13 $m\mathbf{R} \times \mathbf{a}$; torque **15** $T^{2/3}(GM/4\pi^2)^{1/3}$ **17** $4\pi^2 a^3/T^2 G$ **19** $\frac{4\pi^2 (150)^3 10^{27}}{(365 \frac{1}{4})^2 (24)^2 (3600)^2 (6.67) 10^{-11}}$ kg
23 Use Problem 15 **25** $a + c = \frac{1}{C-D}, a - c = \frac{1}{C+D}$, solve for C, D
27 Kepler measures area from focus (sun) **29** Line; $x = 1$
33 $r = 20 - 2t, \theta = \frac{2\pi t}{10}, \mathbf{v} = -2\mathbf{u}_r + (20 - 2t)\frac{2\pi}{10}\mathbf{u}_\theta; \mathbf{a} = (2t - 20)(\frac{2\pi}{10})^2\mathbf{u}_r - 4(\frac{2\pi}{10})\mathbf{u}_\theta; \int_0^{10} |\mathbf{v}| dt$

CHAPTER 13 PARTIAL DERIVATIVES

Section 13.1 Surfaces and Level Curves (page 475)

- 3** x derivatives $\infty, -1, -2, -4e^{-4}$ (flattest) **5** Straight lines **7** Logarithm curves
9 Parabolas **11** No: $f = (x + y)^n$ or $(ax + by)^n$ or any function of $ax + by$ **13** $f(x, y) = 1 - x^2 - y^2$
15 Saddle **17** Ellipses $4x^2 + y^2 = c^2$ **19** Ellipses $5x^2 + y^2 = c^2 + 4cx + x^2$
21 Straight lines not reaching $(1, 2)$ **23** Center $(1, 1); f = x^2 + y^2 - 1$ **25** Four, three, planes, spheres
27 Less than 1, equal to 1, greater than 1 **29** Parallel lines, hyperbolas, parabolas
31 $\frac{d}{dx}: 48x - 3x^2 = 0, x = 16$ hours **33** Plane; planes; 4 left and 3 right (3 pairs)

Section 13.2 Partial Derivatives (page 479)

- 1** $3 + 2xy^2; -1 + 2y^2$ **3** $3x^2y^2 - 2x; 2x^3y - e^y$ **5** $\frac{-2y}{(x-y)^2}; \frac{2x}{(x-y)^2}$ **7** $\frac{-2x}{(x^2+y^2)^2}; \frac{-2y}{(x^2+y^2)^2}$
9 $\frac{x}{x^2+y^2}; \frac{y}{x^2+y^2}$ **11** $\frac{-y}{x^2+y^2}; \frac{x}{x^2+y^2}$ **13** 2, 3, 4 **15** $6(x + iy), 6i(x + iy), -6(x + iy)$
17 $(f = \frac{1}{r}) f_{xx} = \frac{2x^2 - y^2}{r^5}; f_{xy} = \frac{3xy}{r^5}; f_{yy} = \frac{2y^2 - x^2}{r^5}$ **19** $-a^2 \cos ax \cos by, ab \sin ax \sin by, -b^2 \cos ax \cos by$
21 Omit line $x = y$; all positive numbers; $f_x = -2(x - y)^{-3}, f_y = 2(x - y)^{-3}$
23 Omit $z = t$; all numbers; $\frac{-1}{z-t}, \frac{1}{z-t}, \frac{(x-y)}{(z-t)^2}, \frac{(y-x)}{(z-t)^2}$
25 $x > 0, t > 0$ and $x = 0, t > 1$ and $x = -1, -2, \dots, t = e, e^2, \dots; f_x = (\ln t)x^{\ln t - 1}, f_t = (\ln x)t^{\ln x - 1}$
27 $y, x; f = G(x) + H(y)$ **29** $\frac{\partial f}{\partial x} = \frac{\partial(xy)}{\partial x} v(xy) = yv(xy)$
31 $f_{xxx} = 6y^3, f_{yyy} = 6x^3, f_{xxy} = f_{xyx} = f_{yxx} = 18xy^2, f_{yyx} = f_{yxy} = f_{xyy} = 18x^2y$
33 $g(y) = Ae^{cy/7}$ **35** $g(y) = Ae^{cy/2} + Be^{-cy/2}$
37 $f_t = -2f, f_{xx} = f_{yy} = -e^{-2t} \sin x \sin y; e^{-13t} \sin 2x \sin 3y$
39 $\sin(x + t)$ moves left **41** $\sin(x - ct), \cos(x + ct), e^{x-ct}$
43 $(B - A)h_y(C^*) = (B - A)[f_y(b, C^*) - f_y(a, C^*)] = (B - A)(b - a)f_{yx}(c^*, C^*)$; continuous f_{xy} and f_{yx}
45 y converges to b ; inside and stay inside; $d_n = \sqrt{(x_n - a)^2 + (y_n - b)^2} \rightarrow \text{zero}; d_n < \epsilon$ for $n > N$
47 ϵ , less than δ **49** $f(a, b); \frac{1}{x-1}$ or $\frac{1}{(x-1)(y-2)}$ **51** $f(0, 0) = 1; f(0, 0) = 1$; not defined for $x < 0$

Section 13.3 Tangent Planes and Linear Approximations (page 488)

- 1 $z - 1 = y - 1; \mathbf{N} = \mathbf{j} - \mathbf{k}$ 3 $z - 2 = \frac{1}{3}(x - 6) - \frac{2}{3}(y - 3); \mathbf{N} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \mathbf{k}$
 5 $2(x - 1) + 4(y - 2) + 2(z - 1) = 0; \mathbf{N} = 2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ 7 $z - 1 = x - 1; \mathbf{N} = \mathbf{i} - \mathbf{k}$
 9 Tangent plane $2x_0(z - z_0) - 2x_0(x - x_0) - 2y_0(y - y_0) = 0; (0, 0, 0)$ satisfies this equation because $z_0^2 - x_0^2 - y_0^2 = 0$ on the surface; $\cos \theta = \frac{\mathbf{N} \cdot \mathbf{k}}{|\mathbf{N}||\mathbf{k}|} = \frac{-z_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} = \frac{-1}{\sqrt{2}}$ (surface is the 45° cone)
 11 $dz = 3dx - 2dy$ for both; $dz = 0$ for both; $\Delta z = 0$ for $3x - 2y$, $\Delta z = .00029$ for x^3/y^2 ; tangent plane
 13 $z = z_0 + F_x t$; plane $6(x - 4) + 12(y - 2) + 8(z - 3) = 0$; normal line $x = 4 + 6t, y = 2 + 12t, z = 3 + 8t$
 15 Tangent plane $4(x - 2) + 2(y - 1) + 4(z - 2) = 0$; normal line $x = 2 + 4t, y = 1 + 2t, z = 2 + 4t; (0, 0, 0)$ at $t = -\frac{1}{2}$
 17 $dw = y_0 dx + x_0 dy$; product rule; $\Delta w - dw = (x - x_0)(y - y_0)$
 19 $dI = 4000dR + .08dP; dP = \$100; I = (.78)(4100) = \$319.80$
 21 Increase $= \frac{26}{101} - \frac{25}{100} = \frac{3}{404}$, decrease $= \frac{25}{100} - \frac{25}{101} = \frac{1}{404}; dA = \frac{1}{y}dx - \frac{x}{y^2}dy; 3$ 23 $\Delta \theta \approx \frac{-y\Delta x + x\Delta y}{\sqrt{x^2 + y^2}}$
 25 Q increases; $Q_s = -\frac{250}{3}, Q_t = -\frac{5}{3}, P_s = -.2Q_s = \frac{50}{3}, P_t = -.2Q_t = \frac{1}{3}; Q = 50 - \frac{250}{3}(s - .4) - \frac{5}{3}(t - 10)$
 27 $s = 1, t = 10$ gives $Q = 40$: $\begin{matrix} P_s & = & -Q_s & = & sQ_s + Q & = & Q_s + 40 \\ P_t & = & -Q_t & = & sQ_t + 1 & = & Q_t + 1 \end{matrix}$; $Q_s = -20, Q_t = -\frac{1}{2}, P_s = 20, P_t = \frac{1}{2}$
 29 $z - 2 = x - 2 + 2(y - 1)$ and $z - 3 = 4(x - 2) - 2(y - 1); x = 1, y = \frac{1}{2}, z = 0$
 31 $\Delta x = -\frac{1}{2}, \Delta y = \frac{1}{2}; x_1 = \frac{1}{2}, y_1 = -\frac{1}{2}$; line $x + y = 0$
 33 $3a^2\Delta x - \Delta y = -a - a^3$ gives $\Delta y = -\Delta x = \frac{a+a^3}{1+3a^2}$; lemon starts at $(1/\sqrt{3}, -1/\sqrt{3})$
 $-\Delta x + 3a^2\Delta y = a + a^3$
 35 If $x^3 = y$ then $y^3 = x^9$. Then $x^9 = x$ only if $x = 0$ or 1 or -1 (or complex number)
 37 $\Delta x = -x_0 + 1, \Delta y = -y_0 + 2, (x_1, y_1) = (1, 2) = \text{solution}$
 39 $G = H = \frac{x^n}{2x^n - 1}$ 41 $J = \begin{bmatrix} e^x & 0 \\ 1 & e^y \end{bmatrix}, \Delta x = -1 + e^{-x_n}, \Delta y = -1 - (x_n - 1 + e^{-x_n})e^{-y_n}$
 43 $(x_1, y_1) = (0, \frac{5}{4}), (-\frac{3}{4}, \frac{5}{4}), (\frac{3}{4}, 0)$

Section 13.4 Directional Derivatives and Gradients (page 495)

- 1 $\text{grad } f = 2x\mathbf{i} - 2y\mathbf{j}, D_{\mathbf{u}}f = \sqrt{3}x - y, D_{\mathbf{u}}f(P) = \sqrt{3}$
 3 $\text{grad } f = e^x \cos y \mathbf{i} - e^x \sin y \mathbf{j}, D_{\mathbf{u}}f = -e^x \sin y, D_{\mathbf{u}}f(P) = -1$
 5 $f = \sqrt{x^2 + (y - 3)^2}, \text{grad } f = \frac{x}{f}\mathbf{i} + \frac{y-3}{f}\mathbf{j}, D_{\mathbf{u}}f = \frac{x}{f}, D_{\mathbf{u}}f(P) = \frac{1}{\sqrt{5}}$ 7 $\text{grad } f = \frac{2x}{x^2 + y^2}\mathbf{i} + \frac{2y}{x^2 + y^2}\mathbf{j}$
 9 $\text{grad } f = 6x\mathbf{i} + 4y\mathbf{j} = 6\mathbf{i} + 8\mathbf{j} = \text{steepest direction at } P$; level direction $-8\mathbf{i} + 6\mathbf{j}$ is perpendicular; 10, 0
 11 T; F ($\text{grad } f$ is a vector); F; T 13 $\mathbf{u} = (\frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}}), D_{\mathbf{u}}f = \sqrt{a^2 + b^2}$
 15 $\text{grad } f = (e^{x-y}, -e^{x-y}) = (e^{-1}, -e^{-1})$ at $P; \mathbf{u} = (\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}), D_{\mathbf{u}}f = \sqrt{2}e^{-1}$
 17 $\text{grad } f = \mathbf{0}$ at maximum; level curve is one point 19 $\mathbf{N} = (-1, 1, -1), \mathbf{U} = (-1, 1, 2), \mathbf{L} = (1, 1, 0)$
 21 Direction $-\mathbf{U} = (-2, 0, -4)$ 23 $-\mathbf{U} = (\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, \frac{-x^2-y^2}{1-x^2-y^2})$
 25 $f = (x + 2y)$ and $(x + 2y)^2; \mathbf{i} + 2\mathbf{j}$; straight lines $x + 2y = \text{constant}$ (perpendicular to $\mathbf{i} + 2\mathbf{j}$)
 27 $\text{grad } f = \pm(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}); \text{grad } g = \pm(2\sqrt{5}, \sqrt{5}), f = \pm(\frac{x}{\sqrt{5}} - \frac{2y}{\sqrt{5}}) + C, g = \pm(2\sqrt{5}x + \sqrt{5}y) + C$
 29 $\theta = \text{constant}$ along ray in direction $\mathbf{u} = \frac{3\mathbf{i} + 4\mathbf{j}}{5}$; $\text{grad } \theta = \frac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2} = \frac{-4\mathbf{i} + 3\mathbf{j}}{25}$; $\mathbf{u} \cdot \text{grad } \theta = 0$
 31 $\mathbf{U} = (f_x, f_y, f_x^2 + f_y^2) = (-1, -2, 5); -\mathbf{U} = (-1, -2, 5)$; tangent at the point $(2, 1, 6)$
 33 $\text{grad } f$ toward $2\mathbf{i} + \mathbf{j}$ at $P, \mathbf{j}$ at $Q, -2\mathbf{i} + \mathbf{j}$ at $R; (2, \frac{1}{2})$ and $(2\frac{1}{2}, 2)$; largest upper left, smallest lower right;
 $z_{\max} > 9; z$ goes from 2 to 8 and back to 6

- 35 $f = \frac{1}{2}\sqrt{(x-1)^2 + (y-1)^2}$; $(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})_{0,0} = (\frac{-1}{2\sqrt{2}}, \frac{-1}{2\sqrt{2}})$
 37 Figure C now shows level curves; $|\text{grad } f|$ is varying; f could be xy
 39 $x^2 + xy$; e^{x-y} ; no function has $\frac{\partial f}{\partial x} = y$ and $\frac{\partial f}{\partial y} = -x$ because then $f_{xy} \neq f_{yx}$
 41 $\mathbf{v} = (1, 2t)$; $\mathbf{T} = \mathbf{v}/\sqrt{1+4t^2}$; $\frac{d\mathbf{f}}{dt} = \mathbf{v} \cdot (2t, 2t^2) = 2t + 4t^3$; $\frac{d\mathbf{f}}{ds} = (2t + 4t^3)/\sqrt{1+4t^2}$
 43 $\mathbf{v} = (2, 3)$; $\mathbf{T} = \frac{\mathbf{v}}{\sqrt{13}}$; $\frac{d\mathbf{f}}{dt} = \mathbf{v} \cdot (2x_0 + 4t, -2y_0 - 6t) = 4x_0 - 6y_0 - 10t$; $\frac{d\mathbf{f}}{ds} = \frac{d\mathbf{f}/dt}{\sqrt{13}}$
 45 $\mathbf{v} = (e^t, 2e^{2t}, -e^{-t})$; $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$; $\text{grad } f = (\frac{1}{x}, \frac{1}{y}, \frac{1}{z}) = (e^{-t}, e^{-2t}, e^t)$; $\frac{d\mathbf{f}}{dt} = 1 + 2 - 1$, $\frac{d\mathbf{f}}{ds} = \frac{2}{|\mathbf{v}|}$
 47 $\mathbf{v} = (-2 \sin 2t, 2 \cos 2t)$, $\mathbf{T} = (-\sin 2t, \cos 2t)$; $\text{grad } f = (y, x)$, $\frac{d\mathbf{f}}{ds} = -2 \sin^2 2t + 2 \cos^2 2t$, $\frac{d\mathbf{f}}{dt} = \frac{1}{2} \frac{d\mathbf{f}}{ds}$;
 zero slope because $f = 1$ on this path
 49 $z - 1 = 2(x - 4) + 3(y - 5)$; $f = 1 + 2(x - 4) + 3(y - 5)$ 51 $\text{grad } f \cdot \mathbf{T} = 0$; $\mathbf{T}$

Section 13.5 The Chain Rule (page 503)

- 1 $f_y = c f_x = c \cos(x + cy)$ 3 $f_y = 7 f_x = 7e^{x+7y}$ 5 $3g^2 \frac{\partial g}{\partial x} \frac{dx}{dt} + 3g^2 \frac{\partial g}{\partial y} \frac{dy}{dt}$ 7 Moves left at speed 2
 9 $\frac{dx}{dt} = 1$ (wave moves at speed 1)
 11 $\frac{\partial^2}{\partial x^2} f(x + iy) = f''(x + iy)$, $\frac{\partial^2}{\partial y^2} f(x + iy) = i^2 f''(x + iy)$
 so $f_{xx} + f_{yy} = 0$; $(x + iy)^2 = (x^2 - y^2) + i(2xy)$
 13 $\frac{d\mathbf{f}}{dt} = 2x(1) + 2y(2t) = 2t + 4t^3$ 15 $\frac{d\mathbf{f}}{dt} = y \frac{dx}{dt} + x \frac{dy}{dt} = -1$ 17 $\frac{d\mathbf{f}}{dt} = \frac{1}{x+y} \frac{dx}{dt} + \frac{1}{x+y} \frac{dy}{dt} = 1$
 19 $V = \frac{1}{3}\pi r^2 h$, $\frac{dV}{dt} = \frac{2\pi r h}{3} \frac{dr}{dt} + \frac{\pi r^2}{3} \frac{dh}{dt} = 36\pi$
 21 $\frac{dD}{dt} = \frac{90}{\sqrt{90^2 + 90^2}}(60) + \frac{90}{\sqrt{90^2 + 90^2}}(45) = \frac{105}{\sqrt{2}}$ mph; $\frac{dD}{dt} = \frac{60}{\sqrt{45^2 + 60^2}}(60) + \frac{45}{\sqrt{45^2 + 60^2}}(45) \approx 74$ mph
 23 $\frac{d\mathbf{f}}{dt} = u_1 \frac{\partial f}{\partial x} + u_2 \frac{\partial f}{\partial y} + u_3 \frac{\partial f}{\partial z}$ 25 $\frac{\partial f}{\partial t} = 1$ with x and y fixed; $\frac{d\mathbf{f}}{dt} = 6$
 27 $f_t = f_x t + f_y(2t)$; $f_{tt} = f_{xt}t + f_x + 2f_{yt}t + 2f_y = (f_{xx}t + f_{yx}(2t))t + f_x + 2(f_{xy}t + f_{yy}(2t))t + 2f_y$
 29 $\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r} = \frac{\partial f}{\partial x} \cos \theta + \frac{\partial f}{\partial y} \sin \theta$, θ is fixed
 31 $r_{xx} = \frac{1}{\sqrt{x^2 + y^2}} - \frac{x^2}{(x^2 + y^2)^{3/2}} = \frac{y^2}{(x^2 + y^2)^{3/2}}$; $\frac{\partial}{\partial x}(\frac{x}{r}) = \frac{1}{r} - x r^{-2} \frac{\partial r}{\partial x} = \frac{1}{r} - \frac{x^2}{r^3} = \frac{y^2}{r^3}$
 33 $(\frac{\partial z}{\partial x})_y = \frac{1}{\sqrt{1 - (x+y)^2}}$; $(\cos z)(\frac{\partial z}{\partial x})_y = 1$; first answer is also $\frac{1}{\sqrt{1 - \sin^2 z}} = \frac{1}{\cos z}$
 35 $f_r = f_x \cos \theta + f_y \sin \theta$, $f_{r\theta} = -f_x \sin \theta + f_y \cos \theta + f_{xx}(-r \sin \theta \cos \theta) + f_{xy}(-r \sin^2 \theta + r \cos^2 \theta) + f_{yy}(r \cos \theta \sin \theta)$
 37 Yes (with y constant): $\frac{\partial z}{\partial x} = y e^{xy}$, $\frac{\partial x}{\partial z} = \frac{1}{zy} = \frac{1}{y e^{xy}}$ 39 $f_t = f_x x_t + f_y y_t$; $f_{tt} = f_{xx} x_t^2 + 2f_{xy} x_t y_t + f_{yy} y_t^2$
 41 $(\frac{\partial f}{\partial x})_z = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial x} = a - \frac{3}{5}b$; $(\frac{\partial f}{\partial x})_y = a$; $(\frac{\partial f}{\partial z})_x = \frac{\partial f}{\partial y} \frac{\partial y}{\partial z} = \frac{1}{5}b$
 43 1 45 $f = y^2$ so $f_x = 0$, $f_y = 2y = 2r \sin \theta$; $f = r^2$ so $f_r = 2r = 2\sqrt{x^2 + y^2}$, $f_\theta = 0$
 47 $g_u = f_x x_u + f_y y_u = f_x + f_y$; $g_v = f_x x_v + f_y y_v = f_x - f_y$; $g_{uu} = f_{xx} x_u + f_{xy} y_u + f_{yx} x_u + f_{yy} y_u$
 $= f_{xx} + 2f_{xy} + f_{yy}$; $g_{vv} = f_{xx} x_v + f_{xy} y_v - f_{yx} x_v - f_{yy} y_v = f_{xx} - 2f_{xy} + f_{yy}$. Add $g_{uu} + g_{vv}$ 49 False

Section 13.6 Maxima, Minima, and Saddle Points (page 512)

- 1 $(0, 0)$ is a minimum 3 $(3, 0)$ is a saddle point 5 No stationary points 7 $(0, 0)$ is a maximum
 9 $(0, 0, 2)$ is a minimum 11 All points on the line $x = y$ are minima 13 $(0, 0)$ is a saddle point
 15 $(0, 0)$ is a saddle point; $(2, 0)$ is a minimum; $(0, -2)$ is a maximum; $(2, -2)$ is a saddle point
 17 Maximum of area $(12 - 3y)y$ is 12
 19 $2(x + y) + 2(x + 2y - 5) + 2(x + 3y - 4) = 0$ gives $x = 2$;
 $2(x + y) + 4(x + 2y - 5) + 6(x + 3y - 4) = 0$ gives $y = -1$ min because $E_{xx}E_{yy} = (6)(28) > E_{xy}^2 = 12^2$
 21 Minimum at $(0, \frac{1}{2})$; $(0, 1)$; $(0, 1)$

- 23 $\frac{df}{dt} = 0$ when $\tan t = \sqrt{3}$; $f_{\max} = 2$ at $(\frac{1}{2}, \frac{\sqrt{3}}{2})$, $f_{\min} = -2$ at $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$
 25 $(ax + by)_{\max} = \sqrt{a^2 + b^2}$; $(x^2 + y^2)_{\min} = \frac{1}{a^2 + b^2}$ 27 $0 < c < \frac{1}{4}$
 29 The vectors head-to-tail form a 60-60-60 triangle. The outer angle is 120° 31 $2 + \sqrt{3}$; $1 + \sqrt{3}$; $1 + \frac{\sqrt{3}}{10}$
 35 Steiner point where the arcs meet 39 Best point for $p = \infty$ is equidistant from corners
 41 $\text{grad } f = (\sqrt{2} \frac{x-x_1}{d_1} + \frac{x-x_2}{d_2} + \frac{x-x_3}{d_3}, \sqrt{2} \frac{y-y_1}{d_1} + \frac{y-y_2}{d_2} + \frac{y-y_3}{d_3})$; angles are 90-135-135
 43 Third derivatives all 6; $f = \frac{6}{3!}x^3 + \frac{6}{2!}x^2y + \frac{6}{2!}xy^2 + \frac{6}{3!}y^3$
 45 $(\frac{\partial}{\partial x})^n (\frac{\partial}{\partial y})^m \ln(1 - xy)|_{0,0} = n!(n-1)!$ for $m = n > 0$, other derivatives zero; $f = -xy - \frac{x^2y^2}{2} - \frac{x^3y^3}{3} - \dots$
 47 All derivatives are e^2 at $(1,1)$; $f \approx e^2[1 + (x-1) + (y-1) + \frac{1}{2}(x-1)^2 + (x-1)(y-1) + \frac{1}{2}(y-1)^2]$
 49 $x = 1, y = -1$: $f_x = 2, f_y = -2, f_{xx} = 2, f_{xy} = 0, f_{yy} = 2$; series must recover $x^2 + y^2$
 51 Line $x - 2y = \text{constant}$; $x + y = \text{constant}$
 53 $\frac{x^2}{2}f_{xx} + xyf_{xy} + \frac{y^2}{2}f_{yy}|_{0,0}$; $f_{xx} > 0$ and $f_{xx}f_{yy} > f_{xy}^2$ at $(0,0)$; $f_x = f_y = 0$ 55 $\Delta x = -1, \Delta y = -1$
 57 $f = x^2(12 - 4x)$ has $f_{\max} = 16$ at $(2,4)$; line has slope $-4, y = \frac{16}{x^2}$ has slope $\frac{-32}{x^3} = -4$
 59 If the fence were not perpendicular, a point to the left or right would be closer

Section 13.7 Constraints and Lagrange Multipliers (page 519)

- 1 $f = x^2 + (k - 2x)^2$; $\frac{df}{dx} = 2x - 4(k - 2x) = 0$; $(\frac{2k}{5}, \frac{k}{5})$ 3 $\lambda = -4, x_{\min} = 2, y_{\min} = 2$
 5 $\lambda = \frac{1}{3(4)^{1/3}}$; $(x, y) = (\pm 2^{1/6}, 0)$ or $(0, \pm 2^{1/6})$, $f_{\min} = 2^{1/3}$; $\lambda = \frac{1}{3}$; $(x, y) = (\pm 1, \pm 1)$, $f_{\max} = 2$
 7 $\lambda = \frac{1}{2}$, $(x, y) = (2, -3)$; tangent line is $2x - 3y = 13$
 9 $(1 - c)^2 + (-a - c)^2 + (2 - a - b - c)^2 + (2 - b - c)^2$ is minimized at $a = -\frac{1}{2}, b = \frac{3}{2}, c = \frac{3}{4}$
 11 $(1, -1)$ and $(-1, 1)$; $\lambda = -\frac{1}{2}$
 13 f is not a minimum when C crosses to lower level curve; stationary point when C is tangent to level curve
 15 Substituting $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial \lambda} = 0$ and $L = f_{\min}$ leaves $\frac{df_{\min}}{dk} = \lambda$
 17 x^2 is never negative; $(0,0)$; $1 = \lambda(-3y^2)$ but $y = 0$; $g = 0$ has a cusp at $(0,0)$
 19 $2x = \lambda_1 + \lambda_2, 4y = \lambda_1, 2z = \lambda_1 - \lambda_2, x + y + z = 0, x - z = 1$ gives $\lambda_1 = 0, \lambda_2 = 1, f_{\min} = \frac{1}{2}$ at $(\frac{1}{2}, 0, -\frac{1}{2})$
 21 $(1, 0, 0); (0, 1, 0); (\lambda_1, \lambda_2, 0); x = y = 0$ 23 $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$; $\lambda = 0$
 25 $(1, 0, 0), (0, 1, 0), (0, 0, 1)$; at these points $f = 4$ and -2 (min) and 5 (max)
 27 By increasing k , more points are available so $f_{\max}$ goes up. Then $\lambda = \frac{df_{\min}}{dk} \geq 0$
 29 $(0,0)$; $\lambda = 0$; $f_{\min}$ stays at 0
 31 $5 = \lambda_1 + \lambda_2, 6 = \lambda_1 + \lambda_3, \lambda_2 \geq 0, \lambda_3 \leq 0$; subtraction $5 - 6 = \lambda_2 - \lambda_3$ or $-1 \geq 0$ (impossible);
 $x = 2004, y = -2000$ gives $5x + 6y = -1980$
 33 $2x = 4\lambda_1 + \lambda_2, 2y = 4\lambda_1 + \lambda_3, \lambda_2 \geq 0, \lambda_3 \geq 0, 4x + 4y = 40$; max area 100 at $(10,0)(0,10)$; min 25 at $(5,5)$

CHAPTER 14 MULTIPLE INTEGRALS

Section 14.1 Double Integrals (page 526)

- 1 $\frac{8}{3}; \frac{2}{3}$ 3 $1; \ln \frac{3}{2}$ 5 2 7 $\frac{1}{2}$ 9 $\frac{4}{3}$ 11 $\int_{y=1}^2 \int_{x=1}^2 dx dy + \int_{y=2}^4 \int_{x=1}^2 dx dy$
 13 $\int_{y=0}^1 \int_{x=-\frac{1}{2}}^{-\ln y} dx dy$ 15 $\int_{x=0}^1 \int_{y=-\sqrt{x}}^{\sqrt{x}} dy dx$ 17 $\int_0^1 \int_0^{y/2} dx dy = \int_0^{1/2} \int_{2x}^1 dy dx = \frac{1}{4}$
 19 $\int_0^3 \int_{-y}^y dx dy = \int_{-1}^0 \int_{-x}^3 dy dx + \int_0^1 \int_x^3 dy dx = 9$ 21 $\int_0^4 \int_{y/2}^y dx dy + \int_4^8 \int_{y/2}^4 dx dy = \int_0^4 \int_x^{2x} dy dx = 8$
 23 $\int_0^1 \int_0^{bx} dy dx + \int_1^2 \int_0^{b(2-x)} dy dx = \int_0^b \int_{y/b}^{2-(y/b)} dx dy = b$ 25 $f(a, b) - f(a, 0) - f(0, b) + f(0, 0)$

- 27** $\int_0^1 \int_0^1 (2x - 3y + 1) dx dy = \frac{1}{2}$ **29** $\int_a^b f(x) dx = \int_a^b \int_0^{f(x)} 1 dy dx$ **31** $50,000\pi$
33 $\int_1^3 \int_1^2 x^2 dx dy = \frac{14}{3}$ **35** $2 \int_0^{1/\sqrt{2}} \int_0^{\sqrt{1-y^2}} 1 dx dy = \frac{\pi}{4}$
37 $\frac{1}{n^2} \sum_{j=1}^n \sum_{i=1}^n f\left(\frac{i-\frac{1}{2}}{n}, \frac{j-\frac{1}{2}}{n}\right)$ is exact for $f = 1, x, y, xy$ **39** Volume 8.5 **41** Volumes $\ln 2, 2 \ln(1 + \sqrt{2})$
43 $\int_0^1 \int_0^1 x^y dx dy = \int_0^1 \frac{1}{y+1} dy = \ln 2$; $\int_0^1 \int_0^1 x^y dy dx = \int_0^1 \frac{x-1}{\ln x} dx = \ln 2$
45 With long rectangles $\sum y_i \Delta A = \sum \Delta A = 1$ but $\iint y dA = \frac{1}{2}$

Section 14.2 Change to Better Coordinates (page 534)

- 1** $\int_{\pi/4}^{3\pi/4} \int_0^1 r dr d\theta = \frac{\pi}{4}$ **3** S = quarter-circle with $u \geq 0$ and $v \geq 0$; $\int_0^1 \int_0^{\sqrt{1-v^2}} du dv$
5 R is symmetric across the y axis; $\int_0^1 \int_0^{\sqrt{1-v^2}} u du dv = \frac{1}{3}$ divided by area gives $(\bar{u}, \bar{v}) = (4/3\pi, 4/3\pi)$
7 $2 \int_0^{1/\sqrt{2}} \int_{1+x}^{1+\sqrt{1-x^2}} dy dx$; xy region R^* becomes R in the x^*y^* plane; $dx dy = dx^* dy^*$ when region moves
9 $J = \begin{vmatrix} \partial x / \partial r^* & \partial x / \partial \theta^* \\ \partial y / \partial r^* & \partial y / \partial \theta^* \end{vmatrix} = \begin{vmatrix} \cos \theta^* & -r^* \sin \theta^* \\ \sin \theta^* & r^* \cos \theta^* \end{vmatrix} = r^*$; $\int_{\pi/4}^{3\pi/4} \int_0^1 r^* dr^* d\theta^*$
11 $I_y = \iint_R x^2 dx dy = \int_{\pi/4}^{3\pi/4} \int_0^1 r^2 \cos^2 \theta r dr d\theta = \frac{\pi}{16} - \frac{1}{8}$; $I_x = \frac{\pi}{16} + \frac{1}{8}$; $I_0 = \frac{\pi}{8}$
13 $(0,0), (1,2), (1,3), (0,1)$; area of parallelogram is 1
15 $x = u, y = u + 3v + uv$; then $(u, v) = (1,0), (1,1), (0,1)$ give corners $(x, y) = (1,0), (1,5), (0,3)$
17 Corners $(0,0), (2,1), (3,3), (1,2)$; sides $y = \frac{1}{2}x, y = 2x - 3, y = \frac{1}{2}x + \frac{3}{2}, y = 2x$
19 Corners $(1,1), (e^2, e), (e^3, e^3), (e, e^2)$; sides $x = y^2, y = x^2/e^3, x = y^2/e^3, y = x^2$
21 Corners $(0,0), (1,0), (1,2), (0,1)$; sides $y = 0, x = 1, y = 1 + x^2, x = 0$
23 $J = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$, area $\int_0^1 \int_0^1 3 du dv = 3$; $J = \begin{vmatrix} 2e^{2u+v} & e^{2u+v} \\ e^{u+2v} & 2e^{u+2v} \end{vmatrix} = 3e^{3u+3v}$, $\int_0^1 \int_0^1 3e^{3u+3v} du dv = \int_0^1 (e^{3+3v} - e^{3v}) dv = \frac{1}{3}(e^6 - 2e^3 + 1)$
25 Corners $(x, y) = (0,0), (1,0), (1, f(1)), (0, f(0))$; $(\frac{1}{2}, 1)$ gives $x = \frac{1}{2}, y = f(\frac{1}{2})$; $J = \begin{vmatrix} 1 & 0 \\ v f'(u) & f(u) \end{vmatrix} = f(u)$
27 $B^2 = 2 \int_0^{\pi/4} \int_0^{1/\sin \theta} e^{-r^2} r dr d\theta = \int_0^{\pi/4} (e^{-1/\sin^2 \theta} - 1) d\theta$
29 $\bar{r} = \iint r^2 dr d\theta / \iint r dr d\theta = \int_0^\pi \frac{8}{3} a^3 \cos^3 \theta d\theta / \pi a^2 = \frac{32a}{9\pi}$ **31** $\int_0^{2\pi} \int_0^1 r^2 r dr d\theta = \frac{\pi}{2}$
33 Along the right side; along the bottom; at the bottom right corner
35 $\iint xy dx dy = \int_0^1 \int_0^1 (u \cos \alpha - v \sin \alpha)(u \sin \alpha + v \cos \alpha) du dv = \frac{1}{4}(\cos^2 \alpha - \sin^2 \alpha)$
37 $\int_0^{2\pi} \int_4^5 r^2 r^2 r dr d\theta = \frac{2\pi}{6}(5^6 - 4^6)$ **39** $x = \cos \alpha - \sin \alpha, y = \sin \alpha + \cos \alpha$ goes to $u = 1, v = 1$

Section 14.3 Triple Integrals (page 540)

- 1** $\int_0^1 \int_0^z \int_0^y dx dy dz = \frac{1}{6}$
3 $0 \leq y \leq x \leq z \leq 1$ and all other orders xyz, yzx, zxy, zyx ; all six contain $(0,0,0)$; to contain $(1,0,1)$
5 $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 dx dy dz = 8$ **7** $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 dx dy dz = 4$ **9** $\int_{-1}^1 \int_z^1 \int_1^z dx dy dz = \frac{4}{3}$
11 $\int_0^1 \int_0^{2-2z} \int_0^{2-y-2z} dx dy dz = \frac{2}{3}$ **13** $\int_0^{1/2} \int_0^{2-2z} \int_0^{2-y-2z} dx dy dz = \frac{7}{12}$
15 $\int_0^1 \int_0^{1-z} \int_0^{\sqrt{(1-z)^2 - y^2}} dx dy dz = \frac{\pi}{3}$ **17** $\int_0^6 \int_0^1 \int_0^{\sqrt{1-y^2}} dx dy dz = 6\pi$ **19** $\int_0^1 \int_0^1 \int_0^{\sqrt{1-y^2}} dx dy dz = \pi$
21 Corner of cube at $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$; sides $\frac{2}{\sqrt{3}}$; area $\frac{8}{3\sqrt{3}}$
23 Horizontal slices are circles of area $\pi r^2 = \pi(4 - z)$; volume $= \int_0^4 \pi(4 - z) dz = 8\pi$; centroid has $\bar{x} = 0, \bar{y} = 0, \bar{z} = \int_0^4 z\pi(4 - z) dz / 8\pi = \frac{4}{3}$

- 25 $I = \frac{z^2}{2}$ gives zeros; $\frac{\partial I}{\partial x} = \int_0^z \int_0^y f \, dy \, dz$, $\frac{\partial I}{\partial y} = \int_0^z \int_0^x f \, dx \, dz$, $\frac{\partial^2 I}{\partial y \partial x} = \int_0^z f \, dz$
 27 $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (y^2 + z^2) dx \, dy \, dz = \frac{16}{3}$; $\iiint x^2 dV = \frac{8}{3}$; $3 \iiint (x - \frac{x+y+z}{3})^2 dV = \frac{16}{3}$
 29 $\int_0^3 \int_0^2 \int_0^y dx \, dy \, dz = 6$ 31 Trapezoidal rule is second-order; correct for 1, x , y , z , xy , xz , yz , xyz

Section 14.4 Cylindrical and Spherical Coordinates (page 547)

- 1 $(r, \theta, z) = (D, 0, 0)$; $(\rho, \phi, \theta) = (D, \frac{\pi}{2}, 0)$ 3 $(r, \theta, z) = (0, \text{any angle}, D)$; $(\rho, \phi, \theta) = (D, 0, \text{any angle})$
 5 $(x, y, z) = (2, -2, 2\sqrt{2})$; $(r, \theta, z) = (2\sqrt{2}, -\frac{\pi}{4}, 2\sqrt{2})$ 7 $(x, y, z) = (0, 0, -1)$; $(r, \theta, z) = (0, \text{any angle}, -1)$
 9 $\phi = \tan^{-1}(\frac{r}{z})$ 11 45° cone in unit sphere: $\frac{2\pi}{3}(1 - \frac{1}{\sqrt{2}})$ 13 cone without top: $\frac{7\pi}{3}$
 15 $\frac{1}{4}$ hemisphere: $\frac{\pi}{6}$ 17 $\frac{\pi^2}{8}$ 19 Hemisphere of radius π : $\frac{2}{3}\pi^4$ 21 $\pi(R^2 - z^2)$; $4\pi r\sqrt{R^2 - r^2}$
 23 $\frac{2}{3}a^3 \tan \alpha$ (see 8.1.39) 27 $\frac{\partial q}{\partial D} = \frac{\rho - D \cos \phi}{q} = \frac{\text{near side}}{\text{hypotenuse}} = \cos \alpha$
 31 Wedges are not exactly similar; the error is higher order $\Rightarrow$ proof is correct
 33 Proportional to $1 + \frac{1}{h}(\sqrt{a^2 + (D-h)^2} - \sqrt{a^2 + D^2})$
 35 $J = \begin{vmatrix} a & & \\ & b & \\ & & c \end{vmatrix} = abc$; straight edges at right angles 37 $\begin{vmatrix} \cos \theta & -r \sin \theta & 0 \\ \sin \theta & r \cos \theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$
 39 $\frac{8\pi\rho^4}{3}$; $\frac{2}{3}$ 41 ρ^3 ; ρ^2 ; force = 0 inside hollow sphere

CHAPTER 15 VECTOR CALCULUS

Section 15.1 Vector Fields (page 554)

- 1 $f(x, y) = x + 2y$ 3 $f(x, y) = \sin(x + y)$ 5 $f(x, y) = \ln(x^2 + y^2) = 2 \ln r$
 7 $\mathbf{F} = xy\mathbf{i} + \frac{x^2}{2}\mathbf{j}$, $f(x, y) = \frac{x^2 y}{2}$ 9 $\frac{\partial f}{\partial x} = 0$ so f cannot depend on x ; streamlines are vertical ($y = \text{constant}$)
 11 $\mathbf{F} = 3\mathbf{i} + \mathbf{j}$ 13 $\mathbf{F} = \mathbf{i} + 2y\mathbf{j}$ 15 $\mathbf{F} = 2x\mathbf{i} - 2y\mathbf{j}$ 17 $\mathbf{F} = e^{x-y}\mathbf{i} - e^{x-y}\mathbf{j}$
 19 $\frac{dy}{dx} = -1$; $y = -x + C$ 21 $\frac{dy}{dx} = -\frac{x}{y}$; $x^2 + y^2 = C$ 23 $\frac{dy}{dx} = \frac{-x/y^2}{1/y} = \frac{-x}{y}$; $x^2 + y^2 = C$ 25 parallel
 27 $\mathbf{F} = \frac{5x}{r}\mathbf{i} + \frac{5y}{r}\mathbf{j}$ 29 $\mathbf{F} = \frac{-mMG}{r^3}(x\mathbf{i} + y\mathbf{j}) - \frac{mMG}{((x-1)^2 + y^2)^{3/2}}((x-1)\mathbf{i} + y\mathbf{j})$
 31 $\mathbf{F} = \frac{\sqrt{2}}{2}y\mathbf{i} - \frac{\sqrt{2}}{2}x\mathbf{j}$ 33 $\frac{dy}{dx} = \frac{-2}{x^2} = -\frac{1}{2}$; $\frac{dy}{dx} = \frac{x}{\sqrt{x^2-3}} = 2$
 35 $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r} \frac{\partial r}{\partial x} = \frac{\partial f}{\partial r} \frac{x}{r}$; $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial r} \frac{y}{r}$; $f(r) = C$ gives circles
 37 T; F (no equipotentials); T; F (not multiple of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$)
 39 $\mathbf{F}$ and $\mathbf{F} + \mathbf{i}$ and $2\mathbf{F}$ have the same streamlines (different velocities) and equipotentials (different potentials).
 But if f is given, $\mathbf{F}$ must be $\text{grad } f$.

Section 15.2 Line Integrals (page 562)

- 1 $\int_0^1 \sqrt{1^2 + 2^2} dt = \sqrt{5}$; $\int_0^1 2 \, dt = 2$ 3 $\int_0^1 t^2 \sqrt{2} dt + \int_1^2 1 \cdot (2-t) dt = \frac{\sqrt{2}}{3} + \frac{1}{2}$
 5 $\int_0^{2\pi} (-3 \sin t) dt = 0$ (gradient field); $\int_0^{2\pi} -9 \sin^2 t \, dt = -9\pi = -$ area
 7 No, $xy\mathbf{j}$ is not a gradient field; take line $x = t, y = t$ from $(0,0)$ to $(1,1)$ and $\int t^2 dt \neq \frac{1}{2}$
 9 No, for a circle $(2\pi r)^2 \neq 0^2 + 0^2$ 11 $f = x + \frac{1}{2}y^2$; $f(0,1) - f(1,0) = -\frac{1}{2}$
 13 $f = \frac{1}{2}x^2 y^2$; $f(0,1) - f(1,0) = 0$ 15 $f = r = \sqrt{x^2 + y^2}$; $f(0,1) - f(1,0) = 0$
 17 Gradient for $n = 2$; after calculation $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{n-2}{r^n}$
 19 $x = a \cos t, z = a \sin t, ds = a \, dt, M = \int_0^{2\pi} (a + a \sin t)a \, dt = 2\pi a^2$

- 21** $x = a \cos t, y = a \sin t, ds = a dt, M = \int_0^{2\pi} a^3 \cos^2 t dt = \pi a^3, (\bar{x}, \bar{y}) = (0, 0)$ by symmetry
- 23** $\mathbf{T} = \frac{2\mathbf{i}+2\mathbf{j}}{\sqrt{4+4t^2}} = \frac{\mathbf{i}+\mathbf{j}}{\sqrt{1+t^2}}; \mathbf{F} = 3x\mathbf{i} + 4\mathbf{j} = 6t\mathbf{i} + 4\mathbf{j}, ds = 2\sqrt{1+t^2}dt, \mathbf{F} \cdot \mathbf{T}ds = (6t\mathbf{i} + 4\mathbf{j}) \cdot \left(\frac{\mathbf{i}+\mathbf{j}}{\sqrt{1+t^2}}\right) 2\sqrt{1+t^2}dt = 20t dt; \mathbf{F} \cdot d\mathbf{R} = (6t\mathbf{i} + 4\mathbf{j}) \cdot (2dt\mathbf{i} + 2t dt\mathbf{j}) = 20t dt; \text{work} = \int_1^2 20t dt = 30$
- 25** If $\frac{\partial M(y)}{\partial y} = \frac{\partial N(x)}{\partial x}$ then $M = ay + b, N = ax + c$, constants a, b, c
- 27** $\mathbf{F} = 4x\mathbf{j}$ (work = 4 from $(1, 0)$ up to $(1, 1)$) **29** $f = [x - 2y]_{(0,0)}^{(1,1)} = -1$ **31** $f = [xy^2]_{(0,0)}^{(1,1)} = 1$
- 33** Not conservative; $\int_0^1 (ti - tj) \cdot (dt\mathbf{i} + dt\mathbf{j}) = \int 0 dt = 0; \int_0^1 (t^2\mathbf{i} - t\mathbf{j}) \cdot (dt\mathbf{i} + 2t dt\mathbf{j}) = \int_0^1 -t^2 dt = -\frac{1}{3}$
- 35** $\frac{\partial M}{\partial y} = ax, \frac{\partial N}{\partial x} = 2x + b$, so $a = 2, b$ is arbitrary **37** $\frac{\partial M}{\partial y} = 2ye^{-x} = \frac{\partial N}{\partial x}; f = -y^2 e^{-x}$
- 39** $\frac{\partial M}{\partial y} = \frac{-xy}{r^3} = \frac{\partial N}{\partial x}; f = r = \sqrt{x^2 + y^2} = |\mathbf{x}\mathbf{i} + \mathbf{y}\mathbf{j}|$
- 41** $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$ has $\frac{\partial M}{\partial y} = -1, \frac{\partial N}{\partial x} = 1$, no f **43** $2\pi; 0; 0$

Section 15.3 Green's Theorem (page 571)

- 1** $\int_0^{2\pi} (a \cos t) a \cos t dt = \pi a^2; N_x - M_y = 1, \iint dx dy = \text{area } \pi a^2$
- 3** $\int_0^1 x dx + \int_1^0 x dx = 0, N_x - M_y = 0, \iint 0 dx dy = 0$
- 5** $\int x^2 y dx = \int_0^{2\pi} (a \cos t)^2 (a \sin t) (-a \sin t dt) = -\frac{a^4}{4} \int_0^{2\pi} (\sin 2t)^2 dt = -\frac{\pi a^4}{4};$
 $N_x - M_y = -x^2, \iint (-x^2) dx dy = \int_0^{2\pi} \int_0^a -r^2 \cos^2 \theta r dr d\theta = -\frac{\pi a^4}{4}$
- 7** $\int x dy - y dx = \int_0^\pi (\cos^2 t + \sin^2 t) dt = \pi; \iint (1 + 1) dx dy = 2 (\text{area}) = \pi; \int x^2 dy - xy dx = \frac{1}{2} + 1;$
 $\int_0^1 \int_0^1 (2x + x) dx dy = \frac{3}{2}$
- 9** $\frac{1}{2} \int_0^{2\pi} (3 \cos^4 t \sin^2 t + 3 \sin^4 t \cos^2 t) dt = \frac{1}{2} \int_0^{2\pi} 3 \cos^2 t \sin^2 t dt = \frac{3}{2} \frac{\pi}{4}$ (see Answer 5)
- 11** $\int \mathbf{F} \cdot d\mathbf{R} = 0$ around any loop; $\mathbf{F} = \frac{x}{r}\mathbf{i} + \frac{y}{r}\mathbf{j}$ and $\int \mathbf{F} \cdot d\mathbf{R} = \int_0^{2\pi} [-\sin t \cos t + \sin t \cos t] dt = 0;$
 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ gives $\iint 0 dx dy$
- 13** $x = \cos 2t, y = \sin 2t, t$ from 0 to $2\pi; \int_0^{2\pi} -2 \sin^2 2t dt = -2\pi = -2 (\text{area});$
 $\int_0^{2\pi} -2 dt = -4\pi = -2$ times Example 7
- 15** $\int M dy - N dx = \int_0^{2\pi} 2 \sin t \cos t dt = 0; \iint (M_x + N_y) dx dy = \iint 0 dx dy = 0$
- 17** $M = \frac{x}{r}, N = \frac{y}{r}, \int M dy - N dx = \int_0^{2\pi} (\cos^2 t + \sin^2 t) dt = 2\pi; \iint (M_x + N_y) dx dy = \iint \left(\frac{1}{r} - \frac{x^2}{r^3} + \frac{1}{r} - \frac{y^2}{r^3}\right) dx dy =$
 $\iint \frac{1}{r} dx dy = \iint dr d\theta = 2\pi$
- 19** $\int M dy - N dx = \int -x^2 y dx = \int_1^0 -x^2 (1 - x) dx = \frac{1}{12}; \int_0^1 \int_0^{1-y} x^2 dx dy = \frac{1}{12}$
- 21** $\iint (M_x + N_y) dx dy = \iint \text{div } \mathbf{F} dx dy = 0$ between the circles
- 23** Work: $\int a dx + b dy = \iint \left(\frac{\partial b}{\partial x} - \frac{\partial a}{\partial y}\right) dx dy$; Flux: *same integral*
- 25** $g = \tan^{-1}(\frac{y}{x}) = \theta$ is undefined at $(0, 0)$ **27** Test $M_y = N_x : x^2 dx + y^2 dy$ is exact $= d(\frac{1}{3}x^3 + \frac{1}{3}y^3)$
- 29** $\text{div } \mathbf{F} = 2y - 2y = 0; g = xy^2$ **31** $\text{div } \mathbf{F} = 2x + 2y$; no g **33** $\text{div } \mathbf{F} = 0; g = e^x \sin y$
- 35** $\text{div } \mathbf{F} = 0; g = \frac{y^2}{x}$
- 37** $N_x - M_y = -2x, -6xy, 0, 2x - 2y, 0, -2e^{x+y}$; in **31** and **33** $f = \frac{1}{3}(x^3 + y^3)$ and $f = e^x \cos y$
- 39** $\mathbf{F} = (3x^2 - 3y^2)\mathbf{i} - 6xy\mathbf{j}; \text{div } \mathbf{F} = 0$ **41** $f = x^4 - 6x^2 y^2 + y^4; g = 4x^3 y - 4xy^3$
- 43** $\mathbf{F} = e^x \cos y \mathbf{i} - e^x \sin y \mathbf{j}; g = e^x \sin y$
- 45** $N = f(x), \int M dx + N dy = \int_0^1 f(1) dy + \int_1^0 f(0) dy = f(1) - f(0); \iint (N_x - M_y) dx dy =$
 $\iint \frac{\partial f}{\partial x} dx dy = \int_0^1 \frac{\partial f}{\partial x} dx$ (Fundamental Theorem of Calculus)

Section 15.4 Surface Integrals (page 581)

$$1 \mathbf{N} = -2x\mathbf{i} - 2y\mathbf{j} + \mathbf{k}; dS = \sqrt{1+4x^2+4y^2} dx dy; \int_0^{2\pi} \int_0^2 \sqrt{1+4r^2} r dr d\theta = \frac{\pi}{6}(17^{3/2} - 1)$$

$$3 \mathbf{N} = -\mathbf{i} + \mathbf{j} + \mathbf{k}; dS = \sqrt{3} dx dy; \text{area } \sqrt{3}\pi$$

$$5 \mathbf{N} = \frac{-x\mathbf{i}-y\mathbf{j}}{\sqrt{1-x^2-y^2}} + \mathbf{k}; dS = \frac{dx dy}{\sqrt{1-x^2-y^2}}; \int_0^{2\pi} \int_0^{1/\sqrt{2}} \frac{r dr d\theta}{\sqrt{1-r^2}} = \pi(2 - \sqrt{2})$$

$$7 \mathbf{N} = -7\mathbf{j} + \mathbf{k}; dS = 5\sqrt{2} dx dy; \text{area } 5\sqrt{2}A$$

$$9 \mathbf{N} = (y^2 - x^2)\mathbf{i} - 2xy\mathbf{j} + \mathbf{k}; dS = \sqrt{1 + (y^2 - x^2)^2 + 4x^2y^2} dx dy = \sqrt{1 + (y^2 + x^2)^2} dx dy; \int_0^{2\pi} \int_0^1 \sqrt{1+r^4} r dr d\theta = \frac{\pi}{\sqrt{2}} + \frac{\pi \ln(1+\sqrt{2})}{2}$$

$$11 \mathbf{N} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}; dS = 3dx dy; 3(\text{area of triangle with } 2x + 2y \leq 1) = \frac{3}{8}$$

$$13 \pi a \sqrt{a^2 + h^2} \quad 15 \int_0^1 \int_0^{1-y} xy(\sqrt{3} dx dy) = \frac{\sqrt{3}}{24}$$

$$17 \int_0^{2\pi} \int_0^{\pi/4} \sin^2 \phi \cos \phi \sin \theta \cos \theta (\sin \phi d\phi d\theta) = 0 \quad 19 \mathbf{A} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}; \mathbf{B} = \mathbf{j} + \mathbf{k}; \mathbf{N} = -\mathbf{i} - \mathbf{j} + \mathbf{k}; dS = \sqrt{3} du dv$$

$$21 \mathbf{A} = -\sin u(\cos v \mathbf{i} + \sin v \mathbf{j}) + \cos u \mathbf{k}; \mathbf{B} = -(3 + \cos u) \sin v \mathbf{i} + (3 + \cos u) \cos v \mathbf{j};$$

$$\mathbf{N} = -(3 + \cos u)(\cos u \cos v \mathbf{i} + \cos u \sin v \mathbf{j} + \sin u \mathbf{k}); dS = (3 + \cos u) du dv$$

$$23 \iint (-M \frac{\partial f}{\partial x} - N \frac{\partial f}{\partial y} + P) dx dy = \iint (-2x^2 - 2y^2 + z) dx dy = \iint -r^2 (r dr d\theta) = -8\pi$$

$$25 \mathbf{F} \cdot \mathbf{N} = -x + y + z = 0 \text{ on plane}$$

$$27 \mathbf{N} = -\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{F} = (v + u)\mathbf{i} - u\mathbf{j}, \iint \mathbf{F} \cdot \mathbf{N} dS = \iint -v du dv = 0$$

$$29 \iint dS = \int_0^{2\pi} \int_0^{2\pi} (3 + \cos u) du dv = 12\pi^2 \quad 31 \text{ Yes} \quad 33 \text{ No}$$

$$35 \mathbf{A} = \mathbf{i} + f' \cos \theta \mathbf{j} + f' \sin \theta \mathbf{k}; \mathbf{B} = -f \sin \theta \mathbf{j} + f \cos \theta \mathbf{k}; \mathbf{N} = f f' \mathbf{i} - f \cos \theta \mathbf{j} - f \sin \theta \mathbf{k}; dS = |\mathbf{N}| dx d\theta = f(x) \sqrt{1 + f'^2} dx d\theta$$

Section 15.5 The Divergence Theorem (page 588)

$$1 \operatorname{div} \mathbf{F} = 1, \iiint dV = \frac{4\pi}{3} \quad 3 \operatorname{div} \mathbf{F} = 2x + 2y + 2z, \iiint \operatorname{div} \mathbf{F} dV = 0 \quad 5 \operatorname{div} \mathbf{F} = 3, \iiint 3dV = \frac{3}{6} = \frac{1}{2}$$

$$7 \mathbf{F} \cdot \mathbf{N} = \rho^2, \iint_{\rho=a} \rho^2 dS = 4\pi a^4 \quad 9 \operatorname{div} \mathbf{F} = 2z, \int_0^{2\pi} \int_0^{\pi/2} \int_0^a 2\rho \cos \phi (\rho^2 \sin \phi d\rho d\phi d\theta) = \frac{1}{2}\pi a^4$$

$$11 \int_0^a \int_0^a \int_0^a (2x + 1) dx dy dz = a^4 + a^3; -2a^2 + 2a^2 + 0 + a^4 + 0 + a^3$$

$$13 \operatorname{div} \mathbf{F} = \frac{x}{\rho}, \iiint \frac{x}{\rho} dV = 0; \mathbf{F} \cdot \mathbf{n} = x, \iint x dS = 0 \quad 15 \operatorname{div} \mathbf{F} = 1; \iiint 1 dV = \frac{\pi}{3}; \iiint 1 dV = \frac{1}{6}$$

$$17 \operatorname{div} \left(\frac{\mathbf{R}}{\rho^r} \right) = \frac{\operatorname{div} \mathbf{R}}{\rho^r} + \mathbf{R} \cdot \operatorname{grad} \frac{1}{\rho^r} = \frac{3}{\rho^r} - \frac{7}{\rho^r} \mathbf{R} \cdot \operatorname{grad} \rho$$

$$19 \text{ Two spheres, } \mathbf{n} \text{ radial out, } \mathbf{n} \text{ radial in, } \mathbf{n} = \mathbf{k} \text{ on top, } \mathbf{n} = -\mathbf{k} \text{ on bottom, } \mathbf{n} = \frac{x\mathbf{i}+y\mathbf{j}}{\sqrt{x^2+y^2}} \text{ on side;}$$

$$\mathbf{n} = -\mathbf{i}, -\mathbf{j}, -\mathbf{k}, \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \text{ on 4 faces; } \mathbf{n} = \mathbf{k} \text{ on top, } \mathbf{n} = \frac{1}{\sqrt{2}} \left(\frac{x}{r}\mathbf{i} + \frac{y}{r}\mathbf{j} - \mathbf{k} \right) \text{ on cone}$$

$$21 V = \text{cylinder, } \iint \operatorname{div} \mathbf{F} dV = \iint \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy (z \text{ integral} = 1); \iint \mathbf{F} \cdot \mathbf{n} dS =$$

$$\int M dy - N dx, z \text{ integral} = 1 \text{ on side, } \mathbf{F} \cdot \mathbf{n} = 0 \text{ top and bottom; Green's flux theorem.}$$

$$23 \operatorname{div} \mathbf{F} = \frac{-3GM}{a^3} = -4\pi G; \text{ at the center; } \mathbf{F} = 2\mathbf{R} \text{ inside, } \mathbf{F} = 2\left(\frac{a}{\rho}\right)^3 \mathbf{R} \text{ outside}$$

$$25 \operatorname{div} \mathbf{u}_r = \frac{2}{\rho}, q = \frac{2\epsilon_0}{\rho}, \iint \mathbf{E} \cdot \mathbf{n} dS = \iint 1 dS = 4\pi \quad 27 \mathbf{F} (\operatorname{div} \mathbf{F} = 0); \mathbf{F}; \mathbf{T}(\mathbf{F} \cdot \mathbf{n} \leq 1); \mathbf{F}$$

$$29 \text{ Plane circle; top half of sphere; } \operatorname{div} \mathbf{F} = 0$$

Section 15.6 Stokes' Theorem and the Curl of F (page 595)

$$1 \operatorname{curl} \mathbf{F} = \mathbf{i} + \mathbf{j} + \mathbf{k} \quad 3 \operatorname{curl} \mathbf{F} = 0 \quad 5 \operatorname{curl} \mathbf{F} = 0 \quad 7 f = \frac{1}{2}(x + y + z)^2$$

$$9 \operatorname{curl} x^m \mathbf{i} = 0; x^n \mathbf{j} \text{ has zero curl if } n = 0 \quad 11 \operatorname{curl} \mathbf{F} = 2y\mathbf{i}; \mathbf{n} = \mathbf{j} \text{ on circle so } \iint \mathbf{F} \cdot \mathbf{n} dS = 0$$

$$13 \operatorname{curl} \mathbf{F} = 2\mathbf{i} + 2\mathbf{j}, \mathbf{n} = \mathbf{i}, \iint \operatorname{curl} \mathbf{F} \cdot \mathbf{n} dS = \iint 2 dS = 2\pi$$

15 Both integrals equal $\int \mathbf{F} \cdot d\mathbf{R}$; Divergence Theorem, V = region between S and T , always $\text{div curl } \mathbf{F} = 0$

17 Always $\text{div curl } \mathbf{F} = 0$ **19** $f = xz + y$ **21** $f = e^{x-z}$ **23** $\mathbf{F} = y\mathbf{k}$

25 $\text{curl } \mathbf{F} = (a_3b_2 - a_2b_3)\mathbf{i} + (a_1b_3 - a_3b_1)\mathbf{j} + (a_2b_1 - a_1b_2)\mathbf{k}$ **27** $\text{curl } \mathbf{F} = 2\omega\mathbf{k}$; $\text{curl } \mathbf{F} \cdot \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}} = 2\omega/\sqrt{3}$

29 $\mathbf{F} = x(a_3z + a_2y)\mathbf{i} + y(a_1x + a_3z)\mathbf{j} + z(a_1x + a_2y)\mathbf{k}$

31 $\text{curl } \mathbf{F} = -2\mathbf{k}$, $\iint -2\mathbf{k} \cdot \mathbf{R} dS = \int_0^{2\pi} \int_0^{\pi/2} -2 \cos \phi (\sin \phi d\phi d\theta) = -2\pi$; $\int y dx - x dy = \int_0^{2\pi} (-\sin^2 t - \cos^2 t) dt = -2\pi$

33 $\text{curl } \mathbf{F} = 2\mathbf{a}$, $2 \iint (a_1x + a_2y + a_3z) dS = 0 + 0 + 2a_3 \int_0^{2\pi} \int_0^{\pi/2} \cos \phi \sin \phi d\phi d\theta = 2\pi a_3$

35 $\text{curl } \mathbf{F} = -\mathbf{i}$, $\mathbf{n} = \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$, $\iint \mathbf{F} \cdot \mathbf{n} dS = -\frac{1}{\sqrt{3}}\pi r^2$

37 $g = \frac{y^2}{2} - \frac{z^3}{3}$ = stream function; zero divergence

39 $\text{div } \mathbf{F} = \text{div } (\mathbf{V} + \mathbf{W}) = \text{div } \mathbf{V}$ so $y = \text{div } \mathbf{V}$ so $\mathbf{V} = \frac{y^2}{2}\mathbf{j}$ (has zero curl). Then $\mathbf{W} = \mathbf{F} - \mathbf{V} = xy\mathbf{i} - \frac{y^2}{2}\mathbf{j}$

41 $\text{curl } (\text{curl } \mathbf{F}) = \text{curl } (-2y\mathbf{k}) = -2\mathbf{i}$; $\text{grad } (\text{div } \mathbf{F}) = \text{grad } 2x = 2\mathbf{i}$; $\mathbf{F}_{xx} + \mathbf{F}_{yy} + \mathbf{F}_{zz} = 4\mathbf{i}$

43 $\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = \mathbf{a} \sin t$ so $\mathbf{E} = \frac{1}{2}(\mathbf{a} \times \mathbf{R}) \sin t$

45 $\mathbf{n} = \mathbf{j}$ so $\int M dx + P dz = \iint (\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x}) dx dz$ **47** $M_y^* = M_y + M_z f_y + P_y f_x + P_z f_y f_x + P f_{xy}$

49 $\int \mathbf{F} \cdot d\mathbf{R} = \iint \text{curl } \mathbf{F} \cdot \mathbf{n} dS$; $\iint \mathbf{F} \cdot \mathbf{n} dS = \iiint \text{div } \mathbf{F} dV$

CHAPTER 16 MATHEMATICS AFTER CALCULUS

Section 16.1 Linear Algebra (page 602)

1 All vectors $c \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **3** Only $x = 0$ **5** Plane of vectors with $x_1 + x_2 + x_3 = 0$

7 $x_p = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $A(x_p + x_0) = \begin{bmatrix} 3 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **9** $A(x_p + x_0) = b + 0 = b$; another solution

11 $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$; $b = \begin{bmatrix} c \\ c \\ c \end{bmatrix}$

13 $CC^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix}$; $C^T C = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$; (2 by 3) (2 by 3) is impossible

15 Any two are independent **17** C and F have independent columns

19 $\det F = 3$ **21** $F^{-1} = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

23 $\det(F - \lambda I) = \det \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = (2-\lambda)^2 - 1 = 3 - 4\lambda + \lambda^2 = 0$ if $\lambda = 1$ or $\lambda = 3$;

$$F \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}, F \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

25 $y = e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $y = e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $y = \frac{e^t}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{e^{3t}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

27 $\det \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^3 - 3(1-\lambda) + 2 = \lambda^3 - 3\lambda^2 = 0$ if $\lambda = 3$ or $\lambda = 0$ (repeated)

$$29 \det \begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix} = \lambda^2 - 5\lambda = 0 \text{ if } \lambda = 0 \text{ or } \lambda = 5; A \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0 \begin{bmatrix} 2 \\ -1 \end{bmatrix}, A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$31 H = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad 33 \text{ F if } b \neq 0; \text{ T; T; F } (e^{\lambda t} \text{ is not a vector}); \text{ T}$$

Section 16.2 Differential Equations (page 610)

$$1 \ 3Be^{3t} - Be^{3t} = 8e^{3t} \text{ gives } B = 4 : y = 4e^{3t} \quad 3 \ y = 3 - 2t + t^2 \quad 5 \ Ae^t + 4e^{3t} = 7 \text{ at } t = 0 \text{ if } A = 3$$

$$7 \text{ Add } y = Ae^{-t} \text{ because } y' + y = 0; \text{ choose } A = -1 \text{ so } -e^{-t} + 3 - 2t + t^2 = 2 \text{ at } t = 0$$

$$9 \ y = \frac{e^{kt}-1}{k}; y = t; \text{ by l'Hôpital } \lim_{k \rightarrow 0} \frac{e^{kt}-1}{k} = \lim_{k \rightarrow 0} \frac{te^{kt}}{1} = t$$

$$11 \text{ Substitute } y = Ae^t + Bte^t + C \cos t + D \sin t \text{ in equation: } B = 1, C = \frac{1}{2}, D = -\frac{1}{2}, \text{ any } A$$

$$13 \text{ Particular solution } y = Ate^t + Be^t; y' = Ate^t + (A+B)e^t = c(Ate^t + Be^t) + te^t$$

$$\text{gives } A = cA + 1, A + B = cB, A = \frac{1}{1-c}, B = \frac{-1}{(1-c)^2}$$

$$15 \ \lambda^2 e^{\lambda t} + 6\lambda e^{\lambda t} + 5e^{\lambda t} = 0 \text{ gives } \lambda^2 + 6\lambda + 5 = 0, (\lambda + 5)(\lambda + 1) = 0, \lambda = -1 \text{ or } -5$$

$$(\text{both negative so decay}); y = Ae^{-t} + Be^{-5t}$$

$$17 \ (\lambda^2 + 2\lambda + 3)e^{\lambda t} = 0, \lambda = -1 \pm \sqrt{-2} \text{ has imaginary part and negative real part;}$$

$$y = Ae^{(-1+\sqrt{2}i)t} + Be^{(-1-\sqrt{2}i)t}; y = Ce^{-t} \cos \sqrt{2}t + De^{-t} \sin \sqrt{2}t$$

$$19 \ d = 0 \text{ no damping; } d = 1 \text{ underdamping; } d = 2 \text{ critical damping; } d = 3 \text{ overdamping}$$

$$21 \ \lambda = -\frac{b}{2} \pm \frac{\sqrt{b^2-4c}}{2} \text{ is repeated when } b^2 = 4c \text{ and } \lambda = -\frac{b}{2}; (t\lambda^2 + 2\lambda)e^{\lambda t} + b(t\lambda + 1)e^{\lambda t} + cte^{\lambda t} = 0$$

$$\text{when } \lambda^2 + b\lambda + c = 0 \text{ and } 2\lambda + b = 0$$

$$23 \ -a \cos t - b \sin t - a \sin t + b \cos t + a \cos t + b \sin t = \cos t \text{ if } a = 0, b = 1, y = \sin t$$

$$25 \ y = A \cos 3t + B \cos 5t; y'' + 9y = -25B \cos 5t + 9B \cos 5t = \cos 5t \text{ gives } B = \frac{-1}{16};$$

$$y_0 = 0 \text{ gives } A = \frac{1}{16}$$

$$27 \ y = A(\cos \omega t - \cos \omega_0 t), y'' = -A\omega^2 \cos \omega t + A\omega_0^2 \cos \omega_0 t, y'' + \omega_0^2 y = \cos \omega t \text{ gives } A(-\omega^2 + \omega_0^2) = 1;$$

$$\text{breaks down when } \omega^2 = \omega_0^2$$

$$29 \ y = Be^{5t}; 25B + 3B = 1, B = \frac{1}{28} \quad 31 \ y = A + Bt = \frac{1}{2} + \frac{1}{2}t$$

$$33 \ y'' - 25y = e^{5t}; y'' + y = \sin t; y'' = 1 + t; \text{ right side solves homogeneous equation so particular}$$

$$\text{solution needs extra factor } t$$

$$35 \ e^t, e^{-t}, e^{it}, e^{-it} \quad 37 \ y = e^{-2t} + 2te^{-2t}; y(2\pi) = (1 + 4\pi)e^{-4\pi} \approx 0$$

$$39 \ y = (4e^{-rt} - r^2 e^{-4t/r})/(4 - r^2) \rightarrow 1 \text{ as } r \rightarrow 0 \quad 43 \ h \leq 2; h \leq 2.8$$

Section 16.3 Discrete Mathematics (page 615)

$$1 \text{ Two then two then last one; go around hexagon} \quad 3 \text{ Six (each deletes one edge)}$$

$$5 \text{ Connected: there is a path between any two nodes; connecting each new node requires an edge}$$

$$13 \text{ Edge lengths } 1, 2, 4$$

$$15 \text{ No; } 1, 3, 4 \text{ on left connect only to } 2, 3 \text{ on right; } 1, 3 \text{ on right connect only to } 2 \text{ on left} \quad 17 \ 4$$

$$19 \text{ Yes} \quad 21 \text{ F (may loop); T} \quad 25 \ 16$$