
Part Two

Physical
Processes in
Oceanography

8

Small-Scale Mixing Processes

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8.1 Introduction

Forty years ago, the detailed physical mechanisms responsible for the mixing of heat, salt, and other properties in the ocean had hardly been considered. Using profiles obtained from water-bottle measurements, and their variations in time and space, it was deduced that mixing must be taking place at rates much greater than could be accounted for by molecular diffusion. It was taken for granted that the ocean (because of its large scale) must be everywhere turbulent, and this was supported by the observation that the major constituents are reasonably well mixed. It seemed a natural step to define eddy viscosities and eddy conductivities, or mixing coefficients, to relate the deduced fluxes of momentum or heat (or salt) to the mean smoothed gradients of corresponding properties. Extensive tables of these mixing coefficients, K_M for momentum, K_H for heat, and K_S for salinity, and their variation with position and other parameters, were published about that time [see, e.g., Sverdrup, Johnson, and Fleming (1942, p. 482)]. Much mathematical modeling of oceanic flows on various scales was (and still is) based on simple assumptions about the eddy viscosity, which is often taken to have a constant value, chosen to give the best agreement with the observations. This approach to the theory is well summarized in Proudman (1953), and more recent extensions of the method are described in the conference proceedings edited by Nihoul (1975).

Though the preoccupation with finding numerical values of these parameters was not in retrospect always helpful, certain features of those results contained the seeds of many later developments in this subject. The lateral and vertical mixing coefficients evaluated in this way differ by many orders of magnitude, and it was recognized that the much smaller rates of vertical mixing must in some way be due to the smaller scale of the vertical motions. Qualitatively, it was also known that the vertical eddy coefficients tended to be smaller when the density gradients were larger. The analysis of Taylor (1931) had shown that in very stable conditions K_S was smaller than K_M , which he interpreted to mean that the vertical transport of salt requires an intimate mixing between water parcels at different levels, whereas momentum can be transported by wave motion and is less affected by a strong vertical density gradient.

In contrast to these direct considerations of vertical mixing, Iselin (1939a) introduced the far-reaching idea that, because of the vertical stability, virtually all the large-scale mixing in the ocean might be accounted for in terms of lateral mixing (along isopycnals, rather than horizontally). In particular, he pointed to the striking similarity of the T - S relations for a vertical section and a surface section in the North Atlantic, each of which crossed the same isopycnals.

A strong constraint on achieving a fuller understanding of the small-scale mixing processes implicit in early measurements, was the lack of suitable instruments to resolve the scales that are directly involved. Most of the data came from water-bottle samples and widely spaced current meters, and it was tacitly assumed that the smooth profiles drawn through the discrete points actually represented the state of the ocean. Even when continuous temperature profiles became available in the upper layers of the ocean through the development of the bathythermograph, there was a tendency to attribute abrupt changes in slope to malfunctions in the instrument. The parameterization in terms of eddy coefficients implied that turbulence is distributed uniformly through depth, and is maintained by external processes acting on a smaller scale than the flows of interest; but in the absence of techniques to observe the fluctuations, and how they are maintained, little progress could be made.

Many such instruments are already in existence (see chapter 14); their use has rapidly transformed our view of the ocean, and in particular the understanding of the nature of the mixing processes. Temperature, salinity, and velocity fluctuations can be measured down to centimeter scales, and these records show that the distribution of properties is far from smooth. Rapid changes of vertical gradients are common, amounting in many cases to "steps" in the profiles. At some times the temperature and salinity variations are nearly independent, while at others they are closely correlated in a manner that has a profound effect on the vertical fluxes of the two properties (see Section 8.4.2). Viewed on a small scale, the ocean is *not* everywhere turbulent: on the contrary, turbulence in the deep ocean occurs only intermittently and in patches (which are often thin, and elongated horizontally), while the level of fluctuations through most of the volume is very low for most of the time. This is now more clearly recognized to be a consequence of the stable density gradient, which can limit the vertical extent of mixing motions and thus keep the relevant Reynolds numbers very small.

The newly acquired ability to study various mixing processes in the ocean has produced a corresponding increase in activity by theoretical and laboratory modelers in this field. The stimulation has been in both directions: theoreticians have been made aware of striking new observations requiring explanation, and they have developed more and more sophisticated theories and experiments that in turn suggest new observations to test them. Some of the work has required subtle statistical analysis of fluctuating signals, while many of the most exciting developments have been based on identifying individual mixing events (in the laboratory or the ocean), followed by a recognition of their more general significance.

Perhaps the most important factor of all has been the change in attitude to observational oceanography which took place in the early 1960s. Henry Stommel in particular advocated an approach more akin to the formulation and testing of hypotheses in other experimental sciences. Experiments designed to test specific physical ideas in a limited geographical area are now commonplace; but it is easy to forget how recently such uses of ship time have replaced the earlier "expedition" approach, in which the aim was to explore as large an area as possible in a given time (chapter 14).

This chapter will concentrate on the scales of mixing in the ocean, ranging from the smallest that have been studied to those with vertical dimensions of some tens of meters. Vertical-mixing processes will be emphasized, though the effects of quasi-horizontal intrusions near boundaries and across frontal surfaces will also be considered. After a preliminary section introducing ideas that are basic to the whole subject, various mixing phenomena will be identified and discussed in turn, starting with the sea surface and continuing into the interior and finally to the bottom. The grouping of topics within each depth range will be on the basis of the physical processes on which they depend. We shall not attempt to follow the historical order of development or to discuss observations in detail. Where there is a recent good review of a topic available, the reader will be referred to it. The major aims have been to describe the interrelation between theory, observation, and laboratory experiments that has led to the present state of understanding of each process, and to identify the areas still most in need of further work.

8.2 Preliminary Discussion of Various Mechanisms

8.2.1 Classification of Mixing Processes

It is important to keep clearly in mind the various sources of energy that can produce the turbulent motions responsible for mixing in the ocean. The first useful contrast one can make is between mechanically generated turbulence, i.e., that originating in the kinetic energy of motion, by the breakdown of a shear flow for example, and convective turbulence, produced by a distribution of density that is in some sense top heavy. The latter may occur in situations that seem obviously unstable, as when the surface of the sea is cooled [section 8.3.2.(d)], or more subtly, in the interior of the ocean when only one component (salt or heat) is unstably distributed (section 8.4.2) while the net density distribution is "hydrostatically stable."

A second informative classification depends on whether the energy comes from an "external" or "internal" source. In the first case, energy put in at a boundary is used directly to produce mixing in a region extending some distance away from the source. An

example is the mixing through the upper layers of the ocean, and across the seasonal thermocline, caused by the momentum and heat transfers from the wind blowing over the surface. By "internal mixing" is implied a process in which the turbulent energy is both generated and used in the same volume of fluid, which is in the interior well away from boundaries. The mechanisms whereby the energy is ultimately supplied to the interior region must then also be considered carefully. Reviews of mixing processes based on the above classifications or a combination of them have been given by Turner (1973a,b) and Sherman, Imberger and Corcos (1978).

8.2.2 Turbulent Shear Flows

The maintenance of turbulent energy in a shear flow will be introduced briefly by summarizing the results for a "constant stress layer" in a homogeneous fluid flowing over a fixed horizontal boundary. [For a fuller treatment of this subject see chapter 17 and Turner (1973a, chapter 5).]

The boundary stress τ_0 is transmitted to the interior fluid by the so-called Reynolds stresses $\overline{\rho u' w'}$, which arise because of the correlation between the horizontal and vertical components of turbulent velocity u' and w' . The velocity gradient responsible for maintaining the stress can be related to the "friction velocity" u_* defined by $\tau_0 = \rho u_*^2$ ($= -\overline{\rho u' w'}$ in a constant stress layer) using dimensional arguments:

$$\frac{du}{dz} = \frac{u_*}{kz}, \quad (8.1)$$

where z is the distance from the boundary and k a universal constant (the von Karman constant: $k = 0.41$ approximately). Integrating (8.1) leads to the well-known logarithmic velocity profile, which for an aerodynamically rough boundary becomes

$$u = \frac{u_*}{k} \ln \frac{z}{z_0}, \quad (8.2)$$

where z_0 , the roughness length, is related to the geometry of the boundary.

Using (8.1), one can also find the rate of production of mechanical energy per unit mass, ϵ say (which is equal to the rate of dissipation in a locally steady state):

$$\epsilon = u_*^2 \frac{du}{dz} = \frac{u_*^3}{kz}. \quad (8.3)$$

It is also possible to define an eddy viscosity

$$K_M \equiv \frac{\tau_0}{\rho} \frac{du}{dz}, \quad (8.4)$$

which is equal to ku_*z for the logarithmic profile. The relation between the flux and the gradient of a passive

tracer can in some circumstances be predicted by assuming that the turbulent diffusivity (say K_H for heat) is equal to K_M : this procedure makes use of "Reynolds analogy." Notice, however, that the assumption of a constant value of K_M or K_H , by analogy with laminar flows, is already called into question by the above analysis. The logarithmic profile implies that these coefficients are proportional to the distance z from the boundary. In practice they can turn out to be more complicated functions of position, if observations are interpreted in these terms.

8.2.3 Buoyancy Effects and Buoyancy Parameters

As already outlined in section 8.1, vertical mixing in the ocean is dominated by the influence of the (usually stable) density gradients that limit vertical motions. The dynamic effect of the density gradient is contained in the parameter

$$N = \left(-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} \right)^{1/2}, \quad (8.5)$$

the Brunt-Väisälä or (more descriptively) the *buoyancy frequency*, which is the frequency with which a displaced element of fluid will oscillate. The corresponding periods $2\pi/N$ are typically a few minutes in the thermocline, and up to many hours in the weakly stratified deep ocean.

In a shear flow, the kinetic energy associated with the vertical gradient of horizontal velocity du/dz has a destabilizing effect, and the dimensionless ratio

$$Ri = N^2 / \left(\frac{du}{dz} \right)^2 = -g \frac{d\rho/dz}{\rho_0} \left(\frac{du}{dz} \right)^2, \quad (8.6)$$

called the *gradient Richardson number*, gives a measure of the relative importance of the stabilizing buoyancy and destabilizing shear. Of more direct physical significance is the *flux Richardson number* Rf , defined as the ratio of the rate of removal of energy by buoyancy forces to its production by shear. It can be expressed as

$$Rf = \frac{g \overline{\rho' w'}}{\rho u_*^2 (du/dz)} = \frac{K_H}{K_M} Ri \quad (8.7)$$

where ρ' is the density fluctuation and $B = -g \overline{\rho' w'} / \bar{\rho}$ is the buoyancy flux. Note that K_H may be much smaller than K_M in a stratified flow, so that while there is a strict upper limit of unity for Rf in steady conditions with stable stratification, turbulence can persist when $Ri > 1$.

Another "overall" Richardson number expressing the same balance of forces, but involving finite differences rather than gradients, can be written in terms of the overall scales of velocity u and length d imposed by the boundaries:

$$Ri_0 = g \frac{\Delta\rho}{\rho} d / u^2. \quad (8.8)$$

One must always be careful to define precisely what is meant when this term is used.

Two parameters commonly used to compare the relative importance of mechanical and buoyancy terms are expressed in the form of lengths. When the vertical fluxes of momentum $\tau_0 = \rho u_*^2$ and buoyancy B are given, then the Monin-Obukhov length (Monin and Obukhov, 1954)

$$L = \frac{-u_*^3}{kB} \quad (8.9)$$

(where k is the von Karman constant defined in (8.1) is a suitable scaling parameter; it is negative in unstable conditions and positive in stable conditions. This is a measure of the scale at which buoyancy forces become important; as B becomes more negative (larger in the stabilizing sense) buoyancy affects the motions on smaller and smaller scales. If, on the other hand, the rate of energy dissipation ϵ and the buoyancy frequency are known, then dimensional arguments show that the length scale

$$L_0 = \epsilon^{1/2} N^{-3/2}, \quad (8.10)$$

first used by Ozmidov (1965), is the scale of motion above which buoyancy forces are dominant.

The molecular diffusivity κ (of heat, say) and kinematic viscosity ν do not appear in the parameters introduced above (though the processes of dissipation of energy and of buoyancy fluctuations ultimately depend on molecular effects at the smallest scales [section 8.4.1(d)]. When convectively unstable conditions are considered, however, the relevant balance of forces is between the driving effect of buoyancy and the stabilizing influence of the two diffusive processes that act to retard the motion. The parameter expressing this balance, the Rayleigh number

$$Ra = g \frac{\Delta\rho}{\rho} d^3 / \kappa\nu \quad (8.11)$$

does therefore involve κ and ν explicitly. Here $\Delta\rho/\rho = \alpha\Delta T$ is the fractional (destabilizing) density difference between the top and bottom of a layer of fluid of depth d (often due to a temperature difference ΔT , where α is the coefficient of expansion). The Reynolds number $Re = ud/\nu$ and the Prandtl number $Pr = \nu/\kappa$ can also be relevant parameters in both the stable and convectively unstable cases. In particular, it is clear that the Reynolds number based on internal length scales such as (8.9) and (8.10) is much smaller than that defined using the whole depth, which would only be appropriate if the ocean were homogeneous.

8.2.4 Turbulent Mixing and Diffusion in the Horizontal

We mention now several ideas which will not be followed up in detail in this review but which have had an important effect on shaping current theories of mixing in the ocean.

Eckart (1948) pointed to the distinction which should be made between stirring and mixing processes. The former always increase the gradients of any patch of marker moving with the fluid, as it is sheared out by the larger eddies of the motion. True mixing is only accomplished when molecular processes, or much smaller-scale turbulent motions, act to decrease these gradients and spread the marker through the whole of the larger region into which it has been stirred.

The total range of eddy scales, up to that characteristic of the current size of the patch, also play a part in the "neighbor separation" theory of diffusion due to Richardson (1926), which was originally tested in the atmosphere, and applied to the ocean by Richardson and Stommel (1948). Their results imply that the separation of pairs of particles l or equivalently the dispersion $\Sigma = \overline{y^2}$, of a group of particles about its center of gravity, satisfies a relation of the form

$$\frac{\partial\Sigma}{\partial t} \propto \Sigma^{2/3} \propto l^{4/3}. \quad (8.12)$$

The rate of separation and hence the effective diffusivity increases with increasing scale because a larger range of eddy sizes can act on the particles. As Stommel (1949) pointed out, there is no way that (8.12) can be reconciled with an ordinary gradient (Fickian) diffusion theory using a constant eddy diffusivity. He showed, however, that it is explicable using Kolmogorov's theory of the inertial subrange of turbulence. Ozmidov (1965) has demonstrated that (8.12) can be written down directly using a dimensional argument, assuming that the rate of change of Σ (which has the same dimensions as an eddy diffusivity) depends only on the value of Σ and the rate of energy dissipation ϵ :

$$\frac{\partial\Sigma}{\partial t} \propto \epsilon^{1/3} \Sigma^{2/3}. \quad (8.12a)$$

Bowden (1962) summarized the evidence available up to that time to show that this $\Sigma^{2/3}$ (or $l^{4/3}$) relation described the observations well over a wide range of scales, from 10 to 10^8 cm. The dependence on ϵ was not then investigated explicitly, but later experiments have shown that this factor (and hence the rate of diffusion) can vary greatly with the depth below the surface.

The apparent "longitudinal diffusion" produced by a combination of vertical (or lateral) shear and transverse turbulent mixing is another important concept. This originated in a paper by Taylor (1954), who showed that the downstream extension of a cloud of

marked particles due to shear, followed by cross-stream mixing, leads to much larger values of the longitudinal dispersion coefficient D than can be produced by turbulence alone. For homogeneous fluid in a two-dimensional channel it is given by

$$D = 5.9du_* \quad (8.13)$$

where d is the depth and u_* the friction velocity.

Fischer (1973, 1976) has written two excellent reviews of the application of these (and other) results to the interpretation of diffusion processes in open channels and estuaries. The second in particular contains much that is of direct interest to oceanographers (cf. chapter 7).

8.3 Vertical Mixing in the Upper Layers of the Ocean

The major inputs of energy to the ocean come through the air-sea interface, and it is in the near-surface layers that the concept of an "external" mixing process is most clearly applicable. Because of the overall static stability, the direct effects of the energy input is to produce a more homogeneous surface layer, with below it a region of increased density gradient. Thus one must consider in turn the nature of the energy sources at the surface (whether the mixing is due to the mechanical effect of the wind stress, or to the transfer of heat), the influence of these on the motion in the surface layer, and finally the mechanism of mixing across the thermocline below. The variation of these processes with time, which leads to characteristic daily or seasonal changes of the thermocline, will also be considered. Most of the models are one-dimensional in depth, implying that the mixing is uniform in the horizontal, but some individual localized mixing processes will be discussed (see chapter 9 for additional discussion).

8.3.1 Parameterization of Stratified Shear Flows

The earliest theories of flow driven by the stress of the wind acting on the sea surface were based on an extension of the eddy-viscosity assumption. The vertical transports of heat and momentum can be related to the mean gradients of these properties using eddy coefficients that have an assumed dependence on the stability. Munk and Anderson (1948), for example, took K_H and K_M to be particular (different) functions of the gradient Richardson number (8.6), assumed that the (stabilizing) heat flux was constant through the depth, and solved the resultant closed momentum equation numerically to derive the velocity and temperature profiles. More recently, this method has been taken up again, but using more elaborate second-order closure schemes to produce a set of differential equations for the mean fields and the turbulent fluxes in the vertical (see, e.g., Mellor and Durbin, 1975; Launder, 1976; Lumley, 1978).

In this writer's opinion, this method has not yet proved its value for stratified flows in the ocean. The method is relatively complex, and the necessary extrapolations from known flows are so hard to test that the integral methods described in the following sections are at present preferred because they give more direct physical insight. It will, however, be worth keeping in touch with current work in this rapidly developing field.

8.3.2 Mixed-Layer Models

Thorough reviews of the various one-dimensional models of the upper ocean that have appeared (mainly in the past 10 years) have been given by Niiler (1977), and Niiler and Kraus (1977) [and the whole volume edited by Kraus (1977) is very useful]. The treatment here will be more selective: certain features of the models will be isolated, and their relative importance assessed. Laboratory experiments that have played an important role in the development of these ideas will also be discussed (though these have not always proved to be as directly relevant as was originally thought).

The starting point of all these models, which is based on many observations in the ocean and the laboratory, is that the mean temperature and salinity (and hence the density) are nearly uniform within the surface layer. In the case where mixing is driven by the action of the wind stress at the surface, the horizontal velocity in this layer is also assumed to be constant with depth, implying a rapid vertical exchange of horizontal momentum throughout the layer. Another assumption used in most theories is that there is an effectively discontinuous change of these variables across the sea surface and across the lower boundary of the mixed layer (see figure 8.1).

It is implied in what follows that the surface mixing is limited by buoyancy effects, not by rotation (i.e., the "Ekman layer depth" v_* / f is not a relevant scale), but rotation still enters into the calculation of the velocity step across the interface (see section 8.3.3). The Ekman layer depth will be considered explicitly in the context of bottom mixing (section 8.5.1).

Integration of the conservation equations for the heat (or buoyancy) content, and the horizontal momentum equation (in rotating coordinates where appropriate) across such a well-mixed layer, gives expressions for the temperature and bulk horizontal velocity in terms of the exchanges of heat and momentum across the sea surface and the interface below. The surface fluxes are in principle calculable from the external boundary conditions, but the interfacial fluxes are not known a priori. They depend on the rate of deepening, i.e., the rate at which fluid is mixed into the turbulent layer from the stationary region below. A clear understanding of the mechanism of *entrainment* across a density interface, with various kinds of mechanical or convective

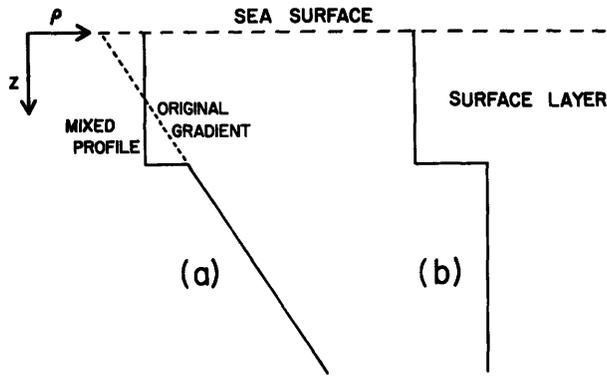


Figure 8.1 Sketch of the two types of density profiles discussed in the text: (a) a well-mixed surface layer with a gradient below it, (b) a two-layer system, with homogeneous water below.

tive energy inputs near the surface, is therefore a crucial part of the problem.

(a) Mixing Driven by a Surface Stress Purely mechanical mixing processes will be discussed first, starting with the case where a constant stress $\tau_0 = \rho_w v_*^2$ is applied at the surface (for example, by the wind), without any transfer of heat. When the water is stratified, a well-mixed layer is produced (see figure 8.1), which at time t has depth h and a density step below it, $\Delta\rho$ say. A dimensional argument suggests that the entrainment velocity $u_e = dh/dt$ can be expressed in the form

$$E_* = \frac{u_e}{v_*} = E_*(Ri_*) \quad (8.14)$$

where

$$Ri_* = \frac{g \Delta\rho h}{\rho_w v_*^2} = \frac{C^2}{v_*^2} \quad (8.15)$$

is an overall Richardson number based on the friction velocity v_* in the water. For the moment dh/dt is taken to be positive, but the general case where h can decrease will be considered in section 8.3.2(d).

Two related laboratory experiments have been carried out to model this process and test these relations. Kato and Phillips (1969) started with a linear salinity gradient (buoyancy frequency N) and applied a stress τ_0 at the surface (figure 8.1a). The depth h of the surface layer is related to $\Delta\rho$ and N by

$$C^2 = g \frac{\Delta\rho}{\rho} h = \frac{1}{2} N^2 h^2 \quad (8.16a)$$

Kantha, Phillips, and Azad (1977) used the same tank (annular in form, to eliminate end effects), but filled it with two layers of different density (figure 8.1b) rather than a gradient. In this case, conservation of buoyancy gives

$$C^2 = g \frac{\Delta\rho}{\rho} h = a \text{ constant.} \quad (8.16b)$$

Thus C^2 and Ri_* are clearly increasing as the layer deepens in the first case (8.16a) whereas they are constant when (8.16b) holds.

Equation (8.14) does not, however, collapse the data of these two experiments onto a single curve: at a given Ri_* and h , E_* was a factor of two larger in the two-layer experiment than with a linear gradient. Price (1979) has proposed that the rate of entrainment should be scaled instead with the mean velocity V of the layer (or more generally, the velocity difference across the interface):

$$\frac{u_e}{V} = E(Ri_V) \quad (8.17)$$

where Ri_V is also defined using V . This is a more appropriate way to describe the process, since other physical effects may intervene between the surface and the interface and V need not be proportional to v_* . (In the experiments described above, side wall friction can dominate, though the method used to correct for this will not be discussed explicitly here.)

The form of Ri_V deduced from the experiments agrees with that previously obtained by Ellison and Turner (1959), which is shown in figure 8.2. [This will also be discussed in another context in section 8.5.2(a).] For the present purpose, we note only that E falls off very rapidly between $0.4 < Ri_V < 1$, so that $Ri_V \approx 0.6$ is a good approximation for Ri_V over the whole range of experimental results. The *assumption* that Ri_V is constant (which was based on an argument about the stability of the layer as a whole) was made by Pollard, Rhines, and Thompson (1973). This was used as the basis of the closure of their mixed-layer model which will be referred to in section 8.3.3.

The formulation in terms of V implies that conservation of momentum is the most important constraint

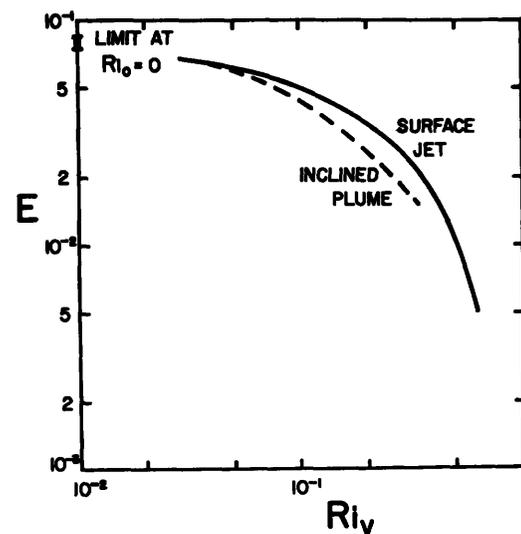


Figure 8.2 The rate of entrainment into a turbulent stratified flow as a function of overall Richardson number, for two types of experiments described by Ellison and Turner (1959).

on the entrainment in these laboratory experiments (and in the analogous oceanic case). The momentum equation

$$\frac{d(hV)}{dt} = v_*^2 = \text{constant}, \quad (8.18)$$

together with the conservation of buoyancy relations (8.16), gives

$$E_* = \frac{u_e}{v_*} = n Ri_v^{1/2} Ri_*^{-1/2} \quad (8.19)$$

where

$$n = \begin{cases} \frac{1}{2} & \text{if } C^2 = \frac{1}{2} N^2 h^2 \text{ (linear stratification)} \\ 1 & \text{if } C^2 = \text{constant (homogeneous lower layer)}. \end{cases}$$

The predicted entrainment rate is a factor of two smaller in the linearly stratified case, as is observed. This difference arises because, in order to maintain Ri_v constant as C^2 increases, the whole layer, as well as the entrained fluid, must be accelerated to a velocity $V = (C^2/Ri_v)^{1/2}$. In the two-layer case, the stress is only required to accelerate entrained fluid to velocity V , which is constant.

Nothing has been said yet about the detailed mechanism of mixing across the interface, in the laboratory or in the ocean. Thorpe (1978a) has shown from measurements in a lake under conditions of surface heating that Kelvin-Helmholtz instability dominates the structure soon after the onset of a wind [section 8.4.1(c)]. Dillon and Caldwell (1978) have demonstrated the importance of a few "catastrophic events," relative to the much slower continuous entrainment processes. We tentatively suggest that these events could be the breakdown of large-scale waves on the interface, a mechanism that has been proposed for the benthic boundary layer (see section 8.5.1).

(b) The Influence of Surface Waves The experiments just described, and the models based on them, imply that the whole effect of the wind stress on the surface is equivalent to that produced by a moving plane, solid boundary. There is only one relevant length scale, the depth of the well-mixed layer. In the ocean, of course, there is a free surface on which waves are generated as well as a current, and this can introduce entirely new physical effects. The presence of waves can modify the heat, momentum, and energy transfer processes [as reviewed by Phillips (1977c)] and individual breaking waves can inject turbulent energy at a much smaller scale. Longuet-Higgins and Turner (1974) and Toba, Tokuda, Okuda, and Kawai (1975) have taken the first steps toward extending wave theories into this turbulent regime.

The interaction between a wind-driven current and surface waves can produce a system of "Langmuir

cells," aligned parallel to the wind. These circulations, extending through the depth of the mixed layer, have long been recognized to have a significant effect on the mixing, and there are many theories purporting to explain the phenomenon [see Pollard (1977) for a review]. It now appears most likely that the generation depends on an instability mechanism, in which there is a positive feedback between the wind-driven current and the cross-wind variation in the Stokes drift associated with an intersecting pattern of two crossed wave trains. Physically, this implies that the vorticity of the shear flow is twisted by the presence of the Stokes drift into the vorticity of the Langmuir circulations. A heuristic model of the process was given by Garrett (1976), but the most complete and satisfactory theory is that of Craik (1977), who has clarified the differences between earlier related models. Faller (1978) has carried out preliminary laboratory experiments that support the main conclusions of this analysis (see also chapter 16).

The effect of density gradients on these circulations has not yet been investigated, so it is not clear whether a mixed layer can be set up by this mechanism under stable conditions. At high wind speeds, however, it seems likely that these organized motions, with horizontal separation determined by the wave field, will have a major influence on both the formation and rate of deepening of the mixed layer. This is an important direction in which the detailed modeling of thermocline mixing processes should certainly be extended.

(c) Input of Turbulent Energy on Smaller Scales The case where kinetic energy is produced at the surface, with turbulence scales much less than that of the mixed layer, has been modeled in the laboratory using an oscillating grid of solid bars. An early application to the ocean was made by Cromwell (1960). The more recent experiments by Thompson and Turner (1975), Hopfinger and Toly (1976), and McDougall (1979), have shown that the turbulent velocity decays rapidly with distance from the grid (like z^{-1}), while its length scale l is proportional to z . The experiments also suggest that the entrainment across an interface below is most appropriately scaled in terms of the velocity and length scales u_1 and l_1 of the turbulence near the interface, rather than using overall parameters such as the velocity of the stirrer and the layer depth. When this is done, the laboratory data are well-described by the relation

$$\frac{u_e}{u_1} = f(Ri_0, Pe), \quad (8.20)$$

where

$$Ri_0 = \frac{g \Delta \rho}{\rho} \frac{l_1}{u_1^2},$$

$Pe = u_1 l / \kappa$ is a Peclet number.

The general form of the curves is similar to those shown in figure 8.2, but there is a distinct difference between the results of experiments using heat and salt as the stratifying agent (reflecting the different molecular diffusivity κ in Pe). They tend to the same form with a small slope at low Ri_0 , where neither buoyancy nor diffusion is important, but diverge at larger Ri_0 to become approximately $u_e/u_1 \propto Ri_0^{-1}$ (heat) and $u_e/u_1 \propto Ri_0^{-3/2}$ (salt). The first form is attractive since it seems to correspond to the prediction of a simple energy argument (see Turner 1973a, chapter 9), but it now appears that one must accept the complications of the general form (8.20), including the fact that molecular processes can affect the structure of the interface and hence the entrainment at low values of Pe (Crapper and Linden, 1974).

The laboratory experiments of Linden (1975) have also shown that the rate of entrainment across an interface, due to stirring with a grid, can be substantially reduced when there is a density gradient below, rather than a second homogeneous layer. This is due to the generation, by the interfacial oscillations, of internal gravity waves that can carry energy away from the interface. The process can have a substantial effect on the mixing in the thermocline below the sharp "interface" itself. It is also a source of wave energy for the deep ocean, though the existence of a mean shear flow in the upper layer probably has an important influence on wave generation as well as on wave breaking (Thorpe, 1978b). It seems likely too that the process of mixing itself is affected in a significant way by the presence of a mean shear. Certainly organized motions in the form of Kelvin-Helmholtz billows occur only with a shear [see section 8.4.1(c)].

(d) The Effect of a Surface Heat Flux Only mechanical energy inputs have so far been considered; the effect of a buoyancy flux will now be added. The discussion will be entirely in terms of heat fluxes, but clearly the increase in salinity due to evaporation should also be taken into account (see, e.g., Niiler and Kraus, 1977).

When there is a net (equivalent) heat input to the sea surface, a stabilizing density gradient is produced that has an inhibiting effect on mixing. Note, however, that penetrating radiation, with simultaneous cooling by evaporation and long-wave radiation right at the surface, produces a localized convective contribution to the turbulence (see Foster, 1971). When there is a net cooling, at night or in the winter, convective motions can extend through the depth of the mixed layer and contribute to the entrainment across the thermocline below. In the latter case, detailed studies of the heat transfer and the motions very near the free surface have been carried out by Foster (1965), McAlister and McLeish (1969) and Katsaros et al. (1977) for example, and a comparison between fresh and salt water has

recently been made by Katsaros (1978).

When there is a constant stabilizing buoyancy flux $B = g\overline{\rho'w'}/\bar{\rho}$ from above (i.e., a constant rate of heating, assumed to be right at the surface), and simultaneously a fixed rate of supply of kinetic energy, there can be a balance between the energy input and the work required to mix the light fluid down. Assuming, as did Kitaigorodskii (1960), that the friction velocity u_* is the parameter determining the rate of working, the depth of the surface layer can become steady at

$$h = au_*^3/B \quad (8.21)$$

while it continues to warm. This argument is closely related to that leading to the Monin-Obukhov length (8.9), and is also a statement of the conservation of energy.

During periods of increasing heating, the equilibrium depth achieved will be continually decreasing, so that the bottom of the mixed layer will rise and leave previously warmed layers behind. The minimum depth coincides with the time of maximum heating (assuming a constant rate of mechanical stirring). As the rate of heating decreases, the interface will descend slowly, while the temperature of the upper layer increases. Finally, when the surface is being cooled, there will be a more rapid cooling and also a deepening of the upper layer. Whether this mixing is "penetrative" or "non-penetrative," i.e., the extent to which convection contributes to entrainment across the thermocline, is the subject of a continuing debate that is summarized in the following section. Molecular effects can also affect the rate of entrainment at low Pe , by altering the shape of the density profile on which the convective turbulence acts.

8.3.3 Energy Arguments Describing the Behavior of the Thermocline

Virtually all the models of the upper mixed layer in current use are based on energy arguments that balance the inputs of kinetic energy against changes in potential energy plus dissipation. They vary in the emphasis they put on different terms in the conservation equations, but some of the conflict between alternative models is resolved by recognizing that different processes may dominate at various stages of the mixing.

Niiler (1975) and de Szoeke and Rhines (1976), for instance, have shown that four distinct dynamic stages can be identified in the case where a wind stress begins to blow over the surface of a linearly stratified ocean (assuming there is no heating or energy dissipation). Using standard notation, the turbulent kinetic energy equation can then be summarized as

$$\frac{1}{2} \frac{\partial h}{\partial t} \left(\begin{array}{ccc} \alpha u_*^2 & + \frac{N^2 h^2}{2} & - |\delta V|^2 \\ A & B & C \end{array} \right) = \begin{array}{c} m u_*^3 \\ D \end{array} \quad (8.22)$$

The terms represent

- A the storage rate of turbulent energy in the mixed layer;
- B the rate of increase of potential energy due to entrainment from below;
- C the rate of production of turbulent mechanical energy by the stress associated with the entrainment across a velocity difference δV ;
- D the rate of production of turbulent mechanical energy by surface processes.

Initially, there is a balance between *A* and *D* and the depth of the layer grows rapidly to a meter or so. After a few minutes, a balance between *B* and *D* is attained, and the mixed layer under typical conditions can grow to about 10 meters within an hour. In the meantime, the mean flow has been accelerating, and the velocity difference across the base of the layer increasing. Following Pollard et al. (1973), it can be deduced from the momentum equations in rotating coordinates that

$$(\delta V)^2 = \frac{2u_*^4}{h^2 f^2} [1 - \cos(ft)], \quad (8.23)$$

where f is the Coriolis parameter. The turbulent energy produced at the base of the mixed layer by this shear can dominate on a time scale of half a pendulum day, and the balance is between *B* and *C*. The depth after this time is of order $u_*/(Nf)^{1/2}$, determined both by the stratification and rotation. [A modification of this argument by Phillips (1977b) leads to the alternative form $u_*/f^{2/3}N^{1/3}$, but observationally it could be difficult to choose between them.] In order to close this model, one still needs to add an independent criterion to relate the entrainment to the shear across the interface. As discussed in section 8.3.2(a), this can be taken as $Ri_v \approx 0.6$ since the mixing rate falls off sharply at this value of the overall Richardson number.

Finally, the intensity of the inertial currents decreases and a slow erosion continues, again as a balance between *B* and *D*. It is this final state that corresponds to the model of the seasonal thermocline introduced by Kraus and Turner (1967), and modified by Turner (1973b). Without considering any horizontal motion, they used the one-dimensional heat and mechanical energy equations, assuming that all the kinetic energy is generated at the surface, and that a constant fraction of this is used for entrainment across the interface below. This model can also deal with the case where the surface is being heated. Gill and Turner (1976) have carried the discussion a stage further, and shown that the cooling period is much better described by assuming that the convectively generated energy is nonpenetrative, i.e., that it contributes very little to the entrainment, only to the cooling of the layer.

As will be apparent from the earlier discussion of individual mixing mechanisms, it is by no means easy

to identify which sources of energy are important, much less to quantify their effects. The most arbitrary feature of mixed-layer models at present is the parameterization of dissipation. It is usually assumed (or implied) that the energy available for entrainment is some fixed fraction of that produced by each type of source, and laboratory and field data are used to evaluate the constants of proportionality. This is the view adopted in the review by Sherman et al. (1978), for example, who added a term representing radiation of wave energy from the base of the layer. They compared the coefficients chosen by various authors, as well as suggesting their own best fit to laboratory experimental data. Another such comparison with laboratory and field data has been made by Niiler and Kraus (1977).

Processes such as the decay of turbulent energy with depth, and the removal by waves propagating through a density gradient below, are not adequately described by such constant coefficients. It is not clear that the shear through the layer is small enough to ignore as a source of extra energy, nor that two-dimensional effects (due to small-scale intrusions in the thermocline) are unimportant. In short, the models seem to have run ahead of the physical understanding on which they should be based. In the following section, some of the theoretical predictions will be compared with observations, consisting usually only of the mixed-layer depth and temperature. There are some good measurements of fluctuating temperature and salinity (Gregg 1976a) but to make further progress, and to distinguish between alternative models, much more detailed measurements of turbulent velocity and structure through the surface mixed layer and the thermocline below will be required. Thorpe (1977) and his coworkers have made the best set of measurements to date, using a Scottish Loch as a large-scale natural laboratory. (The JASIN 1978 experiment should contribute greatly to our understanding of these processes, but none of those results were available at the time of writing.)

8.3.4 Comparison of Models and Observations

In the absence of detailed measurements, the best hope of testing theories lies in choosing simple experimental situations in which as few as possible of the competing processes are active, or where they can be clearly distinguished. It is pointless, for example, to apply a model based on surface inputs alone if the turbulent-energy generation is in fact dominated by shear at the interface.

Price, Mooers, and van Leer (1978) have reported detailed measurements of temperature and horizontal-velocity profiles for two cases of mixed-layer deepening due to storms. They estimated the surface stress from wind observations, and compared model calculations based directly on u_*^2 with those based on the velocity difference (8.23). The predicted responses are quite dif-

ferent in the two cases, and the latter agreed well with the observations. The deepening rate accelerated during the initial rise in wind stress, but decreased abruptly as δV was reduced during the second half of the inertial period, even though u_*^2 continued to increase. They thus found no evidence of deepening driven by wind stress alone on this time scale, although turbulence generated near the surface must still have contributed to keeping the surface layer stirred.

A particularly clear-cut series of observations on convective deepening was reported by Farmer (1975). He has also given an excellent account of the related laboratory and atmospheric observations and models in the convective situation. The convection in the case considered by Farmer was driven by the density increase produced by surface heating of water, which was below the temperature of maximum density in an ice-covered lake. Thus there were no horizontal motions, and no contribution from a wind stress at the surface. From successive temperature profiles he deduced the rate of deepening, and showed that this was on average 17% greater than that corresponding to "nonpenetrative" mixing into a linear density gradient. Thus a small, but not negligible, fraction of the convective energy was used for entrainment. [The numerical values of the energy ratio derived in this and earlier studies will not be discussed here; but note that the relevance of the usual definition has been called into question by Manins and Turner (1978).]

In certain well-documented cases, models developed from that of Kraus and Turner (1967) (using a parameterization in terms of the surface wind stress and the surface buoyancy flux) have given a good prediction of the time-dependent behavior of deep surface mixed layers. Denman and Miyake (1973), for example, were able to simulate the behavior of the upper mixed layer at ocean weather station P over a 2-week period. They used observed values of the wind speed and radiation, and a fixed ratio between the surface energy input and that needed for mixing at the interface.

On the seasonal time scale, Gill and Turner (1976) have systematically compared various models with observations at a North Atlantic weather ship. They concluded that the Kraus-Turner calculation, modified to remove or reduce the penetrative convective mixing during the cooling cycle, gives the best agreement with the observed surface temperature T_s of all the models so far proposed. In particular, it correctly reproduces the phase relations between the dates of maximum heating, maximum surface temperature, and minimum depth, and it predicts a realistic hysteresis loop in a plot of T_s versus total heat content H (i.e., it properly incorporates the asymmetry between heating and cooling periods). This behavior is illustrated in figure 8.3. The model also overcomes a previous difficulty and allows the potential energy to decrease during the cool-

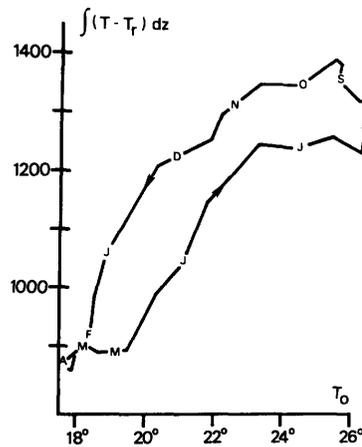


Figure 8.3 The heat content in the surface layer as a function of surface temperature T_0 at ocean weather station Echo. (After Gill and Turner, 1976.) The reference temperature T_r is the mean of the temperature at 250 m and 275 m depth, and the months are marked along the curve.

ing period, instead of increasing continuously as implied by the earlier models.

The mixed-layer depth and the structure of the thermocline are not, however, well predicted by these models; this fact points again to the factors that have been neglected. Niiler (1977) has shown that improved agreement is obtained by empirically allowing the energy available for mixing to decrease as the layer depth increases [though a similar behavior is implied by the use of (8.23); see Thompson (1976) for a comparison of the two types of model]. Direct measurements of the decay of turbulent energy with depth in the mixed layer will clearly be important. In many parts of the ocean it may also be necessary to consider upwelling.

Perhaps the most important deficiency is the neglect of any mixing below the surface layer. There is now strong evidence that the density interface is never really sharp, but has below it a gradient region that is indirectly mixed by the surface stirring. At greater depths too, the density profile is observed to change more rapidly than can be accounted for by advection, so that mixing driven by internal waves, alone or in combination with a shear flow, must become significant. These internal processes are the subject of the following section.

8.4 Mixing in the Interior of the Ocean

The overall properties of the main thermocline apparently can be described rather well in terms of a balance between upwelling w and turbulent diffusion K in the vertical. Munk (1966), for example, after reviewing earlier work, summarized data from the Pacific that show that the T and S distributions can be fitted by exponentials that are solutions of diffusion equations, for example