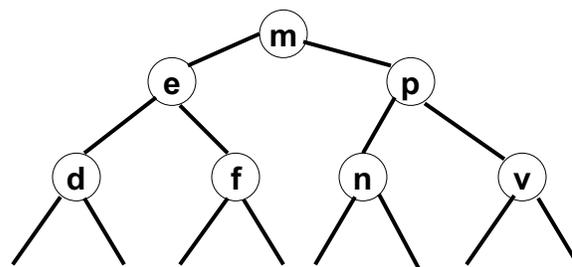


1.00 Lecture 37

Data Structures: TreeMap, HashMap

Binary Trees



<u>Level</u>	<u>Nodes</u>
0	2^0
1	2^1
2	2^2
...	
k	2^k

- Full binary tree has $2^{(k+1)}-1$ nodes
- Maximum of k steps required to find (or not find) a node
 - E.g. 2^{20} nodes, or 1,000,000 nodes, in 20 steps!
- In a binary search tree (but not all types of binary tree):
 - All nodes to left are smaller than parent
 - All nodes to right are larger than parent
 - No ties: each node has a unique key or id

Exercise: Binary Search Tree, Adding Nodes

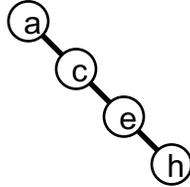
- **Start with an empty binary search tree.**
- **Insert the following nodes while maintaining the binary search tree property:**
 - "b", "q", "t", "d", "a"
- **The first node, "b", will be the root.**
- **Where will the second node, "q", go?**
- **Draw the tree that results with all 5 nodes**

Binary Search Tree (BST)

- **Binary search trees are used to store large amounts of data**
 - High capacity ($\sim 2^k$)
 - Fast access (k steps)
- **Basic tree operations (insert, find, delete) are not difficult to implement**
 - Special cases take some care, as in all data structures
 - And keeping the tree balanced, so that all branches are of comparable length, requires sophistication
 - We won't implement any tree code; we'll use the Java implementation

Java Tree Implementation

- **Trees are efficient if they are balanced**
 - A balanced tree of depth 20 can hold about 2^{20} , or 1,000,000 nodes
 - If the tree were unbalanced, in the worst case it would require 1,000,000 levels to hold 1,000,000 nodes, and 1,000,000 steps to find/insert/delete



- **To prevent unbalance, Java uses a sophisticated binary tree called known as a red-black tree.**
 - Red-black trees automatically rebalance themselves if one branch becomes deeper than a sibling.
 - Other, similar algorithms include AVL trees, 2-3 trees, ...

Keys, Sets, and Maps

- **Every node in a tree has a *key*, which is a unique identifier**
 - If a node contains nothing but the *key*, it is called a `TreeSet`
 - Transit example: gate at Kendall Sq has tree of CharlieCard numbers
 - If a node contains a *key* and a *value*, it is called a `TreeMap`.
 - Phone book example: *key*= your name, *value*= your phone number
 - Trees keep nodes in a defined order, as in a phone book (alphabetical)
- **The *key* is used to look up the *value*.**
 - The *value* is extra data contained in the node indexed by the *key*.
 - Nodes must have unique keys to distinguish between them.
- **Typical methods in a `TreeSet<Integer>` are:**
 - `boolean contains(Integer n)`
 - `Integer first()`
- **The equivalent methods in a `TreeMap<Integer, String>` are:**
 - `boolean containsKey(Integer n)`
 - `String get(Integer n)`
 - `Integer firstKey()`
 - `String firstValue()`

How to Traverse a TreeMap

Given a `TreeMap<Integer, String>`, how would you print out every entry in order?

```
TreeMap<Integer, String> list=  
    new TreeMap<Integer, String> ();  
// add entries  
for (Integer n : list.keySet()) {  
    System.out.println( n + ", " +  
        list.get(n) );  
}
```

Comparable<T>

- Recall the `Comparable` interface from sorting
- In trees, all keys must belong to a single class that implements the `Comparable<T>` interface
 - Or you can supply a `Comparator<T>` to the constructor
- `Comparable<T>` has one method:
 - `int compareTo(T other)`
 - `compareTo` returns:
 - An `int < 0` if (this < other)
 - `0` if (other equals this)
 - An `int > 0` if (this > other)
- Many Java classes already implement `Comparable`, e.g. `String`, `Integer`

Exercise 1: TreeSet

```
public class FullName implements Comparable<FullName> {
    private final String firstName;
    private final String lastName;

    public FullName( String f, String l ) {
        firstName= f;
        lastName= l;
    }

    public String getFirstName() {return firstName;}
    public String getLastName() {return lastName;}

    public int compareTo( FullName fn ) {
        // Complete the compareTo() method
        // Order by last name, then first name
        // Remember String has a compareTo() method already
        // You are comparing pairs of Strings
    }

    public String toString() {
        return firstName + " " + lastName;
    }
}
```

Exercise 1, p.2

```
import java.util.*;

public class FullNameTest {

    public static void main(String[] args) {
        FullName scott= new FullName("Scott", "Stevens");
        FullName ellen= new FullName("Ellen", "Shipps");
        FullName andrea= new FullName("Andrea", "Kondoleon");
        FullName paul= new FullName("Paul", "Stevens");

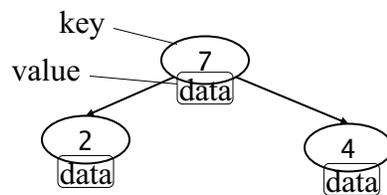
        TreeSet<FullName> names= new TreeSet<FullName>();
        names.add(scott);
        names.add(ellen);
        names.add(andrea);
        names.add(paul);

        for (FullName f : names)
            System.out.println(f);
    }
}
```

Keys and Values

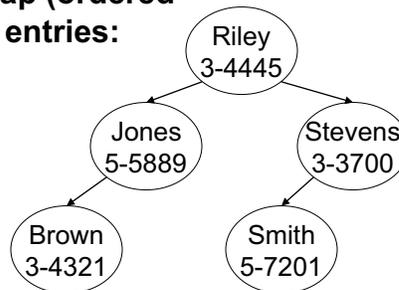
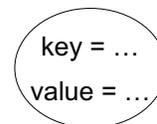
What good is a tree of numbers?

- A “key” in a tree is an ordered value, i.e. a key can be compared with another object of the same type
- A node in an ordered binary tree consists of an ordered key and a value
- All the keys in a tree should be of the same type



Tree Map

- Each node has a key and a value
- Phonebook example:
 - Key: name
 - Value: phone number
- Exercise: Draw the tree map (ordered alphabetically) with these entries:
 - Riley, 3-4445
 - Stevens, 3-3700
 - Smith, 5-7201
 - Jones, 5-5889
 - Brown, 3-4321



Exercise 2: TreeMap

- We use a `TreeMap<FullName, String>` to create a phone book; this code is provided in class `PhoneBook`
 - `FullName` is the key; `String` (phone number) is the value
- We use a loop to display a `JOptionPane` that asks for a full name, in the format: “`firstName lastName`”
 - Your code will try to look up the phone number for this name
- Use the `String` method `split()` to parse the name
 - `split()` takes the delimiter as its argument, e.g., “ ” here (space)
 - `split()` returns an array of `Strings`
- Use the `TreeMap<FullName, String>` method
`String get(FullName fn)`
to return the subscriber entry.
 - `get()` will return the value if the key is found
 - `get()` will return `null` if the key cannot be found.

PhoneBook.java

```
import java.util.*;
import javax.swing.JOptionPane;

public class PhoneBook {
    public static void main(String[] args) {
        FullName scott= new FullName("Scott", "Stevens");
        FullName ellen= new FullName("Ellen", "Shipps");
        FullName pizza= new FullName("Michael", "Pizza");
        FullName paul= new FullName("Paul", "Stevens");
        TreeMap<FullName,String> phones=
            new TreeMap<FullName,String>();

        phones.put(scott, "617-225-7178");
        phones.put(ellen, "781-646-2880");
        phones.put(pizza, "781-648-2000");
        phones.put(paul, "617-498-2142");
    }
}
```

PhoneBook.java, p.2

```
while (true) {
    String text= JOptionPane.showInputDialog(
        "Enter full name");
    if (text.isEmpty())
        break;
    // Your code here
    // Parse the full name ("firstName lastName")
    // Use the get() method with FullName1 key to retrieve
    // the String phone number value.
    // Print out the phone number or "Subscriber unknown"
    // if get() returns null
}
}
```

Exercise 3: Data Structure Efficiency

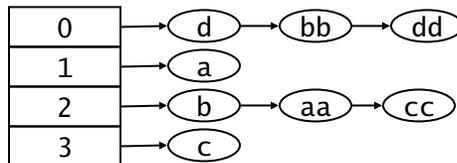
- If you are searching an unordered list of n items for an element, on average how many items will you have to search to find the item:
 - If item is present in the list?
 - If item is not present in the list?
- What happens if the list is ordered?
 - If item is present in the list?
 - If item is not present in the list?
- If the items are stored in a TreeMap how many items will you have to search on average?
 - Whether item is present or not
- Can we do better?

Hashing Illustration

keys = { a, b, c, d, aa, bb, cc, dd }

Hash function: (sum of chars) % 4

a= 97, b= 98, c= 99, d= 100



Hash table (hash map)

- Hashing maps each Object to an index in an array of Node references
- The array contains the “first” reference to a linked list of Objects.
- We traverse the list to add or find Objects that hash to that value
- We keep the lists short, so hash efficiency is close to array index lookup

HashMap

- **HashMap** holds keys and values, similar to **TreeMap**
 - HashMap like a filing cabinet, in which each folder has a tab (hash code) and contains a small number of objects (list)
- **HashMap** provides constant time lookup no matter how many elements it contains.
 - If we have $n = 1,000,000$ items, and t is the time to find one item, then
 - **LinkedList** will take $\sim 500,000t$ ($n/2$) to find an item;
 - **TreeMap** will take $\sim 20t$ ($\log_2 n$)
 - **HashMap** will take $\sim t$
- **Lookup time** depends on having a good `hashCode` () method and is statistical.
- **Elements** in a **HashMap** are **NOT** ordered by anything useful. Storage order is by hash code.

Java hashCode ()

- **Hashing is done in two phases**
 - *hash1* function is responsibility of the key class (the data type being stored in the hash table)
 - In the example, *hash1* is (sum of characters)
 - *hash2* function is responsibility of hash table: it takes *hash1* and maps it to the number of slots in the hash table
 - In the example, *hash2* is (% 4)
- **Hash table class does not know enough to generate a hash code from a particular object**
 - *hash1* should map objects as evenly as possible to different hash values
- **Java Object has int hashCode() method**
 - Thus all Java objects inherit hashCode()
 - Caution: default hashCode() method can return a negative integer
 - We usually take the absolute value of the hashCode()

hashCode ()

- **All hashCode () methods must return the same integer when presented with the same object**

```
if ( o1.equals( o2 ))
    o1.hashCode()==o2.hashCode // must be true
```

 - They cannot return a random integer.
 - If they did, there would be no way to look up a key once it had been inserted in the hash table
- **All hashCode () methods should return a different integer from a different object**
- **Java classes (String, JButton, etc.) have good hashCode () methods.**
- **Object has a terrible hashCode () method.**
 - If you extend Object and you intend to use the new class as a key in a HashMap, you should override the hashCode () method

equals ()

- **equals ()** method returns true if two objects are equivalent
 - Object class default equals () tests if two objects are at same memory location. It doesn't look at their fields.
- **Better equals ()** than default needed in HashMaps to find matching objects
- **equals ()** method in MyClass should use the following pattern:

```
public boolean equals(Object other) {
    if (this == other) return true;
    if (!(other instanceof MyClass) ) return false;
    MyClass otherOne= (MyClass) other;
    if ( /*this and otherOne have equivalent fields*/
        return true;
    else return false;
}
```

Exercise 4: HashPhoneBook

- **FullName** is the same as used in TreeMap.
- **Copy and rename your PhoneBook solution to HashPhoneBook**
 - HashPhoneBook is the same as PhoneBook except it uses a HashMap instead of a TreeMap.
 - Make that one change in your code
- **Run HashPhoneBook**
 - Can you find any subscribers' numbers? Why not?
- **Fix the problem by adding good equals () and hashCode () methods to FullName.**
 - Use the pattern from the previous slide for equals()
 - Use the String hashCode() method within your hashCode()
- **Test using HashPhoneBook.**

Exercise 4

```
public class FullName
    implements Comparable<FullName> {
    private final String firstName;
    private final String lastName;

    public FullName( String f, String l ) {
        firstName= f; lastName= l;
    }

    ...
    // Add overrides for versions in Object for:
    //   public boolean equals( Object o )
    //   public int hashCode()
}
```

Exercise 5: Which Data Structure?

- In programs that must store and retrieve large amounts of data, you typically choose between **TreeMap** and **HashMap**
 - Hashing is faster but does not keep Objects in order
 - Trees are slower but keep Objects in order
- In each of the following cases, select the data structure(s) most appropriate to the application.
 - Planes, runways and gates in a simulation
 - Books in a library
 - Airline reservations for many passengers
 - Events in a data communication system

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