1.020 Ecology II: Engineering for Sustainability
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering

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Problem Set 6 – Resource Allocation
Due: 5PM Weds. April 30, 2008

A. Introduction

The objective of this problem is to allocate agricultural water to maximize profits, subject to environmental constraints.

B. Problem Description

There are 3 farms, all growing rice.
Water is allocated by an irrigation district to each farm to maximize district income (over all 3 farms)
The maximum land available for cultivation is specified for each farm.
Yield is enhanced by fertilizer application.
Fertilizer cost is negligible
Total nitrogen runoff must not exceed a specified threshold.

Variable definitions:

Objective: Maximize $F = \sum_{i=1}^{3} pY_i L_i =$ Net district revenue (\$ season^{-1})

$p =$ Rice price (\$ tonnes^{-1})

$L_i =$ Crop area for Farm *i* (ha)

$Y_i = Y_{0i} + \gamma_i F_i$ = Net yield Farm *i* (tonnes ha^{-1} season^{-1})

$Y_{0i} =$ Nominal yield Farm *i* (tonnes ha^{-1} season^{-1})

$\gamma_i =$ Fertilizer enhanced yield coefficient for Farm *i* (tonnes crop (kg fertilizer)$^{-1}$)

$F_i =$ Amount of fertilizer applied to Farm *i* (kg ha^{-1} season-1)

$N_i = \eta_i F_i L_{\max i}$ = Nitrogen runoff from Farm *i* (kg season^{-1})

$L_{\max i} =$ Maximum land area Farm *i*

[Note: As a simplification, above expression assumes fertilizer is applied to entire farm]

$\eta_i =$ Fraction of applied nitrogen that runs off Farm *i* (unitless)

$R = \sum_{i=1}^{3} N_i =$ Total nitrogen runoff (kg season^{-1})

$W = \sum_{i=1}^{3} W_i L_i =$ Total water used (MCM season^{-1})

$W_i =$ Unit water requirement Farm *i* (MCM ha^{-1} season^{-1})

Resource and environmental constraints:

Water: $W \le W_{avail}$

Land: $L_i \le L_{\max i}$ for each Farm *i*

Nitrogen runoff: $R \le R_{\max}$

C. Inputs

Price $p =$ 200 \$ tonne^{-1}

Nominal rice yield Y_{0i} tonnes ha^{-1}

Farm1	Farm2	Farm3
100	70	90

Fertilizer coefficient γ_i : tonnes kg^{-1}

Farm1	Farm2	Farm3
0.9	1.2	1.1

Nitrate runoff fraction η_i : unitless

Farm1	Farm2	Farm3
0.4	0.35	0.45

Water requirement W_i : m season^{-1}

Farm1	Farm2	Farm3
1.0	1.2	0.9

Resource limits

Resource		Symbol	Value	Units
Water		W_{avail}	2.4	MCM season^{-1}
Land Farm1		$L_{\max,1}$	200	ha
Land Farm2		$L_{\max,2}$	100	ha
Land Farm2		$L_{\max,3}$	150	ha
N Runoff Limit		$R_{\max}$	0-5000	kg season^{-1}

D. Problem Set Tasks

***** All of the information requested below should be compiled in a single MS Word (or equivalent) file with all your team member names clearly identified in the file name and submitted via the 1.020 Stellar site *****

1. Use MATLAB's quadprog function to find the set of crop land areas and amount of fertilizer applied that maximizes revenue when R_{max} = 5000 kg season^{-1}. [HINT: Make sure that you include all land, water, and nitrogen runoff constraints. Also, make sure that the Hessian matrix you construct is symmetric**].**

2. Determine the increase in revenue obtained if R_{max} is reduced. Do this by plotting 1) the revenue and 2) the shadow price of the nitrogen runoff constraint vs. R_{max} over the range $R_{max} = 0$ **to** $R_{max} = 5000$**.** Use the MATLAB function **subplot** to put the two plots in the same figure window.

3. Explain why the shadow price of the runoff constraint increases as the maximum permitted nitrogen increases [Hint: Examine the fertilizer yield coefficients as well as changes in cropland allocations to see how the benefits of fertilizer increase as greater amounts are permitted**].**